COMPLETE SCHOOL ALGEBRA

HAWKES · LUBY · TOUTON

TEACHERS' EDITION

IMPORTANT

THIS teachers' edition is furnished only with the distinct understanding that the person receiving it is a teacher conducting a class in which the corresponding textbook is regularly used. It is understood further that no other person shall be permitted to use this book and that it shall be returned to Ginn and Company as soon as the teacher is through with it.

As it is of the utmost importance that these keys shall not fall into the hands of pupils, it is hoped that all teachers will give the publishers their earnest coöperation in this matter.

GINN AND COMPANY

Boston New York Chicago London
Atlanta Dallas Columbus San Francisco

COMPLETE SCHOOL ALGEBRA

TEACHERS' EDITION

BY

HERBERT E. HAWKES, Ph.D.
PROFESSOR OF MATHEMATICS IN COLUMBIA UNIVERSITY

WILLIAM A. LUBY, A.B.
HEAD OF THE DEPARTMENT OF MATHEMATICS
KANSAS CITY POLYTECHNIC INSTITUTE

AND

FRANK C. TOUTON, Ph.D.
SUPERVISOR OF HIGH SCHOOLS, STATE DEPARTMENT OF
PUBLIC INSTRUCTION, MADISON, WISCONSIN

REVISED EDITION

GINN AND COMPANY
BOSTON · NEW YORK · CHICAGO · LONDON
ATLANTA · DALLAS · COLUMBUS · SAN FRANCISCO

419.11

The Athenæum Press
GINN AND COMPANY · PROPRIETORS · BOSTON · U.S.A.

PREFACE

The busy teacher of high-school algebra often finds that limitations of time forbid the actual working out of all the exercises and problems in a text. This teachers' edition is designed to meet such a condition. In its plan it is brief and schematic rather than complete and detailed. Less time should be required to prepare a lesson from this outline than from a text giving the solutions in full.

For those exercises involving not more than one process in their solution the answers only are given. If two or more processes are required in a solution, an outline is given which covers each step of the work. Moreover, after each indicated step of the work, the result obtained by taking that step is set down. This scheme affords an easy method of tracing a solution.

In the solution of problems the meaning of each letter representing an unknown is clearly stated, the equations representing the conditions of the problems in terms of these letters are given, and the schematic solution of the equation or system always follows.

The authors and publishers desire the coöperation of teachers and school authorities in limiting the circulation of this book to actual teachers of algebra.

COMPLETE SCHOOL ALGEBRA

TEACHERS' EDITION

Page 8

1. Let l be the less number. Then $4l$ represents the greater. Therefore $l + 4l = 120$. Whence $l = 24$, and $4l = 96$.

2. Let n equal the number. Then $7n$ represents seven times the number. Therefore $n + 7n = 216$. Whence $n = 27$.

3. Let n equal the less number. Then $8n$ equals the greater. Therefore $n + 8n = 72$. Whence $n = 8$, and $8n = 64$.

4. Let $x =$ the third number. Then $2x =$ the first, and $4x =$ the second. Therefore $x + 2x + 4x = 252$. Whence $x = 36$, $2x = 72$, and $4x = 144$.

5. Let $x =$ the first number. Then $2x =$ the second, and $6x =$ the third. Therefore $x + 2x + 6x = 117$. Whence $x = 13$, $2x = 26$, and $6x = 78$.

6. Let $x =$ the second number. Then $2x =$ the first number, and $3x =$ the third. Therefore $x + 2x + 3x = 192$. Whence $x = 32$, $2x = 64$, and $3x = 96$.

7. Let $x =$ the first number. Then $5x =$ the second, and $20x =$ the third. Therefore $x + 5x + 20x = 312$. Whence $x = 12$, $5x = 60$, and $20x = 240$.

8. Let $x =$ the first number. Then $3x =$ the second, and $4x =$ the third. Therefore $x + 3x + 4x = 208$. Whence $x = 26$, $3x = 78$, and $4x = 104$.

9. Let $s =$ the son's age in years. Then $3s =$ the father's age in years. Therefore $s + 3s = 44$. Whence $s = 11$, and $3s = 33$.

10. Let $s =$ the side of the square in feet. Then $4s = 160$. Whence $s = 40$.

11. Let $w =$ the width of the rectangle in feet. Then $3w =$ the length. Therefore $2w + 6w = 216$. Whence $w = 27$, and $3w = 81$.

12. Let $s =$ the side of the square in inches. Then the width of the rectangle is s, the length is $2s$, and the perimeter is $6s$. Therefore $6s = 258$. Whence $s = 43$, and the perimeter of the square $= 172$.

Page 12

1. $3a + 4b$.
2. $4b - 3a$.
3. $b^2 - a^2$.
4. $a^2 - b^3$.
5. $3a^2 - 2a^2$.
6. $\frac{a}{b}$.
7. $4a^2b$.
8. $\frac{a+b}{ab}$.
9. $a(2b - c)$.
10. $a(b + c)$.
11. $7x - (a - b)$.
12. $\sqrt{5a} + \sqrt[3]{7b}$.
13. $(x - y)\sqrt{7x}$.
14. $(a + b)^2$.
15. $a - b^2$.
16. $\frac{3ab^2}{4a^3c}$.
17. $\frac{a}{3x} + \frac{4y}{c}$.

Page 14

1. $20 - 5 + 6 - 10 = 11$.
2. $16 - (8 - 2) = 16 - 6 = 10$.
3. $14 - (16 - 8) + (12 - 4)$
$= 14 - 8 + 8 = 14$.
4. $6 \div 3 - 2 = 2 - 2 = 0$.
5. $8 \cdot 6 \div 3 - 10 = 16 - 10 = 6$.
6. $18 \div (2 \cdot 3) = 18 \div 6 = 3$.
7. $(6 - 3)(17 - 2 \cdot 5)$
$= 3(17 - 10) = 3 \cdot 7 = 21$.
8. $23 - 2 \cdot 6 - 4 \div 2 + 16$
$= 23 - 12 - 2 + 16 = 25$.
9. $18 \div (9 - 3) = 18 \div 6 = 3$.
10. $(10 - 3)(16 - 3 \cdot 2 + 8 \div 4)$
$= 7(16 - 6 + 2) = 7 \cdot 12 = 84$.
11. $14 - 3(16 - 2 \cdot 5) \div 6 + 8 \cdot 2$
$= 14 - 3(6) \div 6 + 16$
$= 14 - 3 + 16 = 27$.
12. $(18 - 2) - (4 + 2 \cdot 8 - 18 \div 9) \div 6$
$= 16 - (4 + 16 - 2) \div 6 = 13$.
13. $(16 - 6)(18 - 8) \div 100 \cdot 5 - 5$
$= 10 \cdot 10 \div 100 \cdot 5 - 5$
$= 1 \cdot 5 - 5 = 0$.
14. $(5 + 3)(5 - 3) \div 4 - 3$
$= 8 \cdot 2 \div 4 - 3 = 1$.
15. $3^2 - 2 \cdot 3 \cdot 1 + 1^2 = 9 - 6 + 1 = 4$.
16. $8^2 - 2 \cdot 8 \cdot 3 + 3^2$
$= 64 - 48 + 9 = 25$.
17. $4 \cdot 3 - 2(6 - 2 \cdot 3) + 8$
$= 12 - 2 \cdot 0 + 8 = 20$.
18. $8(12 + 4 + 3 - 1) = 8 \cdot 18 = 144$.
19. $3 + 2 \cdot 4 + (3 + 2)4$
$= 3 + 8 + 20 = 31$.
20. $(8 - 2)3 + 8 - 2 \cdot 3$
$= 18 + 8 - 6 = 20$.
21. $3 \cdot 4 - 6 \cdot 0 + 2 \cdot 5 + 2 \cdot 7^2 - 2 \cdot 8 = 12 - 0 + 10 + 98 - 16 = 104$.
22. $(12 + 24 \times 18 \div 3 + 6)(24 \div 4 + 3 - 2) = (12 + 144 + 6)(7) = 1134$.

Page 15

2. $4a + 3d = 4 \cdot 3 + 3 \cdot 7$
$= 12 + 21$
$= 33$.
3. $ab + cd = 3 \cdot 1 + 5 \cdot 7$
$= 3 + 35$
$= 38$.
4. $c^2 - 3ab = 5^2 - 3 \cdot 3 \cdot 1$
$= 25 - 9$
$= 16$.
5. $abcd - 5f^2 = 3 \cdot 1 \cdot 5 \cdot 7 - 5 \cdot 2^2$
$= 105 - 20$
$= 85$.
6. $\frac{d+c}{f} = \frac{7+5}{2} = 6$.
7. $\frac{cdf}{3f} + \frac{acf}{2a} = \frac{5 \cdot 7 \cdot 2}{3 \cdot 2} + \frac{3 \cdot 5 \cdot 2}{2 \cdot 3} = 16\frac{2}{3}$.
8. $\frac{12b}{f} - \frac{c}{b} = \frac{12 \cdot 1}{2} - \frac{5}{1} = 6 - 5 = 1$.

9. $\frac{1}{c}+\frac{1}{d}+\frac{1}{f}=\frac{1}{5}+\frac{1}{7}+\frac{1}{2}=\frac{14+10+35}{70}=\frac{59}{70}$.

10. $\frac{f^2}{ac}+\frac{6\,cd}{af}=\frac{2^2}{3\cdot 5}+\frac{6\cdot 5\cdot 7}{3\cdot 2}=35\tfrac{4}{15}$.

11. $\frac{a^2+b^2+c^2+d^2}{a+b+c+d}=\frac{3^2+1^2+5^2+7^2}{3+1+5+7}=\frac{9+1+25+49}{16}=5\tfrac{1}{4}$.

12. $5f^3+4f^2-4f-5=5\cdot 2^3+4\cdot 2^2-4\cdot 2-5=40+16-8-5=43$.

13. $3f^5-9f^4+11f^3-11f^2+13f-20$
$=3\cdot 2^5-9\cdot 2^4+11\cdot 2^3-11\cdot 2^2+13\cdot 2-20$
$=96-144+88-44+26-20=2$.

14. $a^f=3^2=9$.

15. $d^f+f^a=7^2+2^3=49+8=57$.

16. $\frac{d+f^c}{3\,c+a}=\frac{7+2^5}{3\cdot 5+3}=\frac{7+32}{15+3}=\frac{39}{18}=2\tfrac{1}{6}$.

17. $\frac{4\,a+3\,b+2\,d}{c+f+2}=\frac{4\cdot 4+3\cdot 0+2\cdot 7}{5+8+2}=\frac{30}{15}=2$.

18. $\frac{b}{a+c+d}=\frac{0}{4+5+7}=0$.

19. $\frac{ab}{c}+\frac{bd}{a}+\frac{bf}{cd}=\frac{4\cdot 0}{5}+\frac{0\cdot 7}{4}+\frac{0\cdot 8}{5\cdot 7}=0$.

20. $\sqrt{a}+\sqrt{2f}$
$=\sqrt{4}+\sqrt{2\cdot 8}$
$=2+4=6$.

21. $c\sqrt{2\,af}=5\sqrt{2\cdot 4\cdot 8}$
$=40$.

22. $\sqrt{c^2-a^2}=\sqrt{5^2-4^2}$
$=\sqrt{25-16}=3$.

23. $(a+f)(c+d)$
$=(4+8)(5+7)=144$.

24. $ab(a+b)$
$=4\cdot 0(4+0)=0$.

25. $(a+c)(a+c)$
$=(4+5)(4+5)=81$.

26. $a+c(a+c)$
$=4+5(4+5)$
$=4+45=49$.

27. $(a+c)a+c$
$=(4+5)4+5$
$=36+5=41$.

28. $a+ca+c$
$=4+5\cdot 4+5$
$=4+20+5=29$.

29. $(f-a)^2$
$=(8-4)^2=4^2=16$.

30. $(d-a)^3$
$=(7-4)^3=3^3=27$.

31. $a^2-2\,ac+c^2$
$=4^2-2\cdot 4\cdot 5+5^2$
$=16-40+25=1$.

32. $a^2+2\,ab+b^2$
$=4^2+2\cdot 4\cdot 0+0^2$
$=16+0+0=16$.

33. $c^3-3\,c^2a+3\,ca^2-a^3$
$=5^3-3\cdot 5^2\cdot 4+3\cdot 5\cdot 4^2-4^3$
$=125-300+240-64=1$.

34. $d\sqrt[3]{2\,af}$
$=7\sqrt[3]{2\cdot 4\cdot 8}$
$=7\cdot 4=28$.

35. $\sqrt{a^2 + ac}$
$= \sqrt[2]{4^2 + 4 \cdot 5}$
$= \sqrt{36} = 6.$

36. $\sqrt[3]{ad + 36}$
$= \sqrt[3]{4 \cdot 7 + 36}$
$= \sqrt[3]{64} = 4.$

37. $13x - 5y = 11.$
$13 \cdot 2 - 5 \cdot 3$
$= 26 - 15 = 11.$ Yes.

38. $7x - 9 = 3x + 25.$
$7 \cdot 8 - 9 = 3 \cdot 8 + 25.$
$56 - 9 = 24 + 25.$
$47 = 49.$ No.

39. $x^2 - 5x + 6 = 0.$
$2^2 - 5 \cdot 2 + 6 = 0.$ Yes.
$3^2 - 5 \cdot 3 + 6 = 0.$ Yes.
$4^2 - 5 \cdot 4 + 6 = 2.$ No.

40. $x^2 - 7x + 12 = 0.$
$2^2 - 7 \cdot 2 + 12 = 2.$ No.
$3^2 - 7 \cdot 3 + 12 = 0.$ Yes.
$4^2 - 7 \cdot 4 + 12 = 0.$ Yes.

41. $2x^2 - 5x - 3 = 0.$
$2 \cdot 4^2 - 5 \cdot 4 - 3 = 9.$ No.
$2 \cdot 3^2 - 5 \cdot 3 - 3 = 0.$ Yes.
$2(\frac{1}{2})^2 - 5(\frac{1}{2}) - 3$
$= \frac{1}{2} - \frac{5}{2} - 3 = -5.$ No.

42. $A = \dfrac{ab}{2} = \dfrac{11 \cdot 14}{2} = 77.$
$\therefore$ the area $= 77$ square inches.

43. (*a*) $A = \pi r^2 = \frac{22}{7} \cdot 7^2 = 154.$
$\therefore$ the area $= 154$ square inches.
(*b*) $\pi r^2 = \frac{22}{7} \cdot (\frac{1}{2})^2 = \frac{11}{14}.$
$\therefore$ the area $= \frac{11}{14}$ square inches.

44. (*a*) $V = \pi r^2 h = \frac{22}{7} \cdot 3^2 \cdot 5$
$= \frac{990}{7} = 141\frac{3}{7}.$
$\therefore$ the volume $= 141\frac{3}{7}$ cubic inches.
(*b*) $\pi r^2 h = \frac{22}{7} \cdot 4^2 \cdot 7 = 352.$
$\therefore$ the volume $= 352$ cubic inches.

45. $S = \dfrac{at^2}{2} = \dfrac{32 \cdot 5^2}{2} = 400.$
$\therefore$ the body falls 400 feet in 5 seconds.

46. $S = \dfrac{at^2}{2} = \dfrac{32 \cdot (16)^2}{2} = 4096.$
$\therefore$ the height of the balloon $= 4096$ feet.

47. $H = \dfrac{d^2 n}{6} = \dfrac{4^2 \cdot 4}{6} = 10\frac{2}{3}.$
$\therefore$ the engine is $10\frac{2}{3}$ horse power.

Page 20

1. (*a*) $14° - 5° = +9°.$
(*b*) $14° - 10° = +4°.$
(*c*) $14° - 18° = -4°.$

2. (*a*) $-16° + 7° = -9°.$
(*b*) $-16° + 12° = -4°.$
(*c*) $-16° + 25° = +9°.$

3. (*a*) $-16° - 7° = -23°.$
(*b*) $-16° - 12° = -28°.$
(*c*) $-16° - 25° = -41°.$

4. $13° + 7° = 20°.$
$20 \cdot 69$ miles $= 1380$ miles.

5. $20° - 5° = 15°.$
$20° - 10° = 10°.$
$20° - 15° = 5°.$
$20° - 20° = 0°.$
$20° - 25° = -5°.$
$20° - 30° = -10°.$
$20 + 15 = 35.$
$35 \div 5 = 7$; after seven days.

6. (*a*) $\begin{cases} +\$4200 = \text{value of property.} \\ -\$2300 = \text{amount of debts.} \end{cases}$
(*b*) $+\$4200 + (-\$2300)$
$= +\$1900.$

7. (a) $\begin{cases} -\$4200 = \text{amount of debts.} \\ +\$2300 = \text{value of property.} \end{cases}$

(b) $-\$4200 + \$2300 = -\$1900.$

8. $-12° + 9° = -3°.$

$-12° + 12° = 0°.$

$-12° + 18° = +6°.$

Page 31

1. $4^2 - (-2)^2 = 16 - 4 = 12.$
2. $3^3 - (-2)^3 = 27 + 8 = 35.$
3. $(5 - 3)(3 + 2) = 2 \cdot 5 = 10.$
4. $(6 - 2)(7 - 3) \div (4 - 9) = 4 \cdot 4 \div (-5) = -3\frac{1}{5}.$
5. $(6 - \frac{1}{2})(5 + \frac{1}{3}) = \frac{11}{2} \cdot \frac{16}{3} = 29\frac{1}{3}.$
6. $(-1)^2 + (-1)^3 + (-2)^2 + (-2)^3 = 1 - 1 + 4 - 8 = -4.$
7. $9 + 3 \cdot 2 + 18 \div (-3) = 9 + 6 - 6 = 9.$
8. $5^2 - 4 \div (-2) \div 6(-3) = 25 - 1 = 24.$
9. $3 \cdot 6 \div 9 - 2 \cdot 6 \div 4 + (-3)^2 = 2 - 3 + 9 = 8.$
10. $3^2 + 2 \cdot 3(-4) \div (-4)^2 = 9 - 24 \div (16) = 7\frac{1}{2}.$
11. $(+2)^2 - 2(2)(-3) + (-3)^2 = 4 + 12 + 9 = 25.$
12. $3(4)^2(-5) - (-5)^3 = 3 \cdot 16(-5) + 125 = -115.$
13. $3^3 + 3(3)^2(-2) + 3(3)(-2)^2 + (-2)^3 = 27 - 54 + 36 - 8 = 1.$
14. $4^3 - 3(4)^2(-3) + 3(4)(-3)^2 - (-3)^3 = 64 + 144 + 108 + 27 = 343.$
15. $3^3 - 3(3)^2(0) + 3(3)(0)^2 - 0^3 = 27 - 0 + 0 - 0 = 27.$
16. $5^2 + 3(5)^2(-4) + 3(5)(-4)^2 - (-4)^3 = 25 - 300 + 240 + 64 = 29.$
17. $y^2 = (-2)^2 = 4.$
18. $y^3 = (-2)^3 = -8.$
19. $y^4 = (-2)^4 = 16.$
20. $y^5 = (-2)^5 = -32.$
21. $2y^2 = 2(-2)^2 = 8.$
22. $2y^3 = 2(-2)^3 = -16.$
23. $5x^2y^2 = 5 \cdot (3)^2(-2)^2 = 180.$
24. $4x^2y^4 = 4(3)^2(-2)^4 = 576.$
25. $x^2 - y^2 = 3^2 - (-2)^2$
    $= 9 - 4 = 5.$
26. $x^3 - y^3 = 3^3 - (-2)^3$
    $= 27 + 8 = 35.$
27. $x^2 + 2xy + y^2$
    $= 3^2 + 2 \cdot 3(-2) + (-2)^2$
    $= 9 - 12 + 4 = 1.$
28. $x^2 - 2xy + y^2$
    $= 3^2 - 2 \cdot 3(-2) + (-2)^2$
    $= 9 + 12 + 4 = 25.$
29. $(x + y)(x - y)$
    $= (3 - 2)(3 + 2)$
    $= 1 \cdot 5 = 5.$
30. $x^3 + 3x^2y + 3xy^2 + y^3$
    $= 3^3 + 3 \cdot 3^2(-2) + 3 \cdot 3(-2)$
    $+ (-2)^3$
    $= 27 - 54 + 36 - 8 = 1.$
31. $4x - 2 = 2x + 8.$
    $4 \cdot 5 - 2 = 2 \cdot 5 + 8.$
    $18 = 18.$ Yes.
32. $3x - 5 = 2x + 8.$
    $3(-9) - 5 = 2(-9) + 8.$
    $-32 = -10.$ No.

33. $x^2 - x - 12 = 0.$
$4^2 - 4 - 12 = 0.$ Yes.
$(-8)^2 - (-8) - 12$
$= 64 + 8 - 12 = 60.$ No.
$(-4)^2 - (-4) - 12$
$= 16 + 4 - 12 = 8.$ No.

34. $3x^2 + 19x = 14.$
$3(\frac{2}{3})^2 + 19(\frac{2}{3})$
$= \frac{4}{3} + \frac{38}{3} = 14.$ Yes.
$3(2)^2 + 19 \cdot 2$
$= 12 + 38 = 50.$ No.
$3(-7)^2 + 19 \cdot (-7) = 14.$ Yes.

35. $27° - 15° = 12°$, noon.
$27° - 27° = 0°$, 4.00 P.M.
$27° - 30° = -3°$, 5.00 P.M.
$27° + 12° = 39°.$
$39 \div 3 = 13.$
13 hours later than 7.00 A.M is 8.00 P.M.

36. $1800 - 1200 + 1750 - 400 + 820 = 2770$, number of feet higher than at first.

37. 300 B.C. $= -300.$
1727 A.D. $= +1727.$

38. -450 means 450 B.C.
$+1917$ means 1917 A.D.
1917 years + 450 years
= 2367 years = difference.

39. $+12 + (+3) = 15$, number of miles per hour toward bow.
$12 + (-3) = 9$, number of miles per hour toward stern.

40. -500 = weight of balloon.
$10(+150) = +1500$ = weight of men.
$+1500$ pounds $-$ 500 pounds
$= +1000$ pounds, weight of men and balloon.

Page 35

1. $a + 4b - c.$
2. $4x - 2b + 5y + 10.$
3. $3ab^2 + 2bx - cy + 4a^2b.$
4. $5x^3y - 5xy^3 + c^3y - 2cy^3.$
5. $6x - 3a + 2b - 5x + 3y = x - 3a + 2b + 3y.$
6. $5a - 4b + 3c + 7b - 2c^2 = 5a + 3b + 3c - 2c^2.$
7. $3a^2 + 2b^2 - 7c^2 - 4b^2 + 5a^2 = 8a^2 - 2b^2 - 7c^2.$
8. $5a^3b - 5ab^3 - 3a^2b + 3ab^2 + 4a^3b + 2ab^3$
$= 9a^3b - 3ab^3 - 3a^2b + 3ab^2.$
9. $11 + 5 - 9 - 3 + 16 - 25 = 32 - 37 = -5.$
10. $16ac - 9xy + 5ac - 2xy - 6ac + 11xy = 15ac.$
11. $7x + 6a - 15a - 8x + 3x = 2x - 9a.$
12. $12y - 17y + 10b + 20y - 25b = 15y - 15b.$
13. $4ab - 8xy - 12ab + 15ab - xy = 7ab - 9xy.$
14. $14x^2 - 13y^2 + x^2 - 5x^2 + y^2 = 10x^2 - 12y^2.$
15. $5z^2 - 4y^2 - 11y^2 + z^2 - 7y^2 = 6z^2 - 22y^2.$
16. $7c^2d - 5a^2b + 6c^2d - a^2b + 9a^2b = 13c^2d + 3a^2b.$
17. $-4a^2b^2 + 6x^2y^2 - 15a^2b^2 + 3x^2y^2 + 0a^2b^2 = -19a^2b^2 + 9x^2y^2.$
18. $-b^3 - 19a + 17b^3 + b^3 - 0b^3 + 13a = 17b^3 - 6a.$
19. $12a^3b + 6c^2d - a^3b + 16a^3b - 13a^3b - 25c^2d = 14a^3b - 19c^2d.$

20. $11\sqrt{a} - 16\sqrt{a} + 21\sqrt{a} - \sqrt{a} = 15\sqrt{a}.$

21. $3\sqrt{x-y} - \sqrt{x-y} + 8\sqrt{x-y} - 7\sqrt{x-y} = 3\sqrt{x-y}.$

22. $3(a+b) - 5(a+b) + 8(a+b) = 6(a+b).$

23. $-7(a-2b) + (a-2b) + 12(a-2b) = 6(a-2b).$

Page 37

1. $\begin{array}{r} x+3 \\ 2x-6 \\ 3x-5 \\ \hline 6x-8 \end{array}$

2. $\begin{array}{r} x-y-3 \\ 2x+y-8 \\ 7x-4y+10 \\ \hline 10x-4y-1 \end{array}$

3. $\begin{array}{r} x+y+z \\ x-3y+4z \\ 3x+4y-7z \\ \hline 5x+2y-2z \end{array}$

4. $\begin{array}{r} 2x+5y-z \\ 3x-8y+6z \\ x-y-z \\ \hline 6x-4y+4z \end{array}$

5. $\begin{array}{r} 3x-1 \\ -2x+4 \\ 6x-8 \\ 3x+3 \\ \hline 10x-2 \end{array}$

6. $\begin{array}{r} x^2-9 \\ -4x^2+16 \\ 8x^2+4 \\ -2x^2+3 \\ \hline 3x^2+14 \end{array}$

7. $\begin{array}{r} 5x+5y \\ 4x-9y+6z \\ 3x-3y-3z \\ \hline 12x-7y+3z \end{array}$

8. $\begin{array}{r} 7x+3z-6y \\ 5x-4z \\ 2x-5z+6y \\ \hline 14x-6z \end{array}$

9. $\begin{array}{r} x^2-x-2 \\ 3x^2-4x+5 \\ 5x^2+8x-7 \\ \hline 9x^2+3x-4 \end{array}$

10. $\begin{array}{r} x^2-4x+1 \\ -2x^2+3x+8 \\ -11x^2+10x-18 \\ \hline -12x^2+9x-9 \end{array}$

11. $\begin{array}{r} -4c^2+3c-5 \\ c^2+c+1 \\ -2c^2+4c+3 \\ -5c^2-2c+5 \\ \hline -10c^2+6c+4 \end{array}$

12. $\begin{array}{r} 4x-5y-5z \\ 6y-2z \\ 7x-6y-9z \\ \hline 11x-5y-16z \end{array}$

13. $\begin{array}{r} x+2z+3y \\ x-3z+y \\ -2x+z-4y \\ \hline 0 \end{array}$

14. $\begin{array}{r} 5x+7z-6y \\ x-11z+2y \\ 9z-5y \\ \hline 6x+5z-9y \end{array}$

15. $\begin{array}{r} 8a-7b-6c \\ -4a-3b+5c \\ 3b+7c \\ \hline 4a-7b+6c \end{array}$

16. $\begin{array}{r} 9ac-bc \\ -3ac+7ab \\ -ac-12ab \\ \hline 5ac-bc-5ab \end{array}$

17. $\begin{array}{r} a^2-4a+10 \\ -6a^2+5a+4 \\ 2a^2+3a-16 \\ \hline -3a^2+4a-2 \end{array}$

18. $\begin{array}{r} -a^2+3a+9 \\ 6a^2-4a-13 \\ 3a^2-a \\ \hline 8a^2-2a-4 \end{array}$

19. $\begin{array}{r} a-x-3c \\ 3a-7x+4 \\ -2a+7c+10 \\ a-2x-c-11 \\ \hline 3a-10x+3c+3 \end{array}$

20. $\begin{array}{r} a+x+c+5 \\ 2a+2x+2c+10 \\ a-x-c-7 \\ -2a-3x+4c+7 \\ \hline 2a-x+6c+15 \end{array}$

21.
$$\begin{array}{r} x - 1 - 3c \\ x - 2c - a \\ - 5 - 3c + a \\ 2x - 7 + 11a \\ \hline 4x - 13 - 8c + 11a \end{array}$$

22.
$$\begin{array}{r} 2x + 3y + 5z \\ x - y + 3z \\ 3x - 2y + 10z \\ \hline 6x + 18z \end{array}$$

23.
$$\begin{array}{r} 4x - 20z \\ 8x + 9y \\ 5z - 3y \\ \hline 12x - 15z + 6y \end{array}$$

24.
$$\begin{array}{r} a - 3(x - y) + z \\ -10a + 4(x - y) + 5 \\ - 2(x - y) + 6 \\ \hline - 9a - (x - y) + z + 11 \end{array}$$

25.
$$\begin{array}{r} a + d + 2(b - c) \\ -12a + 6d + 7(b - c) \\ 11a - 5(b - c) \\ \hline + 7d + 4(b - c) \end{array}$$

26.
$$\begin{array}{r} 7a^2 - 13b^2 + 12c^2 \\ - a^2 + 15b^2 - 7c^2 \\ + 3b^2 + 5c^2 \\ \hline 6a^2 + 5b^2 + 10c^2 \end{array}$$

27.
$$\begin{array}{r} x^2 + 2xy + y^2 \\ - 4x^2 - 5xy - y^2 \\ + 16x^2 - 8xy + y^2 \\ \hline 13x^2 - 11xy + y^2 \end{array}$$

28.
$$\begin{array}{r} 5ab - a^2 + b^2 \\ + 4ab + 5a^2 - 9b^2 \\ - 2ab - 2b^2 \\ \hline 7ab + 4a^2 - 10b^2 \end{array}$$

29.
$$\begin{array}{r} 5x^2 - 6x + 11 \\ - 3x^2 - 4x - 8 \\ + 7x - 18 \\ + 13x \\ \hline 2x^2 + 10x - 15 \end{array}$$

30.
$$\begin{array}{r} 12c^2 - 10bc + 8b^2 \\ - c^2 + bc - 6b^2 + c^3 \\ - 11c^3 \\ \hline 11c^2 - 9bc + 2b^2 - 10c^3 \end{array}$$

31.
$$\begin{array}{r} 4x^2y - xy + y^2 \\ - 2x^2y - 3xy + 4xy^2 \\ + x^2y + 4y^2 + 3xy^2 \\ \hline 3x^2y - 4xy + 5y^2 + 7xy^2 \end{array}$$

32.
$$\begin{array}{r} \frac{a}{2} + \frac{b}{3} - \frac{c}{4} \\ - \frac{2a}{3} + \frac{b}{2} + c \\ \frac{3a}{4} - b + \frac{c}{2} + 7 \\ \hline \frac{7a}{12} - \frac{b}{6} + \frac{5c}{4} + 7 \end{array}$$

33. $ax + 3x = (a + 3)x.$

34. $2ax + x = (2a + 1)x.$

35. $3cx - x = (3c - 1)x.$

36. $ax + bx + cx = (a + b + c)x.$

37. $2ax - 3x + bx = (2a - 3 + b)x.$

38. $3ax - 4cx + x = (3a - 4c + 1)x.$

39. $3ax - bx - x + a^2x = (3a - b - 1 + a^2)x.$

40. $bx - 5cx - x - 4bx = (-3b - 5c - 1)x.$

41. $a(b + c) + 3(b + c) = (a + 3)(b + c).$

42. $4(a - x) - 5b(a - x) = (4 - 5b)(a - x).$

43. $8a(a + 3b) - 1(a + 3b) = (8a - 1)(a + 3b).$

44. $7b(x^2 + y^2) - a(x^2 + y^2) + (x^2 + y^2) = (7b - a + 1)(x^2 + y^2).$

Page 42

1. $x + 5 = 7.$
Subtracting 5 in both members,
$x = 2.$

2. $y - 9 = 6.$
Adding 9 in both members,
$y = 15.$

3. $4x - 6 = 14.$
Adding 6 in both members,
$4x = 20.$
Whence $x = 5.$

4. $4x + 2 = 3x - 4.$
Subtracting $3x$ and 2 in both members, $x = -6.$

5. $5x - 2 = 4x + 5.$
Subtracting $4x$ and adding 2 in both members,
$x = 7.$

6. $5p = 3p + 8.$
Subtracting $3p$ in both members,
$2p = 8.$
Whence $p = 4.$

7. $4x + 2 = 2x.$
Subtracting $2x$ and 2 in both members, $2x = -2.$
Whence $x = -1.$

8. $3n = n + 3.$
$2n = 3.$
$n = \frac{3}{2}.$

9. $4x + 2 = x + 5.$
$3x = 3.$
$x = 1.$

10. $5x + 6 = 3x + 10.$
$2x = 4.$
$x = 2.$

11. $4x - 7 = 2x + 3.$
$2x = 10.$
$x = 5.$

12. $3k - 5 = 7 - k.$
$4k = 12.$
$k = 3.$

13. $6 - a = 9 - 4a.$
$3a = 3.$
$a = 1.$

14. $2x = 1 - x.$
$3x = 1.$
$x = \frac{1}{3}.$

15. $5 + 3y = y + 4.$
$2y = -1.$
$y = -\frac{1}{2}.$

16. $2x + 8 = 5x - 1.$
$9 = 3x.$
$x = 3.$

17. $5 + 2q = 5q - 1.$
$6 = 3q.$
$q = 2.$

18. $2 - 3n = n + 1.$
$1 = 4n.$
$n = \frac{1}{4}.$

19. $4x - 2 = 7 + x.$
$3x = 9.$
$x = 3.$

20. $4x + 3 - x + 5 = x - 10.$
Combining, $3x + 8 = x - 10.$
$2x = -18.$
$x = -9.$

21. $3 + 2z - 1 = 12 - 3z.$
$2z + 2 = 12 - 3z.$
$5z = 10.$
$z = 2.$

22. $2x + 3x + x = x + 9 + 2x.$
$$6x = 3x + 9.$$
$$3x = 9.$$
$$x = 3.$$

23. $4y - 3 + 3y - 4 = 6y - 8.$
$$7y - 7 = 6y - 8.$$
$$y = -1.$$

24. $36 - 8x + 4 - 9x = 2x + 10.$
$$40 - 17x = 2x + 10.$$
$$30 = 19x.$$
$$x = \tfrac{30}{19}.$$

25. $3x + 12x + 17 - x + 4x = 107.$
$$18x + 17 = 107.$$
$$18x = 90.$$
$$x = 5.$$

26. $4k - 3 = 5k - 16 - 43k - 71.$
$$4k - 3 = -38k - 87.$$
$$42k = -84.$$
$$k = -2.$$

Page 46

1. Let $x =$ the number of papers sold by second boy.
Then $x + 5 =$ the number of papers sold by first boy.
But by the conditions of the problem,
$$x + x + 5 = 67.$$
Whence $x = 31;\ x + 5 = 36.$

2. Let $x =$ the number of students in smaller group.
Then $x + 3 =$ the number of students in larger group.
But by the conditions of the problem,
$$x + x + 3 = 51.$$
Whence $x = 24;\ x + 3 = 27.$

3. Let $x =$ the less number.
Then $5x =$ the greater number.
But $x + 5x = 30.$
Whence $x = 5;\ 5x = 25.$

4. Let $x =$ the less number.
Then $3x =$ the greater number.
But $x + 3x = 72.$
Whence $x = 18;\ 3x = 54.$

5. Let $x =$ the less number.
Then $5x =$ the greater number.
But $5x - x = 52.$
Whence $x = 13;\ 5x = 65.$

6. Let $x =$ the number of cars in shorter train.
Then $2x =$ the number of cars in longer train.
But $x + 2x = 72.$
Whence $x = 24;\ 2x = 48.$

7. Let $x =$ the side in feet.
Then $x + x + x + x = 24.$
Whence $x = 6.$

8. Let $x =$ the width in feet.
Then $3x =$ the length in feet.
But $x + 3x + x + 3x = 64.$
Whence $x = 8;\ 3x = 24.$

9. Let $x =$ the breadth in feet.
Then $5x =$ the length in feet.
But $x + 5x + x + 5x = 78.$
Whence $x = 6\frac{1}{2};\ 5x = 32\frac{1}{2}.$

10. Let $x =$ the width in feet.
Then $x + 12 =$ the length in feet.
But $x + x + 12 + x + x + 12 = 96.$
Whence $x = 18;\ x + 12 = 30.$

11. Let $x =$ the length in feet of the shorter piece.
Then $x + 8 =$ the length in feet of the longer piece.
But $x + x + 8 = 36.$
Whence $x = 14;\ x + 8 = 22.$

12. Let $x =$ the length in inches of the shorter piece.
Then $3x + 14 =$ the length in inches of the longer piece.
But $x + 3x + 14 = 120.$
Whence $x = 26\frac{1}{2};\ 3x + 14 = 93\frac{1}{2}.$

13. Let $x =$ the first number.
Then the other numbers will be $2x$ and $8x$.
But $x + 2x + 8x = 22.$
Whence $x = 2;\ 2x = 4;\ 8x = 16.$

14. Let $x =$ the third number.
Then $2x =$ the first number,
and $4x =$ the second number.
But $x + 2x + 4x = 63.$
Whence $x = 9;\ 2x = 18;\ 4x = 36.$

15. Let $x =$ the number of men in the first squad.
Then $x + 5 =$ the number in the second squad,
and $x + 5 + 10 = x + 15$, the number in the third squad.
But $x + x + 5 + x + 15 = 95.$
Whence $x = 25;\ x + 5 = 30;\ x + 15 = 40.$

16. Let $x =$ the number of cars in the shorter section.
Then $2x + 2 =$ the number of cars in the longer section.
But $x + 2x + 2 = 62.$
Whence $x = 20;\ 2x + 2 = 42.$

17. Let $x =$ the less number.
Then $x + 1 =$ the greater number.
But $x + x + 1 = 45.$
Whence $x = 22;\ x + 1 = 23.$

18. Let $x =$ the less number.
Then $x + 2 =$ the greater number.
But $x + x + 2 = 36.$
Whence $x = 17;\ x + 2 = 19.$

19. Let x, $x + 1$, $x + 2$, be the numbers.
Then $x + x + 1 + x + 2 = 72.$
Whence $x = 23;\ x + 1 = 24;\ x + 2 = 25.$

20. Let x, $x + 2$, $x + 4$, be the numbers.
Then $x + x + 2 + x + 4 = 48.$
Whence $x = 14;\ x + 2 = 16;\ x + 4 = 18.$

21. Let x, $x + 2$, $x + 4$, $x + 6$, $x + 8$, be the numbers.
Then $x + x + 2 + x + 4 + x + 6 + x + 8 = 85.$
Whence $x = 13;\ x + 2$, etc. $= 15, 17, 19, 21.$

22. Let $x =$ the length in inches of the shorter piece.
Then $x + 1$ and $x + 2 =$ the lengths in inches of the other pieces.
But $x + x + 1 + x + 2 = 36.$
Whence $x = 11;\ x + 1 = 12;\ x + 2 = 13.$

Page 51

1. $\begin{array}{r} a + 3 \\ a + 2 \\ \hline 1 \end{array}$

2. $\begin{array}{r} a - 6 \\ a - 4 \\ \hline -2 \end{array}$

3. $\begin{array}{r} 3a + 7 \\ 2a - 3 \\ \hline a + 10 \end{array}$

4. $\begin{array}{r} -4a + 9 \\ 3a + 7 \\ \hline -7a + 2 \end{array}$

5. $\begin{array}{r} -2x + 5 \\ 2x - 5 \\ \hline -4x + 10 \end{array}$

6. $\begin{array}{r} 3x + 6 \\ 6x - 3 \\ \hline -3x + 9 \end{array}$

7. $\begin{array}{r} 4x - 9 \\ 9x + 4 \\ \hline -5x - 13 \end{array}$

8. $\begin{array}{r} 4a - 2 \\ 4a \phantom{{}-2} \\ \hline -2 \end{array}$

9. $\begin{array}{r} 5c \\ 2ab - 5c \\ \hline -2ab + 10c \end{array}$

10. $\begin{array}{r} 5xy \phantom{{}+a} \\ 3xy - a \\ \hline 2xy + a \end{array}$

11. $\begin{array}{r} x + 4 \\ 0 \\ \hline x + 4 \end{array}$

12. $\begin{array}{r} 0 \\ 2x - 5 \\ \hline -2x + 5 \end{array}$

13. $$\begin{array}{r} 2x - 2y + z \\ x - 2y - 3z \\ \hline x \qquad + 4z \end{array}$$

14. $$\begin{array}{r} 4x - 5y - 3z \\ 4x - 8y + 2z \\ \hline 3y - 5z \end{array}$$

15. $$\begin{array}{r} 4a + b + 2c \\ 3a - 2b \qquad \\ \hline a + 3b + 2c \end{array}$$

16. $$\begin{array}{r} 3a - 5b \qquad \\ 5a - 4b + 3c \\ \hline -2a - b - 3c \end{array}$$

17. $$\begin{array}{r} 5x^2 + 8x - 2 \\ 3x^2 - x - 5 \\ \hline 2x^2 + 9x + 3 \end{array}$$

18. $$\begin{array}{r} -5x^2 + 8x - 13 \\ 7x^2 - 4x + 11 \\ \hline -12x^2 + 12x - 24 \end{array}$$

19. $$\begin{array}{r} -x - y + b \\ a - x + y \qquad \\ \hline -a \qquad - 2y + b \end{array}$$

20. $$\begin{array}{r} 4a - b + 5c \qquad \\ 3a - 2b + c + 6 \\ \hline a + b + 4c - 6 \end{array}$$

21. $$\begin{array}{r} a - 3b \qquad\qquad \\ 2a + 2b - 2c - 4 \\ \hline -a - 5b + 2c + 4 \end{array}$$

22. $$\begin{array}{r} -3a + 4x - 3c - 11 \\ 2a - 3x + 4c \qquad \\ \hline -5a + 7x - 7c - 11 \end{array}$$

23. $$\begin{array}{r} c - a - b \\ -c + a + b \\ \hline 2c - 2a - 2b \end{array}$$

24. $$\begin{array}{r} -7a^2 + 8ab + 3b^2 \\ 2a^2 - 3ab + b^2 \\ \hline -9a^2 + 11ab + 2b^2 \end{array}$$

25. $$\begin{array}{r} 4x^2y - 5xy^2 - y^3 \qquad \\ -3x^2y - 3xy^2 \qquad + 3x^3 \\ \hline 7x^2y - 2xy^2 - y^3 - 3x^3 \end{array}$$

26. $$\begin{array}{r} 3a^3 + 4c^3 + 5a^2c + 2ac^2 \\ a^3 - c^3 - 3a^2c + 3ac^2 \\ \hline 2a^3 + 5c^3 + 8a^2c - ac^2 \end{array}$$

27. $$\begin{array}{r} 2x + 5y - 3z \\ x - 2y + z \\ \hline x + 7y - 4z \end{array}$$

28. $$\begin{array}{r} 5x - 2y - 4z \\ 7x - 9y - 3z \\ \hline -2x + 7y - z \end{array}$$

29. $$\begin{array}{r} 6a - 3b \qquad \\ 5a - 4b + 6c \\ \hline a + b - 6c \end{array}$$

30. $$\begin{array}{r} 5 \qquad\qquad\qquad \\ a - b + 2c \\ \hline 5 - a + b - 2c \end{array}$$

31. $$\begin{array}{r} 2bx - c \\ ax^2 + bx + c \\ \hline -ax^2 + bx - 2c \end{array}$$

32. $$\begin{array}{r} 3xy - z \qquad\qquad \\ 3ab + c \\ \hline 3xy - z - 3ab - c \end{array}$$

33. $$\begin{array}{r} 0 \qquad\qquad \\ 2x - 4y - z \\ \hline -2x + 4y + z \end{array}$$

34. $$\begin{array}{r} 2c^2 - 3ac + 4a^2 \\ 3c^2 - 2ac + a^2 \\ \hline -c^2 - ac + 3a^2 \end{array}$$

35. $$\begin{array}{r} xy^2 + x^2y - z \\ -3xy^2 + 2x^2y \qquad \\ \hline +4xy^2 - x^2y - z \end{array}$$

36. $$\begin{array}{r} x - 2y - z \\ 3x - 2y - z \\ \hline -2x \qquad\qquad \end{array}$$

37. $$\begin{array}{r} x^2 - 7x - 10 \\ 3x^2 + 14x - 8 \\ \hline -2x^2 - 21x - 2 \end{array}$$

38. $$\begin{array}{r} x - y - z \\ 5x + 3y - 8z \\ \hline -4x - 4y + 7z \end{array}$$

39. $$\begin{array}{r} 4 - 8x^2 \qquad \\ -6 + x^2 + 5x \\ \hline 10 - 9x^2 - 5x \end{array}$$

40. $$\begin{array}{r} a^4 + a^2c^2 + c^4 \\ -4a^4 - 3a^2c^2 + 2c^4 \\ \hline 5a^4 + 4a^2c^2 - c^4 \end{array}$$

41. $$\begin{array}{r} 3a + 5b - c \\ 0 \qquad\qquad \\ \hline 3a + 5b - c \end{array}$$

42. $$\begin{array}{r} 4a + 6b - 8y \qquad \\ a \qquad\qquad\qquad - 12 \\ \hline 3a + 6b - 8y + 12 \end{array}$$

43.
$$\begin{array}{r} a^2 + 2\,ab - 1 \\ a^2 + 12\,ab - 20 \\ \hline 2\,a^2 + 14\,ab - 21 \end{array} \qquad \begin{array}{r} a^2 + 13\,ab - 30 \\ 2\,a^2 + 14\,ab - 21 \\ \hline -\,a^2 - ab - 9 \end{array}$$

44.
$$\begin{array}{r} a - 3\,b + c \\ 4\,a + 5\,b - 6\,c + 4 \\ \hline 5\,a + 2\,b - 5\,c + 4 \end{array} \qquad \begin{array}{r} a - b + c - x \\ 5\,a + 2\,b - 5\,c + 4 \\ \hline -4\,a - 3\,b + 6\,c - x - 4 \end{array}$$

45.
$$\begin{array}{r} 5\,x + 3\,x^2y - 15\,xy^2 \\ -6\,x + 7\,x^2y - 12\,xy^2 \\ \hline -x + 10\,x^2y - 27\,xy^2 \\ 11\,x - 5\,x^2y + 7\,xy^2 \\ \hline -12\,x + 15\,x^2y - 34\,xy^2 \end{array}$$

46.
$$\begin{array}{r} 4\,abc^2 - 3\,ab^2c + 2\,a^2bc \phantom{{}- 4\,a^2b^2c} \\ 6\,abc^2 - 5\,ab^2c \phantom{{}+ 2\,a^2bc} - 4\,a^2b^2c \\ \hline 10\,abc^2 - 8\,ab^2c + 2\,a^2bc - 4\,a^2b^2c \\ 2\,abc^2 + 7\,ab^2c - 3\,a^2bc \phantom{{}- 4\,a^2b^2c} \\ \hline 8\,abc^2 - 15\,ab^2c + 5\,a^2bc - 4\,a^2b^2c \end{array}$$

47.
$$\begin{array}{r} 3\,x - 4\,xy - 2\,z \\ -3\,x + 7\,xy - 4\,z \\ \hline 3\,xy - 6\,z \\ 9\,x - 2\,xy + 4\,z - 7\,a^2bc \\ \hline -9\,x + 5\,xy - 10\,z + 7\,a^2bc \end{array} \qquad \begin{array}{r} 5\,z - 2\,xy - a^2bc \\ -z - 6\,a^2bc + 9\,x \\ \hline 4\,z - 2\,xy - 7\,a^2bc + 9\,x \end{array}$$

48.
$$\begin{array}{r} 4\,x - 3\,y + 6 \\ 3\,x + 5\,y - 10 \\ \hline 7\,x + 2\,y - 4 \end{array}$$

49.
$$\begin{array}{r} 7\,c + 5\,d - e \\ 4\,c + 5\,d - 9\,e \\ \hline 3\,c + 8\,e \end{array}$$

50.
$$\begin{array}{r} x^2 + 2\,x + 5 \\ 2\,x^2 + x - 10 \\ \hline 3\,x^2 + 3\,x - 5 \\ x^2 - 5\,x + 3 \\ \hline 2\,x^2 + 8\,x - 8 \end{array}$$

51.
$$\begin{array}{r} x + 3\,y - 2\,z \\ 4\,x - 5\,y + 3\,z \\ \hline 5\,x - 2\,y + z \\ 3\,x - 2\,y - 6\,z \\ \hline 2\,x + 7\,z \end{array}$$

52.
$$\begin{array}{r} 5\,x + 3\,y - z \\ 4\,y + 7\,z \\ \hline 5\,x + 7\,y + 6\,z \\ x - y + 3\,z \\ \hline 4\,x + 8\,y + 3\,z \end{array}$$

53.
$$\begin{array}{r} 4\,x - 3\,y + 7 \\ 4\,x - 8 \\ \hline 8\,x - 3\,y - 1 \\ 2\,x - 5\,y - 4 \\ \hline 6\,x + 2\,y + 3 \end{array}$$

54.
$$\begin{array}{r} 3\,c - 5\,d - e \\ 5\,c + 6\,d + 11\,e \\ \hline -2\,c - 11\,d - 12\,e \\ 5\,c + 4\,e \\ \hline -7\,c - 11\,d - 16\,e \end{array}$$

55.
$$\begin{array}{r} 3\,a + 3\,b - 4\,c \\ x + 4\,a - 3\,b - 11\,c - 4 \\ \hline -x - a + 6\,b + 7\,c + 4 \end{array} \qquad \begin{array}{r} -3\,b - 3\,c - 4 \\ -8\,c + 4\,a + x \\ \hline -3\,b - 11\,c - 4 + 4\,a + x \end{array}$$

56.
$$\begin{array}{r} x^3 - c^3 + 3\,cx^2 - 3\,c^2x \\ x^3 + c^3 - 4\,cx^2 - 2\,c^2x \\ \hline -2\,c^3 + 7\,cx^2 - c^2x \end{array}$$

Page 58

1. $$4x - 1 = 3x + 3.$$
Transposing, $$4x - 3x = 3 + 1.$$
Combining, $$x = 4.$$

2. $$4x + 5 = x - 10.$$
Transposing, $$4x - x = -10 - 5.$$
Combining, $$3x = -15.$$
Whence $$x = -5.$$

3. $$-6y - 2 = 1 - 9y.$$
Transposing, $$9y - 6y = 2 + 1.$$
Combining, $$3y = 3.$$
Whence $$y = 1.$$

4. $$6x + 3 - 2x = 27.$$
Combining, $$4x + 3 = 27.$$
Transposing and combining, $$4x = 24.$$
Whence $$x = 6.$$

5. $$4y + 3 = 2 + 5y.$$
Transposing, $$3 - 2 = 5y - 4y.$$
Combining, $$y = 1.$$

6. $$4k + 1 - 2k = 4.$$
Combining, $$2k + 1 = 4.$$
Transposing and combining, $$2k = 3.$$
Whence $$k = \tfrac{3}{2}.$$

7. $$16 + 4x - 4 = x + 12.$$
Combining, $$12 + 4x = x + 12.$$
Transposing and combining, $$3x = 0.$$
Whence $$x = 0.$$

8. $$\begin{aligned} 4x - 15 &= 15 - x - 5. \\ 4x - 15 &= 10 - x. \\ 5x &= 25. \\ x &= 5. \end{aligned}$$

9. $$\begin{aligned} 5n + 11 - 2n &= 6 - 4. \\ 3n + 11 &= 2. \\ 3n &= -9. \\ n &= -3. \end{aligned}$$

10. $$\begin{aligned} 2x - 1 &= 29 + 7x. \\ -5x &= 30. \\ x &= -6. \end{aligned}$$

11. $$\begin{aligned} 3k + 9 + 5k - 33 &= 0. \\ 8k - 24 &= 0. \\ 8k &= 24. \\ k &= 3. \end{aligned}$$

12. $3h - 20 + 8h - 24 = 0.$
$$11h - 44 = 0.$$
$$11h = 44.$$
$$h = 4.$$

13. $6x - 13 + 2x + 3 = 0.$
$$8x - 10 = 0.$$
$$8x = 10.$$
$$x = \tfrac{10}{8} = \tfrac{5}{4}.$$

14. $x + 2 - 5x = -9x + 12.$
$$-4x + 2 = -9x + 12.$$
$$5x = 10.$$
$$x = 2.$$

15. $4x + 3 = 15 + 2x + 6.$
$$4x + 3 = 2x + 21.$$
$$2x = 18.$$
$$x = 9.$$

16. $3y + 5 + y + 3 = 0.$
$$4y + 8 = 0.$$
$$4y = -8.$$
$$y = -2.$$

17. $9 - 8h + 2 = 3 - 4h.$
$$11 - 8h = 3 - 4h.$$
$$8 = 4h.$$
$$h = 2.$$

18. $3 - 5x + 2 = 5 + 7x.$
$$5 - 5x = 5 + 7x.$$
$$0 = 12x.$$
$$x = 0.$$

19. $8x - 3x = 15x + 4 - 13x.$
$$5x = 2x + 4.$$
$$3x = 4.$$
$$x = \tfrac{4}{3}.$$

20. $8 + 7y - 13 = y - 27 - 5y.$
$$7y - 5 = -27 - 4y.$$
$$11y = -22.$$
$$y = -2.$$

21. $3x - 15 - 10x - 9 + 16x - 21 = 0.$
$$9x - 45 = 0.$$
$$9x = 45.$$
$$x = 5.$$

22. $18 + 5x - 6 - 2x + 1 + 3x - 25 = 0.$
$$6x - 12 = 0.$$
$$6x = 12.$$
$$x = 2.$$

23. $0 = 18 - 4x + 27 + 9x - 3 + 16x.$
$$0 = 42 + 21x.$$
$$-21x = 42.$$
$$x = -2.$$

24. $5n - 8 + 4n + 5 = 7n - 3 - 2n + 5.$
$$9n - 3 = 5n + 2.$$
$$4n = 5.$$
$$n = \tfrac{5}{4}.$$

Page 61

1. Let $n =$ the number.
Then $n + 16 = 37.$
Whence $n = 21.$

2. Let $n =$ the number.
Then $n - 23 = 9.$
Whence $n = 32.$

3. Let $n =$ the number.
Then $n + 28 = 15.$
Whence $n = -13.$

4. Let $n =$ the number.
Then $n - 15 = 24.$
Whence $n = 39.$

5. Let $n =$ the number.
Then $2n - 13 = 49.$
Whence $n = 31.$

6. Let $n =$ the number.
Then $2n + 16 = 12.$
Whence $n = -2.$

7. Let $n =$ the number.
Then $3n - 16 = n.$
Whence $n = 8.$

8. Let $n =$ the number.
Then $3n - 23 = 2n - 8.$
Whence $n = 15.$

9. Let $n =$ the number.
Then $5n + 12 = 8n - 6.$
Whence $n = 6.$

10. Let $n =$ the number.
Then $4n - 7 = 2n + 21.$
Whence $n = 14.$

11. Let $n =$ the number.
Then $90 - n = n - 16.$
Whence $n = 53.$

12. Let $x =$ the number.
Then
$$x - 16 = 2(52 - x)$$
$$= 104 - 2x.$$
Whence $x = 40.$

13. Let $n =$ the number.
Then $14 + n = 58 - n.$
Whence $n = 22.$

14. Let $n =$ the number.
Then $2n + 9 = 5n - 12.$
Whence $n = 7.$

15. Let $n =$ the number.
Then $5n + 16 = 3n + 10.$
Whence $n = -3.$

17. Let $n =$ the less number.
Then $n + 12 =$ the greater number,
and $n + n + 12 = 58.$
Whence $n = 23;\ n + 12 = 35.$

18. Let $n =$ the less number.
Then $n + 42 =$ the greater number,
and $n + n + 42 = 8.$
Whence $n = -17;\ n + 42 = 25.$

19. Let $n =$ the first number.
Then $n + 5 =$ the second number,
and $n + n + 5 = 21.$
Whence $n = 8;\ n + 5 = 13.$

20. Let $n =$ the less number.
Then $n + 41 =$ the greater number,
and $n + n + 41 = 85.$
Whence $n = 22;\ n + 41 = 63.$

21. Let $n =$ the first number.
Then $n + 5 =$ the second number,
and $n + 17 =$ the third number.
But $n + n + 5 + n + 17 = 55.$
Whence $n = 11;\ n + 5 = 16;\ n + 17 = 28.$

22. Let $n =$ the first number.
Then $n - 10 =$ the second number,
and $2n - 2 =$ the third number.
But $n + n - 10 + 2n - 2 = 16.$
Whence $n = 7;\ n - 10 = -3;\ 2n - 2 = 12.$

23. Let n, $n + 1$, $n + 2$ be the three numbers in question.
Then $n + n + 1 + n + 2 = 90.$
Whence $n = 29;\ n + 1 = 30;\ n + 2 = 31.$

24. Let n, $n + 2$, $n + 4$ be the three numbers in question.
Then $n + n + 2 + n + 4 = 33.$
Whence $n = 9;\ n + 2 = 11;\ n + 4 = 13.$

25. Let $x =$ the width in feet.
Then $3x =$ the length in feet,
and $x + 3x + x + 3x = 32.$
Whence $x = 4;\ 3x = 12.$

26. Let $x =$ the width in feet.
Then $2x =$ the length in feet,
and $x + 2x + x + 2x = 102.$
Whence $x = 17;\ 2x = 34.$

27. Let $x =$ the width in feet.
Then $2x + 7 =$ the length in feet,
and $x + 2x + 7 + x + 2x + 7 = 110.$
Whence $x = 16;\ 2x + 7 = 39.$

28. Let $x =$ the bookkeeper's salary in dollars.
Then $\frac{2}{3}x =$ the stenographer's salary in dollars,
and $x + \frac{2}{3}x = 40 - 5.$
$\frac{5}{3}x = 35,$
and, dividing by $\frac{5}{3}$, $x = 21;$
$\frac{2}{3}x = 14.$

29. Let $x =$ James's allowance in cents.
Then $2x =$ Elizabeth's allowance in cents,
and $2x + 50 =$ John's allowance in cents.
But $x + 2x + 2x + 50 = 150.$
Whence $x = 20;\ 2x = 40;\ 2x + 50 = 90.$

30. Let $x =$ the number of courses laid the first day.
Then $x - 2 =$ the number laid the second day,
and $x - 4,\ x - 6,\ x - 8,\ x - 10$, etc.
$=$ the number laid on each succeeding day.
But $x + x - 2 + x - 4 + x - 6 + x - 8 + x - 10 = 120.$
Whence $x = 25.$

Page 66

1. $14 - (6 - 3) - 5 = 14 - 6 + 3 - 5 = 6.$
2. $10 + (7 - 4) - (9 - 7) = 10 + 7 - 4 - 9 + 7 = 11.$
3. $(7 - 5 + 2) - (6 - 4) + 12 = 7 - 5 + 2 - 6 + 4 + 12 = 14.$
4. $11a - (4a - 9a) + (6a - a) = 11a - 4a + 9a + 6a - a = 21a.$
5. $(2b - 5a) - (4a - b - 7a) = 2b - 5a - 4a + b + 7a = 3b - 2a.$
6. $a - (b - a) + (2b - 3c) = a - b + a + 2b - 3c = 2a + b - 3c.$
7. $a - b - (c - d) + (a - b) - (b - c)$
$= a - b - c + d + a - b - b + c = 2a - 3b + d.$
8. $(x - y) - (2y - 3x) + (x - 4y) = x - y - 2y + 3x + x - 4y = 5x - 7y.$
9. $x - (x - y - 2z) - (3z + y + 4) + (x - 6)$
$= x - x + y + 2z - 3z - y - 4 + x - 6 = x - z - 10.$
10. $7 - [8 - (3 - 10)] - (13 - 25) = 7 - [8 - 3 + 10] - 13 + 25$
$= 7 - 8 + 3 - 10 - 13 + 25 = 4.$
11. $8 - [5 - (4 - 6)] - (6 - 15) = 8 - [5 - 4 + 6] - 6 + 15$
$= 8 - 5 + 4 - 6 - 6 + 15 = 10.$
12. $12 - (8 - 17) - [(4 - 1) - 8] = 12 - 8 + 17 - 4 + 1 + 8 = 26.$
13. $16 - [3 - (2 - 5)] + (3 - 5) - (8 - 4)$
$= 16 - [3 - 2 + 5] + 3 - 5 - 8 + 4$
$= 16 - 3 + 2 - 5 + 3 - 5 - 8 + 4 = 4.$
14. $a + [2a - (3a - 2b)] + (3b - 2a)$
$= a + [2a - 3a + 2b] + 3b - 2a$
$= a + 2a - 3a + 2b + 3b - 2a = 5b - 2a.$
15. $a + [5a - (3x - 2a)] - (4a - 3x)$
$= a + 5a - 3x + 2a - 4a + 3x = 4a.$
16. $2a - [6a - (5x - 4a)] + (8a - 7x) = 2a - [6a - 5x + 4a] + 8a - 7$
$= 2a - 6a + 5x - 4a + 8a - 7x = -2x.$

17. Two; two or three; three or four; one; two; two; two; two.

18. $(5x - 6y) - [-4x - (4z - y) - 2z]$
$= 5x - 6y - [-4x - 4z + y - 2z]$
$= 5x - 6y + 4x + 4z - y + 2z = 9x - 7y + 6z.$

19. $[3x - (2y + z)] - [-(3y - 2x) + 5x]$
$= 3x - 2y - z - [-3y + 2x + 5x]$
$= 3x - 2y - z + 3y - 2x - 5x = -4x + y - z.$

20. $[(a + 3) - (x - 5)] - [a + 3 + (x - 5)]$
$= a + 3 - x + 5 - a - 3 - x + 5$
$= -2x + 10.$

21. $7 - [-6 - \{-4 + (9 - 10)\} + 11] = 7 - [-6 - \{-4 + 9 - 10\} + 11]$
$= 7 - [-6 + 4 - 9 + 10 + 11]$
$= 7 + 6 - 4 + 9 - 10 - 11 = -3.$

22. $5a - [2a + (-3a + 4b) - (a - 8b) + 4a]$
$= 5a - [2a - 3a + 4b - a + 8b + 4a]$
$= 5a - 2a + 3a - 4b + a - 8b - 4a$
$= 3a - 12b.$

23. $2x - 3y - [\{3z - 7x - (y - 4z) - 9x\} + z]$
$= 2x - 3y - [3z - 7x - y + 4z - 9x + z]$
$= 2x - 3y - 3z + 7x + y - 4z + 9x - z$
$= 18x - 2y - 8z.$

24. $\{4a - [2a - (8a + 2b) + 4] - (4b - 6)\}$
$= \{4a - [2a - 8a - 2b + 4] - 4b + 6\}$
$= \{4a - 2a + 8a + 2b - 4 - 4b + 6\}$
$= 10a - 2b + 2.$

25. $-5x + [15x - \{11x - (2x - 7x - 4) - 3x\} - 22]$
$= -5x + 15x - \{11x - 2x + 7x + 4 - 3x\} - 22$
$= -5x + 15x - 11x + 2x - 7x - 4 + 3x - 22$
$= -3x - 26.$

26. $(4y - 7x) - \{3x - [4x - (7y - 4x) - (-5y + 3x)]\}$
$= 4y - 7x - \{3x - [4x - 7y + 4x + 5y - 3x]\}$
$= 4y - 7x - \{3x - 4x + 7y - 4x - 5y + 3x\}$
$= 4y - 7x - 3x + 4x - 7y + 4x + 5y - 3x$
$= 2y - 5x.$

27. $[(a + b) + c] = [a + b + c]$; $[(a + b) - c] = [a + b - c].$

28. $[4x + (3z - 5y)] = [4x + 3z - 5y]$; $[4x - (3z - 5y)] = [4x - 3z + 5y].$

29. $[(a - b) + (b - 2a)] = [a - b + b - 2a] = -a$; $[(a - b) - (b - 2a)]$
$= [a - b - b + 2a] = [3a - 2b].$

30. $[(3a - 2b) + (2b - 3a)] = [3a - 2b + 2b - 3a] = 0$; $[(3a - 2b)$
$- (2b - 3a)] = [3a - 2b - 2b + 3a] = [6a - 4b].$

31. $[(a-2b)+(3c-d)]=[a-2b+3c-d]$; $[(a-2b)-(3c-d)]$
$=[a-2b-3c+d]$.

32. $[(4x-3)+(5y-7)]=[4x-3+5y-7]=[4x+5y-10]$;
$[(4x-3)-(5y-7)]=[4x-3-5y+7]=[4x-5y+4]$.

33. $[(x^2-a^2)+(y^2-2a^2)]=[x^2-a^2+y^2-2a^2]=[x^2+y^2-3a^2]$;
$[(x^2-a^2)-(y^2-2a^2)]=[x^2-a^2-y^2+2a^2]=[x^2-y^2+a^2]$.

Page 69

1. $x^2-a^2-2a=+(x^2)-(a^2+2a)$.

2. $12a+x^2-9-4a^2=+(x^2)-(4a^2-12a+9)$.

3. $y^2-9b^2+6ab-a^2=+(y^2)-(9b^2-6ab+a^2)$.

4. $10ab+x^2-a^2-25b^2=+(x^2)-(a^2-10ab+25b^2)$.

5. $x^2-b^2-4b^2+y^2-a^2-2xy=+(x^2-2xy+y^2)-(a^2+5b^2)$.

6. $-4ab+x^2-4b^2+y^2-a^2-2xy$
$=+(x^2-2xy+y^2)-(a^2+4ab+4b^2)$.

7. $16x^2-a^2-16xy-4+4a+4y^2$
$=+(16x^2-16xy+4y^2)-(a^2-4a+4)$.

8. $x^2-b^2-10xy+12ab-36a^2+25y^2$
$=+(x^2-10xy+25y^2)-(b^2-12ab+36a^2)$.

9. $2ab-a^2+x^2+2xy-b^2+y^2=+(x^2+2xy+y^2)-(a^2-2ab+b^2)$.

10. $x^2-16xy-a^2+16a+64y^2-64$
$=+(x^2-16xy+64y^2)-(a^2-16a+64)$.

11. (1) $x^2-a^2-2a=x^2-(a^2+2a)$.

11. (2) $12a+x^2-9-4a^2=12a+x^2-(9+4a^2)$.

11. (3) $y^2-9b^2+6ab-a^2=y^2-9b^2-(-6ab+a^2)$.

11. (4) $10ab+x^2-a^2-25b^2=10ab+x^2-(a^2+25b^2)$.

12. (5) $x^2-b^2-4b^2+y^2-a^2-2xy=x^2-5b^2-(-y^2+a^2+2xy)$.

12. (6) $-4ab+x^2-4b^2+y^2-a^2-2xy$
$=-4ab+x^2-4b^2-(-y^2+a^2+2xy)$.

12. (7) $16x^2-a^2-16xy-4+4a+4y^2$
$=16x^2-a^2-16xy-(4-4a-4y^2)$.

12. (8) $x^2-b^2-10xy+12ab-36a^2+25y^2$
$=x^2-b^2-10xy-(-12ab+36a^2-25y^2)$.

12. (9) $2ab-a^2+x^2+2xy-b^2+y^2$
$=2ab-a^2+x^2-(-2xy+b^2-y^2)$.

12. (10) $x^2-16xy-a^2+16a+64y^2-64$
$=x^2-16xy-a^2-(-16a-64y^2+64)$.

Page 73

1. $$\begin{array}{l} x+2 \\ 2 \\ \hline 2x+4 \end{array}$$

2. $$\begin{array}{l} x-4 \\ x \\ \hline x^2-4x \end{array}$$

3. $$\begin{array}{l} x^2+5 \\ 2x \\ \hline 2x^3+10x \end{array}$$

4. $$\begin{array}{l} 3x^2+4 \\ 2x \\ \hline 6x^3+8x \end{array}$$

5. $$\begin{array}{l} x^2-2x \\ x^2 \\ \hline x^4-2x^3 \end{array}$$

6. $$\begin{array}{l} y^2-3y+2 \\ 3y \\ \hline 3y^3-9y^2+6y \end{array}$$

7. $$\begin{array}{l} y^3-5y^2+3y-1 \\ y^2 \\ \hline y^5-5y^4+3y^3-y^2 \end{array}$$

8. $$\begin{array}{l} 5x^2-2x-4 \\ x^2 \\ \hline 5x^4-2x^3-4x^2 \end{array}$$

9. $$\begin{array}{l} -4x^2+6x-5 \\ 6x^2 \\ \hline -24x^4+36x^3-30x^2 \end{array}$$

10. $$\begin{array}{l} 2x^2-3x-2 \\ -4x^2 \\ \hline -8x^4+12x^3+8x^2 \end{array}$$

11. $$\begin{array}{l} x^3-3x^2-4 \\ -5x^4 \\ \hline -5x^7+15x^6+20x^4 \end{array}$$

12. $$\begin{array}{l} 2x-3x^2-2x^3 \\ -2x^3 \\ \hline -4x^4+6x^5+4x^6 \end{array}$$

13. $$\begin{array}{l} 7xy-x+y \\ 3xy \\ \hline 21x^2y^2-3x^2y+3xy^2 \end{array}$$

14. $$\begin{array}{l} x^2-2xy+y^2 \\ -3xy \\ \hline -3x^3y+6x^2y^2-3xy^3 \end{array}$$

15. $$\begin{array}{l} a^4-a^2b^2+b^4 \\ -a^2b^2 \\ \hline -a^6b^2+a^4b^4-a^2b^6 \end{array}$$

16. $$\begin{array}{l} -a^2x^2-2ax+7b^2 \\ -4abx \\ \hline 4a^3bx^3+8a^2bx^2-28ab^3x \end{array}$$

17. $$\begin{array}{l} 7x^3-8x^2+12x-6 \\ -4x^3 \\ \hline -28x^6+32x^5-48x^4+24x^3 \end{array}$$

18. $$\begin{array}{l} -9a^2-12ax+42x^2 \\ 3ax^3 \\ \hline -27a^3x^3-36a^2x^4+126ax^5 \end{array}$$

19. $7(2x-3)=14x-21$.

20. $5x(x-y)=5x^2-5xy$.

21. $-8(3x-7)=-24x+56$.

22. $-9(-4a+b)=36a-9b$.

23. $-3x(2x-7)=-6x^2+21x$.

24. $6x^2(9x^3-4x)=54x^5-24x^3$.

25. $-3(x^2-2x-6)=-3x^2+6x+18$.

26. $5xy(x^2-6x+9)=5x^3y-30x^2y+45xy$.

27. $-3x(ax-bx-3cx^2)=-3ax^2+3bx^2+9cx^3$.

28. $(x^3-ax+a^2)(-2a^2x)=-2a^2x^4+2a^3x^2-2a^4x$.

29. $-7ab(ax^2-bx+c)=-7a^2bx^2+7ab^2x-7abc$.

30. $4x^2(-3x+7x^2-x^4)=-12x^3+28x^4-4x^6$.

Page 74

1. $$\begin{array}{r} x+4 \\ x+3 \\ \hline x^2+4x \\ +3x+12 \\ \hline x^2+7x+12 \end{array}$$

2. $$\begin{array}{r} 2x+3 \\ x+3 \\ \hline 2x^2+3x \\ +6x+9 \\ \hline 2x^2+9x+9 \end{array}$$

3. $$\begin{array}{r} 4x+7 \\ 3x+2 \\ \hline 12x^2+21x \\ +8x+14 \\ \hline 12x^2+29x+14 \end{array}$$

4. $$\begin{array}{r} 3x-5 \\ 3x+8 \\ \hline 9x^2-15x \\ +24x-40 \\ \hline 9x^2+9x-40 \end{array}$$

5. $$\begin{array}{r} 3x-2 \\ 2x+3 \\ \hline 6x^2-4x \\ +9x-6 \\ \hline 6x^2+5x-6 \end{array}$$

6. $$\begin{array}{r} 6-4a \\ -7+5a \\ \hline -42+28a \\ +30a-20a^2 \\ \hline -42+58a-20a^2 \end{array}$$

7. $$\begin{array}{r} 2x+y \\ x+3y \\ \hline 2x^2+xy \\ +6xy+3y^2 \\ \hline 2x^2+7xy+3y^2 \end{array}$$

8. $$\begin{array}{r} 2x-3y \\ 3x-2y \\ \hline 6x^2-9xy \\ -4xy+6y^2 \\ \hline 6x^2-13xy+6y^2 \end{array}$$

9. $$\begin{array}{r} 3x-2 \\ 2x-3 \\ \hline 6x^2-4x \\ -9x+6 \\ \hline 6x^2-13x+6 \end{array}$$

10. $$\begin{array}{r} -3x+11a \\ 5x-a \\ \hline -15x^2+55ax \\ +3ax-11a^2 \\ \hline -15x^2+58ax-11a^2 \end{array}$$

11. $$\begin{array}{l} ax-bx \\ cx+dx \\ \hline acx^2-bcx^2+adx^2-bdx^2 \end{array}$$

12. $$\begin{array}{l} -cx+d \\ -cx^2+bx \\ \hline c^2x^3-cdx^2-bcx^2+bdx \end{array}$$

13. $$\begin{array}{l} 4x-3y^2 \\ 6x+5y \\ \hline 24x^2-18xy^2+20xy-15y^3 \end{array}$$

14. $$\begin{array}{r} x^2-5x+6 \\ x-3 \\ \hline x^3-5x^2+6x \\ -3x^2+15x-18 \\ \hline x^3-8x^2+21x-18 \end{array}$$

15. $$\begin{array}{r} 3x^2-3x-7 \\ 2x+4 \\ \hline 6x^3-6x^2-14x \\ +12x^2-12x-28 \\ \hline 6x^3+6x^2-26x-28 \end{array}$$

16. $$\begin{array}{r} x^2-xy+y^2 \\ x+y \\ \hline x^3-x^2y+xy^2 \\ +x^2y-xy^2+y^3 \\ \hline x^3+y^3 \end{array}$$

17. $$\begin{array}{r} a^2x^2-2a^2x+4a^2 \\ ax+2a \\ \hline a^3x^3-2a^3x^2+4a^3x \\ +2a^3x^2-4a^3x+8a^3 \\ \hline a^3x^3+8a^3 \end{array}$$

18. $$\begin{array}{r} 3x^3-x^2-5x \\ 2x^3-5x^2 \\ \hline 6x^6-2x^5-10x^4 \\ -15x^5+5x^4+25x^3 \\ \hline 6x^6-17x^5-5x^4+25x^3 \end{array}$$

19.
$$\begin{array}{r} 2x^2 - 7x + 12 \\ x^2 - 3x - 5 \\ \hline 2x^4 - 7x^3 + 12x^2 \\ - 6x^3 + 21x^2 - 36x \\ - 10x^2 + 35x - 60 \\ \hline 2x^4 - 13x^3 + 23x^2 - x - 60 \end{array}$$

20.
$$\begin{array}{r} x^2 - 2x - 3 \\ x^2 - 2x - 3 \\ \hline x^4 - 2x^3 - 3x^2 \\ - 2x^3 + 4x^2 + 6x \\ - 3x^2 + 6x + 9 \\ \hline x^4 - 4x^3 - 2x^2 + 12x + 9 \end{array}$$

21.
$$\begin{array}{r} 3x^2 - 5x - 1 \\ 2x - 3 \\ \hline 6x^3 - 10x^2 - 2x \\ - 9x^2 + 15x + 3 \\ \hline 6x^3 - 19x^2 + 13x + 3 \end{array}$$

22.
$$\begin{array}{r} x^2 - 3x - 2 \\ x^2 - 2x + 3 \\ \hline x^4 - 3x^3 - 2x^2 \\ - 2x^3 + 6x^2 + 4x \\ + 3x^2 - 9x - 6 \\ \hline x^4 - 5x^3 + 7x^2 - 5x - 6 \end{array}$$

23.
$$\begin{array}{r} t^2 - 3t + 2 \\ t^2 - 3t + 2 \\ \hline t^4 - 3t^3 + 2t^2 \\ - 3t^3 + 9t^2 - 6t \\ + 2t^2 - 6t + 4 \\ \hline t^4 - 6t^3 + 13t^2 - 12t + 4 \end{array}$$

24.
$$\begin{array}{r} -y^2 + 3y - 2 \\ -y^2 + 3y - 2 \\ \hline y^4 - 3y^3 + 2y^2 \\ - 3y^3 + 9y^2 - 6y \\ + 2y^2 - 6y + 4 \\ \hline y^4 - 6y^3 + 13y^2 - 12y + 4 \end{array}$$

25.
$$\begin{array}{r} -2x^2 + 4x - 5 \\ -2x^2 + 4x - 5 \\ \hline 4x^4 - 8x^3 + 10x^2 \\ - 8x^3 + 16x^2 - 20x \\ + 10x^2 - 20x + 25 \\ \hline 4x^4 - 16x^3 + 36x^2 - 40x + 25 \end{array}$$

26.
$$\begin{array}{r} 3x^2 - 4x + 7 \\ 3x^2 - 4x + 7 \\ \hline 9x^4 - 12x^3 + 21x^2 \\ - 12x^3 + 16x^2 - 28x \\ + 21x^2 - 28x + 49 \\ \hline 9x^4 - 24x^3 + 58x^2 - 56x + 49 \end{array}$$

Page 77

1.
$$\begin{array}{r} 3x^2 - 5x - 2 \\ 2x - 3 \\ \hline 6x^3 - 10x^2 - 4x \\ - 9x^2 + 15x + 6 \\ \hline 6x^3 - 19x^2 + 11x + 6 \end{array}$$

2.
$$\begin{array}{r} 3x^2 + 2x - 7 \\ 2x^2 - 5x - 3 \\ \hline 6x^4 + 4x^3 - 14x^2 \\ - 15x^3 - 10x^2 + 35x \\ - 9x^2 - 6x + 21 \\ \hline 6x^4 - 11x^3 - 33x^2 + 29x + 21 \end{array}$$

3.
$$\begin{array}{r} x^2 + 2x - 4 \\ x^2 + 2x + 4 \\ \hline x^4 + 2x^3 - 4x^2 \\ + 2x^3 + 4x^2 - 8x \\ + 4x^2 + 8x - 16 \\ \hline x^4 + 4x^3 + 4x^2 - 16 \end{array}$$

4.
$$\begin{array}{r} 3a^2 - 8a - 1 \\ a^2 + 2a - 3 \\ \hline 3a^4 - 8a^3 - a^2 \\ + 6a^3 - 16a^2 - 2a \\ - 9a^2 + 24a + 3 \\ \hline 3a^4 - 2a^3 - 26a^2 + 22a + 3 \end{array}$$

5.
$$\begin{array}{rrrrr}
\multicolumn{5}{l}{2x^2 - 7x - 2} \\
\multicolumn{5}{l}{2x^2 - 7x - 2} \\ \hline
4x^4 & -14x^3 & -4x^2 & & \\
 & -14x^3 & +49x^2 & +14x & \\
 & & -4x^2 & +14x & +4 \\ \hline
4x^4 & -28x^3 & +41x^2 & +28x & +4
\end{array}$$

6.
$$\begin{array}{rrrrrr}
\multicolumn{6}{l}{2x^3 - 3x + 2} \\
\multicolumn{6}{l}{2x^3 - 3x + 2} \\ \hline
4x^6 & -6x^4 & +4x^3 & & & \\
 & -6x^4 & & +9x^2 & -6x & \\
 & & +4x^3 & & -6x & +4 \\ \hline
4x^6 & -12x^4 & +8x^3 & +9x^2 & -12x & +4
\end{array}$$

7.
$$\begin{array}{rrrrr}
\multicolumn{5}{l}{a^2 - \frac{a}{2} + \frac{1}{4}} \\
\multicolumn{5}{l}{a^2 - a + 1} \\ \hline
a^4 & -\frac{a^3}{2} & +\frac{a^2}{4} & & \\
 & -a^3 & +\frac{a^2}{2} & -\frac{a}{4} & \\
 & & +a^2 & -\frac{a}{2} & +\frac{1}{4} \\ \hline
a^4 & -\frac{3a^3}{2} & +\frac{7a^2}{4} & -\frac{3a}{4} & +\frac{1}{4}
\end{array}$$

8.
$$\begin{array}{rrrrr}
\multicolumn{5}{l}{x^2 - xy + y^2} \\
\multicolumn{5}{l}{x^2 + xy + y^2} \\ \hline
x^4 & -x^3y & +x^2y^2 & & \\
 & +x^3y & -x^2y^2 & +xy^3 & \\
 & & +x^2y^2 & -xy^3 & +y^4 \\ \hline
x^4 & & +x^2y^2 & & +y^4
\end{array}$$

9.
$$\begin{array}{rrrrrr}
\multicolumn{6}{l}{3x^3 + 5x^2 - x + 2} \\
\multicolumn{6}{l}{x^2 - 2x + 3} \\ \hline
3x^5 & +5x^4 & -x^3 & +2x^2 & & \\
 & -6x^4 & -10x^3 & +2x^2 & -4x & \\
 & & +9x^3 & +15x^2 & -3x & +6 \\ \hline
3x^5 & -x^4 & -2x^3 & +19x^2 & -7x & +6
\end{array}$$

10.
$$\begin{array}{rrrrrr}
\multicolumn{6}{l}{-a^3 - 2a^2 + a - 3} \\
\multicolumn{6}{l}{-3a^2 + 2a - 1} \\ \hline
3a^5 & +6a^4 & -3a^3 & +9a^2 & & \\
 & -2a^4 & -4a^3 & +2a^2 & -6a & \\
 & & +a^3 & +2a^2 & -a & +3 \\ \hline
3a^5 & +4a^4 & -6a^3 & +13a^2 & -7a & +3
\end{array}$$

11.
$$\begin{array}{rrrrrr}
\multicolumn{6}{l}{-a^3 + 3a + 6} \\
\multicolumn{6}{l}{-3a^2 + 4a - 5} \\ \hline
3a^5 & & -9a^3 & -18a^2 & & \\
 & -4a^4 & & +12a^2 & +24a & \\
 & & +5a^3 & & -15a & -30 \\ \hline
3a^5 & -4a^4 & -4a^3 & -6a^2 & +9a & -30
\end{array}$$

12. $$\begin{array}{llllll}
\multicolumn{6}{l}{x^3 - x - 5} \\
\multicolumn{6}{l}{3x^2 - 2x - 4} \\ \hline
3x^5 & & -3x^3 & -15x^2 & & \\
 & -2x^4 & & +2x^2 & +10x & \\
 & & -4x^3 & & +4x & +20 \\ \hline
3x^5 & -2x^4 & -7x^3 & -13x^2 & +14x & +20
\end{array}$$

13. $$\begin{array}{llllll}
\multicolumn{6}{l}{-x^3 + x^2 + 3x - 6} \\
\multicolumn{6}{l}{-x^2 - 3x + 5} \\ \hline
x^5 & -x^4 & -3x^3 & +6x^2 & & \\
 & +3x^4 & -3x^3 & -9x^2 & +18x & \\
 & & -5x^3 & +5x^2 & +15x & -30 \\ \hline
x^5 & +2x^4 & -11x^3 & +2x^2 & +33x & -30
\end{array}$$

14. $(6x - 4a)0 = 0.$

15. $$\begin{array}{lllllll}
\multicolumn{7}{l}{a^3 - 5a^2 + 4a + 7} \\
\multicolumn{7}{l}{a^3 + a^2 - a + 3} \\ \hline
a^6 & -5a^5 & +4a^4 & +7a^3 & & & \\
 & a^5 & -5a^4 & +4a^3 & +7a^2 & & \\
 & & -a^4 & +5a^3 & -4a^2 & -7a & \\
 & & & +3a^3 & -15a^2 & +12a & +21 \\ \hline
a^6 & -4a^5 & -2a^4 & +19a^3 & -12a^2 & +5a & +21
\end{array}$$

16. $$\begin{array}{lllllll}
\multicolumn{7}{l}{2x^3 - 5x^2 - 9x + 8} \\
\multicolumn{7}{l}{8x^3 + 5x - 3} \\ \hline
16x^6 & -40x^5 & -72x^4 & +64x^3 & & & \\
 & & 10x^4 & -25x^3 & -45x^2 & +40x & \\
 & & & -6x^3 & +15x^2 & +27x & -24 \\ \hline
16x^6 & -40x^5 & -62x^4 & +33x^3 & -30x^2 & +67x & -24
\end{array}$$

17. $$\begin{array}{lllll}
\multicolumn{5}{l}{x^2 - 2xy + 3y^2} \\
\multicolumn{5}{l}{x^2 + 2xy + 3y^2} \\ \hline
x^4 & -2x^3y & +3x^2y^2 & & \\
 & +2x^3y & -4x^2y^2 & +6xy^3 & \\
 & & +3x^2y^2 & -6xy^3 & +9y^4 \\ \hline
x^4 & & +2x^2y^2 & & +9y^4
\end{array}$$

18. $$\begin{array}{llllll}
\multicolumn{6}{l}{4xy - 5x^2y} \\
\multicolumn{6}{l}{x^2y - xy^2} \\ \hline
\multicolumn{6}{l}{4x^3y^2 - 5x^4y^2 - 4x^2y^3 + 5x^3y^3} \\
\multicolumn{6}{l}{3x^2y - 2xy^2} \\ \hline
12x^5y^3 & -15x^6y^3 & -12x^4y^4 & +15x^5y^4 & & \\
 & & -8x^4y^4 & +10x^5y^4 & +8x^3y^5 & -10x^4y^5 \\ \hline
12x^5y^3 & -15x^6y^3 & -20x^4y^4 & +25x^5y^4 & +8x^3y^5 & -10x^4y^5
\end{array}$$

19.
$$\begin{array}{l}
x^2 - xy - xz - yz + y^2 + z^2 \\
x + y + z \\
\hline
x^3 - x^2y - x^2z - xyz + xy^2 + xz^2 \\
\quad + x^2y \qquad - xyz - xy^2 \qquad - y^2z + y^3 + yz^2 \\
\qquad + x^2z - xyz \qquad - xz^2 + y^2z \qquad - yz^2 + z^3 \\
\hline
x^3 \qquad - 3\,xyz \qquad + y^3 \qquad + z^3
\end{array}$$

20.
$$\begin{array}{l}
a + b + c \\
a + b + c \\
\hline
a^2 + ab + ac \\
\qquad ab \qquad + b^2 + bc \\
\qquad\qquad ac \qquad + bc + c^2 \\
\hline
a^2 + 2\,ab + 2\,ac + b^2 + 2\,bc + c^2
\end{array}$$

21.
$$\begin{array}{l}
2\,a - 3\,b + 4\,c \\
2\,a - 3\,b + 4\,c \\
\hline
4\,a^2 - 6\,ab + 8\,ac \\
\quad - 6\,ab \qquad + 9\,b^2 - 12\,bc \\
\qquad + 8\,ac \qquad - 12\,bc + 16\,c^2 \\
\hline
4\,a^2 - 12\,ab + 16\,ac + 9\,b^2 - 24\,bc + 16\,c^2
\end{array}$$

22.
$$\begin{array}{l}
c + d - \frac{1}{2} \\
c + d - \frac{1}{2} \\
\hline
c^2 + cd - \dfrac{c}{2} \\
\quad + cd \qquad + d^2 - \dfrac{d}{2} \\
\qquad - \dfrac{c}{2} \qquad - \dfrac{d}{2} + \dfrac{1}{4} \\
\hline
c^2 + 2\,cd - c + d^2 - d + \frac{1}{4}
\end{array}$$

23.
$$\begin{array}{l}
2\,a - 4\,b + 3 \\
2\,a - 4\,b + 3 \\
\hline
4\,a^2 - 8\,ab + 6\,a \\
\quad - 8\,ab \qquad + 16\,b^2 - 12\,b \\
\qquad + 6\,a \qquad - 12\,b + 9 \\
\hline
4\,a^2 - 16\,ab + 12\,a + 16\,b^2 - 24\,b + 9
\end{array}$$

24.
$$\begin{array}{l}
a - 2\,b + 3\,c - 4\,d \\
a - 2\,b + 3\,c - 4\,d \\
\hline
a^2 - 2\,ab + 3\,ac - 4\,ad \\
\quad - 2\,ab \qquad + 4\,b^2 - 6\,bc + 8\,bd \\
\qquad + 3\,ac \qquad - 6\,bc \qquad + 9\,c^2 - 12\,cd \\
\qquad\quad - 4\,ad \qquad + 8\,bd \qquad - 12\,cd + 16\,d^2 \\
\hline
a^2 - 4\,ab + 6\,ac - 8\,ad + 4\,b^2 - 12\,bc + 16\,bd + 9\,c^2 - 24\,cd + 16\,d^2
\end{array}$$

25.
$$\begin{array}{l}
x+y+z \\
x+y+z \\ \hline
x^2+xy+xz \\
\quad +xy+y^2+yz \\
\qquad +xz+yz+z^2 \\ \hline
x^2+2xy+2xz+y^2+2yz+z^2 \\
x+y+z \\ \hline
x^3+2x^2y+2x^2z+xy^2+2xyz+xz^2 \\
\quad +x^2y+2xy^2+2xyz+y^3+2y^2z+yz^2 \\
\qquad +x^2z+2xyz+2xz^2+y^2z+2yz^2+z^3 \\ \hline
x^3+3x^2y+3x^2z+3xy^2+6xyz+3xz^2+y^3+3y^2z+3yz^2+z^3
\end{array}$$

26.
$$\begin{array}{l}
x+2y \\
x+2y \\ \hline
x^2+2xy \\
\quad +2xy+4y^2 \\ \hline
x^2+4xy+4y^2 \\
x^2-4xy+4y^2 \\ \hline
\qquad 8xy \qquad (Ans.)
\end{array}
\qquad\qquad
\begin{array}{l}
x-2y \\
x-2y \\ \hline
x^2-2xy \\
\quad -2xy+4y^2 \\ \hline
x^2-4xy+4y^2
\end{array}$$

27.
$$\begin{array}{r}
x-3 \\
x-3 \\ \hline
x^2-3x \\
-3x+9 \\ \hline
x^2-6x+9 \\
x-3 \\ \hline
x^3-6x^2+9x \\
-3x^2+18x-27 \\ \hline
x^3-9x^2+27x-27 \\
-x^3+9x^2-27x+27 \\ \hline
0 \quad (Ans.)
\end{array}
\qquad\qquad
\begin{array}{r}
3-x \\
3-x \\ \hline
9-3x \\
-3x+x^2 \\ \hline
9-6x+x^2 \\
3-x \\ \hline
27-18x+3x^2 \\
-9x+6x^2-x^3 \\ \hline
27-27x+9x^2-x^3
\end{array}$$

28.
$$\begin{array}{l}
x+y \\
x+y \\ \hline
x^2+xy \\
\quad +xy+y^2 \\ \hline
x^2+2xy+y^2 \\
x+y \\ \hline
x^3+2x^2y+xy^2 \\
\quad +x^2y+2xy^2+y^3 \\ \hline
x^3+3x^2y+3xy^2+y^3 \\
x^3-3x^2y+3xy^2-y^3 \\ \hline
2x^3 \qquad +6xy^2 \qquad (Ans.)
\end{array}
\qquad\qquad
\begin{array}{l}
x-y \\
x-y \\ \hline
x^2-xy \\
\quad -xy+y^2 \\ \hline
x^2-2xy+y^2 \\
x-y \\ \hline
x^3-2x^2y+xy^2 \\
\quad -x^2y+2xy^2-y^3 \\ \hline
x^3-3x^2y+3xy^2-y^3
\end{array}$$

29.
$$\begin{array}{l}
2x-3a\\
2x-3a\\ \hline
4x^2-6ax\\
\qquad-6ax+9a^2\\ \hline
4x^2-12ax+9a^2\\
2x-3a\\ \hline
8x^3-24ax^2+18a^2x\\
\qquad-12ax^2+36a^2x-27a^3\\ \hline
8x^3-36ax^2+54a^2x-27a^3\\
-8x^3+36ax^2-54a^2x+27a^3\\ \hline
16x^3-72ax^2+108a^2x-54a^3 \ (Ans.)
\end{array}$$

$$\begin{array}{l}
3a-2x\\
3a-2x\\ \hline
9a^2-6ax\\
\qquad-6ax+4x^2\\ \hline
9a^2-12ax+4x^2\\
3a-2x\\ \hline
27a^3-36a^2x+12ax^2\\
\qquad-18a^2x+24ax^2-8x^3\\ \hline
27a^3-54a^2x+36ax^2-8x^3
\end{array}$$

30.
$$\begin{array}{l}
ax-5ay\\
ax-5ay\\ \hline
a^2x^2-5a^2xy\\
\qquad-5a^2xy+25a^2y^2\\ \hline
a^2x^2-10a^2xy+25a^2y^2\\
ax-5ay\\ \hline
a^3x^3-10a^3x^2y+25a^3xy^2\\
\qquad-5a^3x^2y+50a^3xy^2-125a^3y^3\\ \hline
a^3x^3-15a^3x^2y+75a^3xy^2-125a^3y^3\\
a^3x^3-3a^3x^2y+3a^3xy^2-a^3y^3\\ \hline
\qquad-12a^3x^2y+72a^3xy^2-124a^3y^3 \ (Ans.)
\end{array}$$

$$\begin{array}{l}
ax-ay\\
ax-ay\\ \hline
a^2x^2-a^2xy\\
\qquad-a^2xy+a^2y^2\\ \hline
a^2x^2-2a^2xy+a^2y^2\\
ax-ay\\ \hline
a^3x^3-2a^3x^2y+a^3xy^2\\
\qquad-a^3x^2y+2a^3xy^2-a^3y^3\\ \hline
a^3x^3-3a^3x^2y+3a^3xy^2-a^3y^3
\end{array}$$

31.
$$\begin{array}{l}
4x-3y\\
4x-3y\\ \hline
16x^2-12xy\\
\qquad-12xy+9y^2\\ \hline
16x^2-24xy+9y^2\\
4x^2+20xy+25y^2\\ \hline
12x^2-44xy-16y^2 \ (Ans.)
\end{array}$$

$$\begin{array}{l}
2x+5y\\
2x+5y\\ \hline
4x^2+10xy\\
\qquad+10xy+25y^2\\ \hline
4x^2+20xy+25y^2
\end{array}$$

32. From Ex. 27
$$\begin{array}{r}
(x-3)^3=x^3-9x^2+27x-27\\
4x^2-4x+1\\ \hline
x^3-13x^2+31x-28 \ (Ans.)
\end{array}$$

$$\begin{array}{r}
2x-1\\
2x-1\\ \hline
4x^2-2x\\
-2x+1\\ \hline
4x^2-4x+1
\end{array}$$

33.
$$\begin{array}{l}
2x^2-3a\\
2x^2+3a\\ \hline
4x^4-6ax^2\\
\qquad+6ax^2-9a^2\\ \hline
4x^4\qquad\qquad-9a^2\\
2x^2\qquad\qquad+3a\\ \hline
8x^6\qquad\qquad-18a^2x^2+12ax^4-27a^3
\end{array}$$

34.
$$\begin{array}{l}
x^3 - 5 \\
x^3 + 4 \\ \hline
x^6 - 5x^3 \\
\quad + 4x^3 - 20 \\ \hline
x^6 - \quad x^3 - 20 \\
\quad + \quad x^5 + x^2 \\ \hline
x^6 - \quad x^5 - x^3 - x^2 - 20
\end{array}$$

35.
$$\begin{array}{r}
a^2 + a + 3 \quad\quad\quad\quad \\
-a^2 + a - 3 \quad\quad\quad\quad \\ \hline
-a^4 - a^3 - 3a^2 \quad\quad\quad\quad \\
+a^3 + \quad a^2 + 3a \quad\quad \\
-3a^2 - 3a - 9 \\ \hline
-a^4 - 5a^2 \quad\quad\quad - 9
\end{array}$$

36.
$$\begin{array}{l}
a - x^2 + 3 \\
a + x^2 - 3 \\ \hline
a^2 - ax^2 + 3a \\
\quad + ax^2 \quad\quad - x^4 + 3x^2 \\
\quad\quad\quad - 3a \quad\quad + 3x^2 - 9 \\ \hline
a^2 \quad\quad\quad\quad\quad - x^4 + 6x^2 - 9
\end{array}$$

37.
$$\begin{array}{l}
x^2 + x - a - 2 \\
-x^2 + x - a + 2 \\ \hline
-x^4 - x^3 + ax^2 + 2x^2 \\
\quad + x^3 \quad + \quad x^2 - \quad ax - 2x \\
\quad\quad - ax^2 \quad\quad - \quad ax \quad\quad + a^2 + 2a \\
\quad\quad\quad + 2x^2 \quad\quad + 2x \quad\quad - 2a - 4 \\ \hline
-x^4 \quad\quad + 5x^2 - 2ax \quad\quad + a^2 \quad\quad - 4
\end{array}$$

38.
$$\begin{array}{r}
2x-3 \quad\quad\quad\quad\quad \\
3x^2-5x \quad\quad\quad\quad \\ \hline
6x^3- \quad 9x^2 \quad\quad\quad\quad \\
-10x^2+15x \quad\quad \\ \hline
6x^3-19x^2+15x \quad\quad \\
8x^3-25x^2-15x+36 \\ \hline
-2x^3+ \quad 6x^2+30x-36
\end{array} \quad (Ans.)$$

$$\begin{array}{r}
4x^2-3x \quad\quad\quad\quad \\
2x-7 \quad\quad\quad\quad \\ \hline
8x^3- \quad 6x^2 \quad\quad\quad\quad \\
-28x^2+21x \quad\quad \\ \hline
8x^3-34x^2+21x \quad\quad \\
9x^2-36x+36 \\ \hline
8x^3-25x^2-15x+36
\end{array}$$

$$\begin{array}{r}
3x-6 \quad\quad \\
3x-6 \quad\quad \\ \hline
9x^2-18x \quad\quad \\
-18x+36 \\ \hline
9x^2-36x+36
\end{array}$$

Page 79

1. $2(x+3) = 12.$

Removing parenthesis,

$$2x + 6 = 12.$$
$$2x = 6.$$
$$x = 3.$$

2. $5(x-1) = 30.$

Removing parenthesis,

$$5x - 5 = 30.$$
$$5x = 35.$$
$$\boldsymbol{x = 7.}$$

3. $4(x+6) = 16.$

$$4x + 24 = 16.$$
$$4x = -8.$$
$$\boldsymbol{x = -2.}$$

4. $2(3-x) + 1 = 2.$

$$6 - 2x + 1 = 2.$$
$$5 = 2x.$$
$$\boldsymbol{x = \tfrac{5}{2}.}$$

5. $$\begin{aligned} 5(x-3)+14&=4.\\ 5x-15+14&=4.\\ 5x&=5.\\ x&=1.\end{aligned}$$

6. $$\begin{aligned} 5(x-7)+8x&=4.\\ 5x-35+8x&=4.\\ 13x-35&=4.\\ 13x&=39.\\ x&=3.\end{aligned}$$

7. $$\begin{aligned} 4(4x-1)+3&=x.\\ 16x-4+3&=x.\\ 16x-1&=x.\\ 15x&=1.\\ x&=\tfrac{1}{15}.\end{aligned}$$

8. $$\begin{aligned} 16+2(4y-7)-12y&=0.\\ 16+8y-14-12y&=0.\\ 2-4y&=0.\\ 2&=4y.\\ y&=\tfrac{1}{2}.\end{aligned}$$

9. $$\begin{aligned} 5x-4(4-x)-11&=0.\\ 5x-16+4x-11&=0.\\ 9x&=27.\\ x&=3.\end{aligned}$$

10. $$\begin{aligned} 2(x+1)-3&=3(x-1).\\ 2x+2-3&=3x-3.\\ 2&=x.\end{aligned}$$

11. $$\begin{aligned} 4(2x-5)+15&=3(x+10).\\ 8x-20+15&=3x+30.\\ 5x&=35.\\ x&=7.\end{aligned}$$

12. $$\begin{aligned} 3(x+6)+8&=5(6+x).\\ 3x+18+8&=30+5x.\\ -2x&=4.\\ x&=-2.\end{aligned}$$

13. $$\begin{aligned} 7(x+5)&=4(x+8)+3.\\ 7x+35&=4x+32+3.\\ 3x&=0.\\ x&=0.\end{aligned}$$

14. $$\begin{aligned} 7(y-2)-2(3+y)&=0.\\ 7y-14-6-2y&=0.\\ 5y&=20.\\ y&=4.\end{aligned}$$

15. $$\begin{aligned} 9y-3(2y-4)&=2(5-y)+7.\\ 9y-6y+12&=10-2y+7.\\ 5y&=5.\\ y&=1.\end{aligned}$$

16. $$\begin{aligned} 2n-9(2n+4)&=2(n-9).\\ 2n-18n-36&=2n-18.\\ -18n&=18.\\ n&=-1.\end{aligned}$$

17. $$\begin{aligned} 7x-12-2(x-5)&=x-14.\\ 7x-12-2x+10&=x-14.\\ 4x&=-12.\\ x&=-3.\end{aligned}$$

18. $$\begin{aligned} 5(3h-1)-7h&=3(h+7)-1.\\ 15h-5-7h&=3h+21-1.\\ 5h&=25.\\ h&=5.\end{aligned}$$

20. $$\begin{aligned} (x+3)^2-(x+5)^2&=-40.\\ x^2+6x+9-x^2-10x-25&=-40.\\ 24&=4x.\\ x&=6.\end{aligned}$$

21. $$\begin{aligned} (x+2)^2-(x-4)^2+48&=0.\\ x^2+4x+4-x^2+8x-16+48&=0.\\ 12x&=-36.\\ x&=-3.\end{aligned}$$

22.
$$(2x-4)(3x-6)=6x^2+72.$$
$$6x^2-24x+24=6x^2+72.$$
$$-24x=48.$$
$$x=-2.$$

23.
$$(x+4)(x+6)=(x+18)(x+13).$$
$$x^2+10x+24=x^2+31x+234.$$
$$-21x=210.$$
$$x=-10.$$

24.
$$(h+2)(h+3)=(h-5)(h-2).$$
$$h^2+5h+6=h^2-7h+10.$$
$$12h=4.$$
$$h=\tfrac{1}{3}.$$

25. $(k-7)(5+k)-(k-5)(k+7)+5=0.$
$$(k^2-2k-35)-(k^2+2k-35)+5=0.$$
$$k^2-2k-35-k^2-2k+35+5=0.$$
$$-4k+5=0.$$
$$4k=5.$$
$$k=\tfrac{5}{4}.$$

Page 83

1. Let $x=$ the greater number.
Then $49-x=$ the less number,
and $2x=7+5(49-x)$.
Whence $x=36$; $49-x=13$.

2. Let $x=$ the less number.
Then $45-x=$ the greater number,
and $10x=300-5(45-x)$.
Whence $x=15$; $45-x=30$.

3. Let $x=$ the first number.
Then $15-x=$ the second number,
and $2x-10(15-x)=0$.
Whence $x=12\tfrac{1}{2}$; $15-x=2\tfrac{1}{2}$.

4. Let $x=$ the less part.
Then $75-x=$ the greater part,
and $29+4x=3(75-x)$.
Whence $x=28$; $75-x=47$.

5. Let x, $x+1$, be the integers.
Then $2x+4(x+1)=106$.
Whence $x=17$; $x+1=18$.

6. Let x, $x+1$, be the integers.

Then $4(x+1)-3x=17.$

Whence $x=13$; $x+1=14.$

7. Let $x=$ the first number.

Then $15-x=$ the second number,

and $7x=54+10(15-x).$

Whence $x=12$; $15-x=3.$

8. Let $x=$ the first number.

Then $15-x=$ the second number,

and $2x=240+5(15-x).$

Whence $x=45$; $15-x=-30.$

9. Let $x=$ the greater side.

Since the sum of the two sides = half the perimeter,

then $45-x=$ the less side,

and $5x+4(45-x)=207.$

Whence $x=27$; $45-x=18.$

10. Let x, $x+1$, be the integers.

Then $(x+1)^2-x^2=35.$

Whence $x=17$; $x+1=18.$

11. Let x, $x+1$, be the integers.

Then $(x+1)^2-x^2=71.$

Whence $x=35$; $x+1=36.$

12. Let x, $x+2$, be the integers.

Then $(x+2)^2-x^2=104.$

Whence $x=25$; $x+2=27.$

13. Let x, $x+1$, be the integers.

Then $x^2+(x+1)^2=2(x+1)^2-17.$

Whence $x=8$; $x+1=9.$

14. Let x, $x+2$, be the integers.

Then $(x+2)^2-x^2=48.$

Whence $x=11$; $x+2=13.$

15. Let x, $x+2$, be the integers.

Then $x(x+2)=(x+2)^2-42.$

Whence $x=19$; $x+2=21.$

16. Let x, $x+2$, be the integers.

Then $x(x+2)=44+x^2.$

Whence $x=22$; $x+2=24.$

17. Let $b =$ B's age in years.
Then $3b =$ A's age in years,
and $b + 10 =$ C's age in years.
But $b + 3b + b + 10 = 45.$
Whence $b = 7;\ 3b = 21;\ b + 10 = 17.$

18. Let $b =$ B's age in years, now.
Then $2b =$ A's age in years, now,
and $2b + 7 =$ C's age in years, now.
Then $b + 6$, $2b + 6$, $2b + 13$, will represent their respective ages 6 years hence.
Therefore $b + 6 + 2b + 6 + 2b + 13 = 85.$
Whence $b = 12;\ 2b = 24;\ 2b + 7 = 31.$

19. Let $b =$ B's age in years, now.
Then $b + 10 =$ A's age in years, now,
and $b - 6 =$ C's age in years, now.
Then $b - 6$, $b + 4$, $b - 12$, will represent their respective ages 6 years ago.
Therefore $b - 6 + b + 4 + b - 12 = 40.$
Whence $b = 18;\ b + 10 = 28;\ b - 6 = 12.$

20. Let $b =$ B's age in years, now.
Then $2b + 2 =$ A's age in years, now,
and $2b - 5 =$ C's age in years, now.
Then $b + 6$, $2b + 8$, $2b + 1$, will represent their respective ages 6 years hence.
Therefore $b + 6 + 2b + 8 + 2b + 1 = 75.$
Whence $b = 12;\ 2b + 2 = 26;\ 2b - 5 = 19.$

21. Let $x =$ the number of years.
Then $50 - x =$ A's age x years ago,
and $36 - x =$ B's age x years ago.
Therefore $50 - x = 3\,(36 - x).$
Whence $x = 29.$

22. Let $x =$ the number of years.
Then $19 + x =$ A's age x years hence,
and $54 + x =$ B's age x years hence.
If A is half as old as B, then B is twice as old as A.
That is, $54 + x = 2\,(19 + x).$
Whence $x = 16.$

23. Let $x =$ B's age in years, now.

Then $x + 30 =$ A's age in years, now.

Then $x + 20$, $x + 50$, will represent their respective ages 20 years hence.

Therefore $x + 50 = 2(x + 20)$.

Whence $x = 10$; $x + 30 = 40$.

24. Let $x =$ B's age in years, now.

Then $3x =$ A's age in years, now.

Then $x + 15$, $3x + 15$, will represent their respective ages 15 years hence.

Therefore $3x + 15 = 2(x + 15)$.

Whence $x = 15$; $3x = 45$.

25. Let $x =$ B's age in years, now.

Then $2x + 8 =$ A's age in years, now.

Then $x - 16$, $2x - 8$, will represent their respective ages 16 years ago.

Therefore $2x - 8 = 4(x - 16)$.

Whence $x = 28$; $2x + 8 = 64$.

27. Let $s =$ the side of the square in rods.

Then $s + 15 =$ the length of the rectangle in inches,

and $s - 10 =$ the width of the rectangle in inches.

Therefore $s^2 = (s + 15)(s - 10)$.

Whence $s = 30$ rods, the side of the square;

$s + 15 = 45$ rods, $s - 10 = 20$ rods, the sides of the rectangle.

28. Let $x =$ the breadth in feet.

Then $3x - 3 =$ the length in feet,

and $2x + 2(3x - 3) = 210$.

Whence $x = 27$; $3x - 3 = 78$.

29. Let $x =$ the breadth in feet.

Then $2x + 6 =$ the length in feet,

and $2x + 2(2x + 6) = 228$.

Whence $x = 36$; $2x + 6 = 78$.

30. Let $x =$ the breadth in feet.

Then $x + 20 =$ the length in feet.

But $2x + 2(x + 20) = 240$ (feet).

Whence $x = 50$; $x + 20 = 70$.

31. Let $x =$ the breadth in yards.

Then $2x - 5 =$ the length in yards,

and $2x + 2(2x - 5) = 260$ (yards).

Whence $x = 45$; $2x - 5 = 85$.

32. Let $x =$ the width of the flower bed in feet.

Then $24 - 2x$ and $30 - 2x =$ the sides of the grassplot,

and $2(24 - 2x) + 2(30 - 2x) =$ the perimeter of the grassplot.

But the perimeter $= \frac{1}{2}(24 + 24 + 30 + 30) = \frac{1}{2}(108) = 54.$

Therefore $2(24 - 2x) + 2(30 - 2x) = 54.$

Whence $x = 6\frac{3}{4}.$

34. Let $d =$ the number of dimes.

Then $35 - d =$ the number of quarters,

and $10d + 25(35 - d) = 650.$

Whence $d = 15;\ 35 - d = 20.$

35. Let $n =$ the number of nickels.

Then $30 - n =$ the number of dimes,

and $5n + 10(30 - n) = 260.$

Whence $n = 8;\ 30 - n = 22.$

36. Let $d =$ the number of dimes.

Then $d + 5 =$ the number of nickels,

and $2(2d + 5) =$ the number of quarters.

But $10d + 5(d + 5) + (25)(2)(2d + 5) = 1080.$

Whence $d = 7;\ d + 5 = 12;\ 2(2d + 5) = 38.$

Page 92

1.
$$\begin{array}{l|l} x^2 + 10x + 24 & x + 4 \\ \underline{x^2 + 4x} & \overline{x + 6} \\ \qquad 6x + 24 & \\ \qquad \underline{6x + 24} & \end{array}$$

Check.
$$\begin{array}{r} x + 4 \\ \underline{x + 6} \\ x^2 + 4x \\ \underline{+ 6x + 24} \\ x^2 + 10x + 24 \end{array}$$

2.
$$\begin{array}{l|l} x^2 - 2x - 15 & x + 3 \\ \underline{x^2 + 3x} & \overline{x - 5} \\ \quad - 5x - 15 & \\ \quad \underline{- 5x - 15} & \end{array}$$

Check.
$$\begin{array}{r} x + 3 \\ \underline{x - 5} \\ x^2 + 3x \\ \underline{- 5x - 15} \\ x^2 - 2x - 15 \end{array}$$

3.
$$\begin{array}{l|l} x^2 + x - 6 & x - 2 \\ \underline{x^2 - 2x} & \overline{x + 3} \\ \qquad 3x - 6 & \\ \qquad \underline{3x - 6} & \end{array}$$

Check.
$$\begin{array}{r} x - 2 \\ \underline{x + 3} \\ x^2 - 2x \\ \underline{+ 3x - 6} \\ x^2 + x - 6 \end{array}$$

4.
$$\begin{array}{l|l}
3x^2+5x+2 & x+1 \\ \cline{2-2}
\underline{3x^2+3x} & 3x+2 \\
\quad 2x+2 & \\
\quad \underline{2x+2} &
\end{array}$$

5.
$$\begin{array}{l|l}
2a^2-3a-2 & 2a+1 \\ \cline{2-2}
\underline{2a^2+\ \ a} & a-2 \\
\quad -4a-2 & \\
\quad \underline{-4a-2} &
\end{array}$$

6.
$$\begin{array}{l|l}
5x^2-22x+8 & x-4 \\ \cline{2-2}
\underline{5x^2-20x} & 5x-2 \\
\quad -\ 2x+8 & \\
\quad \underline{-\ 2x+8} &
\end{array}$$

7.
$$\begin{array}{l|l}
6x^2+19x-7 & 3x-1 \\ \cline{2-2}
\underline{6x^2-\ \ 2x} & 2x+7 \\
\quad 21x-7 & \\
\quad \underline{21x-7} &
\end{array}$$

8.
$$\begin{array}{l|l}
3x^2-\ \ ax-2a^2 & x-a \\ \cline{2-2}
\underline{3x^2-3ax} & 3x+2a \\
\quad 2ax-2a^2 & \\
\quad \underline{2ax-2a^2} &
\end{array}$$

9.
$$\begin{array}{l|l}
4x^2-8ax+3a^2 & 2x-3a \\ \cline{2-2}
\underline{4x^2-6ax} & 2x-\ a \\
\quad -2ax+3a^2 & \\
\quad \underline{-2ax+3a^2} &
\end{array}$$

10.
$$\begin{array}{l|l}
x^3-8x^2+\ 6x+12 & x-2 \\ \cline{2-2}
\underline{x^3-2x^2} & x^2-6x-6 \\
\quad -6x^2+\ 6x & \\
\quad \underline{-6x^2+12x} & \\
\qquad -\ 6x+12 & \\
\qquad \underline{-\ 6x+12} &
\end{array}$$

11.
$$\begin{array}{l|l}
8x^3-12x^2+6x-1 & 2x-1 \\ \cline{2-2}
\underline{8x^3-\ 4x^2} & 4x^2-4x+1 \\
\quad -\ 8x^2+6x & \\
\quad \underline{-\ 8x^2+4x} & \\
\qquad 2x-1 & \\
\qquad \underline{2x-1} &
\end{array}$$

12.
$$\begin{array}{l|l}
x^3\qquad -11x-6 & x+3 \\ \cline{2-2}
\underline{x^3+3x^2} & x^2-3x-2 \\
\quad -3x^2-11x & \\
\quad \underline{-3x^2-\ 9x} & \\
\qquad -\ 2x-6 & \\
\qquad \underline{-\ 2x-6} &
\end{array}$$

Check.
$$\begin{array}{r}
x^2-3x-2 \\
\underline{x\ +3\qquad\quad} \\
x^3-3x^2-\ 2x\qquad \\
\underline{+3x^2-\ 9x-6} \\
x^3\qquad\quad -11x-6
\end{array}$$

13.
$$\begin{array}{l|l}
x^3\qquad -14x-8 & x-4 \\ \cline{2-2}
\underline{x^3-4x^2} & x^2+4x+2 \\
\quad 4x^2-14x & \\
\quad \underline{4x^2-16x} & \\
\qquad 2x-8 & \\
\qquad \underline{2x-8} &
\end{array}$$

14.
$$\begin{array}{l|l}
x^3-2x^2-5x+6 & x-3 \\ \cline{2-2}
\underline{x^3-3x^2} & x^2+x-2 \\
\quad x^2-5x & \\
\quad \underline{x^2-3x} & \\
\qquad -2x+6 & \\
\qquad \underline{-2x+6} &
\end{array}$$

15.
$$\begin{array}{l|l}
x^3\qquad -5x+2 & x^2+2x-1 \\ \cline{2-2}
\underline{x^3+2x^2-\ \ x} & x-2 \\
\quad -2x^2-4x+2 & \\
\quad \underline{-2x^2-4x+2} &
\end{array}$$

16.
$$\begin{array}{l|l}
x^3\qquad -11x+6 & x^2+3x-2 \\ \cline{2-2}
\underline{x^3+3x^2-\ 2x} & x-3 \\
\quad -3x^2-\ 9x+6 & \\
\quad \underline{-3x^2-\ 9x+6} &
\end{array}$$

17.
$$
\begin{array}{rrrrr|l}
x^4 & & -\ 11x^2 & +\ 2x & +\ 12 & x^2+2x-4 \\
\cline{6-6}
x^4 & +\ 2x^3 & -\ 4x^2 & & & x^2-2x-3 \\
\cline{1-3}
 & -\ 2x^3 & -\ 7x^2 & +\ 2x & & \\
 & -\ 2x^3 & -\ 4x^2 & +\ 8x & & \\
\cline{2-5}
 & & -\ 3x^2 & -\ 6x & +\ 12 & \\
 & & -\ 3x^2 & -\ 6x & +\ 12 & \\
\cline{3-5}
\end{array}
$$

Check.
$$
\begin{array}{rrrrr}
x^2 & +\ 2x & -\ 4 & & \\
x^2 & -\ 2x & -\ 3 & & \\
\hline
x^4 & +\ 2x^3 & -\ 4x^2 & & \\
 & -\ 2x^3 & -\ 4x^2 & +\ 8x & \\
 & & -\ 3x^2 & -\ 6x & +\ 12 \\
\hline
x^4 & & -\ 11x^2 & +\ 2x & +\ 12
\end{array}
$$

18.
$$
\begin{array}{rrrr|l}
x^3 & & & +\ 8 & x+2 \\
\cline{5-5}
x^3 & +\ 2x^2 & & & x^2-2x+4 \\
\cline{1-2}
 & -\ 2x^2 & & & \\
 & -\ 2x^2 & -\ 4x & & \\
\cline{2-4}
 & & 4x & +\ 8 & \\
 & & 4x & +\ 8 & \\
\cline{3-4}
\end{array}
$$

Check.
$$
\begin{array}{rrrr}
x^2 & -\ 2x & +\ 4 & \\
x & +\ 2 & & \\
\hline
x^3 & -\ 2x^2 & +\ 4x & \\
 & +\ 2x^2 & -\ 4x & +\ 8 \\
\hline
x^3 & & & +\ 8
\end{array}
$$

19.
$$
\begin{array}{rrrr|l}
8x^3 & & & +\ 1 & 2x+1 \\
\cline{5-5}
8x^3 & +\ 4x^2 & & & 4x^2-2x+1 \\
\cline{1-2}
 & -\ 4x^2 & & & \\
 & -\ 4x^2 & -\ 2x & & \\
\cline{2-4}
 & & 2x & +\ 1 & \\
 & & 2x & +\ 1 & \\
\cline{3-4}
\end{array}
$$

20.
$$
\begin{array}{rrrr|l}
27x^3 & & & +\ 8a^3 & 3x+2a \\
\cline{5-5}
27x^3 & +\ 18x^2a & & & 9x^2-6ax+4a^2 \\
\cline{1-2}
 & -\ 18x^2a & & & \\
 & -\ 18x^2a & -\ 12a^2x & & \\
\cline{2-4}
 & & 12a^2x & +\ 8a^3 & \\
 & & 12a^2x & +\ 8a^3 & \\
\cline{3-4}
\end{array}
$$

21.
$$
\begin{array}{rrrr|l}
ax & +\ 3a & -\ bx & -\ 3b & x+3 \\
\cline{5-5}
ax & +\ 3a & & & a-b \\
\hline
 & & -\ bx & -\ 3b & \\
 & & -\ bx & -\ 3b & \\
\cline{3-4}
\end{array}
$$

22.
$$
\begin{array}{rrrr|l}
3ax & -\ ay & -\ 6bx & +\ 2by & -a+2b \\
\cline{5-5}
3ax & & -\ 6bx & & -3x+y \\
\hline
 & -\ ay & & +\ 2by & \\
 & -\ ay & & +\ 2by & \\
\cline{2-4}
\end{array}
$$

23.
$$
\begin{array}{l|l}
28\,ax + 9\,ny - 21\,ay - 12\,nx & -4\,x + 3\,y \\
\underline{28\,ax \qquad\quad - 21\,ay \qquad\qquad} & \overline{-7\,a + 3\,n} \\
\quad + 9\,ny \qquad\quad - 12\,nx & \\
\quad \underline{+ 9\,ny \qquad\quad - 12\,nx} &
\end{array}
$$

Page 93

1.
$$
\begin{array}{l|l}
6\,x^2 - 13\,x + 6 & 2\,x - 3 \\
\underline{6\,x^2 - 9\,x \qquad} & \overline{3\,x - 2} \\
\qquad - 4\,x + 6 & \\
\qquad \underline{- 2\,x + 6} &
\end{array}
$$

Check.
$$
\begin{array}{r}
2\,x - 3 \\
\underline{3\,x - 2} \\
6\,x^2 - 9\,x \phantom{{}+6} \\
\underline{- 4\,x + 6} \\
6\,x^2 - 13\,x + 6
\end{array}
$$

2.
$$
\begin{array}{l|l}
25\,x^4 + 30\,x^2 - 7 & 5\,x^2 + 7 \\
\underline{25\,x^4 + 35\,x^2 \qquad} & \overline{5\,x^2 - 1} \\
\qquad - 5\,x^2 - 7 & \\
\qquad \underline{- 5\,x^2 - 7} &
\end{array}
$$

3.
$$
\begin{array}{l|l}
6\,x^2 + 11\,x - 35 & 2\,x + 7 \\
\underline{6\,x^2 + 21\,x \qquad} & \overline{3\,x - 5} \\
\qquad - 10\,x - 35 & \\
\qquad \underline{- 10\,x - 35} &
\end{array}
$$

4.
$$
\begin{array}{l|l}
12\,a^2 + 19\,a - 21 & 4\,a - 3 \\
\underline{12\,a^2 - 9\,a \qquad} & \overline{3\,a + 7} \\
\qquad\quad 28\,a - 21 & \\
\qquad\quad \underline{28\,a - 21} &
\end{array}
$$

5.
$$
\begin{array}{l|l}
x^3 - 2\,x^2 + 4\,x - 8 & x - 2 \\
\underline{x^3 - 2\,x^2 \qquad\qquad} & \overline{x^2 + 4} \\
\qquad\qquad 4\,x - 8 & \\
\qquad\qquad \underline{4\,x - 8} &
\end{array}
$$

6.
$$
\begin{array}{l|l}
a^3 - 3\,a^2b + 3\,ab^2 - b^3 & a - b \\
\underline{a^3 - a^2b \qquad\qquad} & \overline{a^2 - 2\,ab + b^2} \\
\quad - 2\,a^2b & \\
\quad \underline{- 2\,a^2b + 2\,ab^2 \qquad} & \\
\qquad\qquad ab^2 - b^3 & \\
\qquad\qquad \underline{ab^2 - b^3} &
\end{array}
$$

7.
$$
\begin{array}{l|l}
x^3 - 15\,x^2 + 65\,x + 63 & x - 7 \\
\underline{x^3 - 7\,x^2 \qquad\qquad} & \overline{x^2 - 8\,x + 9 + \dfrac{126}{x - 7}} \\
\quad - 8\,x^2 + 65\,x & \\
\quad \underline{- 8\,x^2 + 56\,x \qquad} & \\
\qquad\qquad + 9\,x + 63 & \\
\qquad\qquad \underline{+ 9\,x - 63} & \\
\qquad\qquad\qquad + 126 &
\end{array}
$$

Check.
$$
\begin{array}{r}
x^2 - 8\,x + 9 \phantom{{}-63} \\
\underline{x \phantom{{}^2} - 7 \qquad\qquad\quad} \\
x^3 - 8\,x^2 + 9\,x \phantom{{}-63} \\
- 7\,x^2 + 56\,x - 63 \\
\underline{126} \\
x^3 - 15\,x^2 + 65\,x + 63
\end{array}
$$

8.
$$\begin{array}{l|l}
-25x^3+5x^2+5x-1 & 5x^2-1 \\ \cline{2-2}
\underline{-25x^3\qquad\quad+5x} & -5x+1 \\
\qquad\quad 5x^2\qquad-1 & \\
\qquad\quad \underline{5x^2\qquad-1} &
\end{array}$$

9.
$$\begin{array}{l|l}
2x^3-14x^2+14x+12 & 2x-4 \\ \cline{2-2}
\underline{2x^3-\ \ 4x^2} & x^2-5x-3 \\
\quad -10x^2+14x & \\
\quad \underline{-10x^2+20x} & \\
\qquad\qquad -\ \ 6x+12 & \\
\qquad\qquad \underline{-\ \ 6x+12} &
\end{array}$$

10.
$$\begin{array}{l|l}
6x^3\qquad\quad -18x+12 & 3x-3 \\ \cline{2-2}
\underline{6x^3-6x^2} & 2x^2+2x-4 \\
\qquad 6x^2-18x & \\
\qquad \underline{6x^2-\ \ 6x} & \\
\qquad\qquad -12x+12 & \\
\qquad\qquad \underline{-12x+12} &
\end{array}$$

11.
$$\begin{array}{l|l}
4x^3\qquad\quad +16x+12 & x^2+x-3 \\ \cline{2-2}
\underline{4x^3+4x^2-12x} & 4x-4+\dfrac{32x}{x^2+x-3} \\
\quad -4x^2+28x+12 & \\
\quad \underline{-4x^2-\ \ 4x+12} & \\
\qquad\qquad 32x &
\end{array}$$

Check.
$$\begin{array}{r}
x^2+x-3 \\
\underline{4x-4\qquad} \\
4x^3+4x^2-12x\qquad\quad \\
\underline{-4x^2-\ \ 4x+12} \\
4x^3\qquad\quad -16x+12 \\
\underline{+32x\qquad} \\
4x^3\qquad\quad +16x+12
\end{array}$$

12.
$$\begin{array}{l|l}
6x^3-2x^2+10x+18 & 3x^2-4x+9 \\ \cline{2-2}
\underline{6x^3-8x^2+18x} & 2x+2 \\
\qquad 6x^2-\ \ 8x+18 & \\
\qquad \underline{6x^2-\ \ 8x+18} &
\end{array}$$

13.
$$\begin{array}{l|l}
3a^3+28a^2+29a-140 & 3a-5 \\ \cline{2-2}
\underline{3a^3-\ \ 5a^2} & a^2+11a+28 \\
\qquad 33a^2 & \\
\qquad \underline{33a^2-55a} & \\
\qquad\qquad\quad 84a-140 & \\
\qquad\qquad\quad \underline{84a-140} &
\end{array}$$

14.
$$\begin{array}{l|l}
-6x^3+23x^2-37x-24 & 2x-3 \\ \cline{2-2}
\underline{-6x^3+\ \ 9x^2} & -3x^2+7x-8+\dfrac{-48}{2x-3} \\
\qquad 14x^2 & \\
\qquad \underline{14x^2-21x} & \\
\qquad\qquad -16x-24 & \\
\qquad\qquad \underline{-16x+24} & \\
\qquad\qquad\qquad -48 &
\end{array}$$

Check.
$$\begin{array}{r}
-3x^2+7x-8 \\
\underline{2x\ -3\qquad\quad} \\
-6x^3+14x^2-16x\qquad\quad \\
+\ \ 9x^2-21x+24 \\
\underline{-48} \\
-6x^3+23x^2-37x-24
\end{array}$$

15.
$$\begin{array}{l|l}
2x^2+10xy-4x-20y & 2x+10y \\ \cline{2-2}
\underline{2x^2+10xy} & x-2 \\
\qquad -4x & \\
\qquad \underline{-4x-20y} &
\end{array}$$

16.
$$\begin{array}{l|l}
3cx+6ac+8ax+4x^2 & 3c+4x \\ \cline{2-2}
\underline{3cx\qquad\qquad\quad +4x^2} & x+2a \\
\qquad 6ac+8ax & \\
\qquad \underline{6ac+8ax} &
\end{array}$$

17.
$$\begin{array}{l|l}
12a^3-53a^2+53a+8 & 4a^2-7a-1 \\ \cline{2-2}
\underline{12a^3-21a^2-\ \ 3a} & 3a-8 \\
\quad -32a^2+56a+8 & \\
\quad \underline{-32a^2+56a+8} &
\end{array}$$

18.
$$\begin{array}{l|l}
-15a^3+56a^2-99a+70 & 3a^2-7a+10 \\ \cline{2-2}
\underline{-15a^3+35a^2-50a} & -5a+7 \\
\qquad 21a^2-49a+70 & \\
\qquad \underline{21a^2-49a+70} &
\end{array}$$

19.
$$\begin{array}{l|l}
3x^4+11x^3-3x^2+17x-4 & -3x^2+x-4 \\ \cline{2-2}
\underline{3x^4-\quad x^3+4x^2} & -x^2-4x+1 \\
\qquad 12x^3-7x^2+17x & \\
\qquad \underline{12x^3-4x^2+16x} & \\
\qquad\qquad -3x^2+x-4 & \\
\qquad\qquad \underline{-3x^2+x-4} &
\end{array}$$

20.
$$\begin{array}{l|l}
6an+15xy-10ny-9ax & -2n+3x \\ \cline{2-2}
\underline{6an\qquad\qquad\qquad -9ax} & -3a+5y \\
\qquad 15xy-10ny & \\
\qquad \underline{15xy-10ny} &
\end{array}$$

21.
$$\begin{array}{l|l}
a^4 + 11a^3 + 23a^2 - 55a - 140 & a^2 - 5 \\ \cline{2-2}
\underline{a^4 \qquad\quad - 5a^2} & a^2 + 11a + 28 \\
\quad 11a^3 + 28a^2 & \\
\quad \underline{11a^3 \qquad\quad - 55a \qquad\;} & \\
\qquad\qquad 28a^2 \qquad - 140 & \\
\qquad\qquad \underline{28a^2 \qquad - 140} &
\end{array}$$

22.
$$\begin{array}{l|l}
25x^3 - 10x^2 + 36x + 72 & 5x - 6 \\ \cline{2-2}
\underline{25x^3 - 30x^2} & 5x^2 + 4x + 12 + \dfrac{144}{5x - 6} \\
\qquad 20x^2 & \\
\qquad \underline{20x^2 - 24x\quad} & \\
\qquad\qquad 60x + 72 & \\
\qquad\qquad \underline{60x - 72} & \\
\qquad\qquad\qquad 144 &
\end{array}$$

Check.
$$\begin{array}{r}
5x^2 + 4x + 12 \\
\underline{5x - 6 \qquad\qquad} \\
25x^3 + 20x^2 + 60x \qquad \\
- 30x^2 - 24x - 72 \\
\underline{+ 144} \\
25x^3 - 10x^2 + 36x + 72
\end{array}$$

23.
$$\begin{array}{l|l}
a^4 + 4a^3 + 6a^2 + 4a + 1 & a^2 + 2a + 1 \\ \cline{2-2}
\underline{a^4 + 2a^3 + a^2} & a^2 + 2a + 1 \\
\qquad 2a^3 + 5a^2 & \\
\qquad \underline{2a^3 + 4a^2 + 2a} & \\
\qquad\qquad a^2 + 2a + 1 & \\
\qquad\qquad \underline{a^2 + 2a + 1} &
\end{array}$$

24.
$$\begin{array}{l|l}
x^4 - 4x^3 - 31x^2 + 40x + 21 & x^2 - 7x - 3 \\ \cline{2-2}
\underline{x^4 - 7x^3 - 3x^2} & x^2 + 3x - 7 \\
\qquad 3x^3 - 28x^2 & \\
\qquad \underline{3x^3 - 21x^2 - 9x} & \\
\qquad\qquad - 7x^2 + 49x + 21 & \\
\qquad\qquad \underline{- 7x^2 + 49x + 21} &
\end{array}$$

25.
$$\begin{array}{l|l}
a^4 - 8a^3 + 24a^2 - 32a + 16 & a^2 - 4a + 4 \\ \cline{2-2}
\underline{a^4 - 4a^3 + 4a^2} & a^2 - 4a + 4 \\
\quad - 4a^3 + 20a^2 & \\
\quad \underline{- 4a^3 + 16a^2 - 16a} & \\
\qquad\qquad 4a^2 - 16a + 16 & \\
\qquad\qquad \underline{4a^2 - 16a + 16} &
\end{array}$$

26.
$$\begin{array}{l|l}
10x^4 - 27x^3 - 42x^2 + 224x - 419 & 2x^2 - 5x + 9 \\ \cline{2-2}
\underline{10x^4 - 25x^3 + 45x^2} & 5x^2 - x - 46 + \dfrac{3x-5}{2x^2-5x+9} \\ \cline{2-2}
\quad - 2x^3 - 87x^2 & \\
\quad \underline{- 2x^3 + 5x^2 - 9x} & \\
\qquad - 92x^2 + 233x & \\
\qquad \underline{- 92x^2 + 230x - 414} & \\
\qquad\qquad 3x - 5 &
\end{array}$$

27.
$$\begin{array}{l|l}
ax - 2bx + 3cx - 6cy + 4by - 2ay & -x + 2y \\ \cline{2-2}
\underline{ax \qquad\qquad\qquad\qquad\qquad - 2ay} & -a + 2b - 3c \\ \cline{2-2}
\quad - 2bx + 3cx - 6cy + 4by & \\
\quad \underline{- 2bx \qquad\qquad + 4by} & \\
\qquad 3cx - 6cy & \\
\qquad \underline{3cx - 6cy} &
\end{array}$$

28.
$$\begin{array}{l|l}
x^4 \qquad + a^2x^2 \qquad\qquad + a^4 & x^2 - ax + a^2 \\ \cline{2-2}
\underline{x^4 - ax^3 + a^2x^2} & x^2 + ax + a^2 \\ \cline{2-2}
\quad ax^3 & \\
\quad \underline{ax^3 \qquad - a^2x^2 + a^3x} & \\
\qquad a^2x^2 - a^3x & \\
\qquad \underline{a^2x^2 - a^3x + a^4} &
\end{array}$$

29.
$$\begin{array}{l|l}
a^4 \qquad + 3a^2b^2 \qquad\qquad + 4b^4 & a^2 - ab + 2b^2 \\ \cline{2-2}
\underline{a^4 - a^3b + 2a^2b^2} & a^2 + ab + 2b^2 \\ \cline{2-2}
\quad a^3b + a^2b^2 & \\
\quad \underline{a^3b - a^2b^2 + 2ab^3} & \\
\qquad 2a^2b^2 - 2ab^3 & \\
\qquad \underline{2a^2b^2 - 2ab^3 + 4b^4} &
\end{array}$$

30.
$$\begin{array}{l|l}
x^6 \qquad\qquad - 4x^3y^3 \qquad\qquad + 8y^6 & x^2 + 2xy + 2y^2 \\ \cline{2-2}
\underline{x^6 + 2x^5y + 2x^4y^2} & x^4 - 2x^3y + 2x^2y^2 - 4xy^3 + 4y^4 \\ \cline{2-2}
\quad - 2x^5y - 2x^4y^2 - 4x^3y^3 & \\
\quad \underline{- 2x^5y - 4x^4y^2 - 4x^3y^3} & \\
\qquad 2x^4y^2 & \\
\qquad \underline{2x^4y^2 + 4x^3y^3 + 4x^2y^4} & \\
\qquad\quad - 4x^3y^3 - 4x^2y^4 & \\
\qquad\quad \underline{- 4x^3y^3 - 8x^2y^4 - 8xy^5} & \\
\qquad\qquad 4x^2y^4 + 8xy^5 & \\
\qquad\qquad \underline{4x^2y^4 + 8xy^5 + 8y^6} &
\end{array}$$

31.
$$
\begin{array}{l|l}
9a^4 \qquad + 29a^2b^2 \qquad + 49b^4 & 3a^2 + 4ab + 7b^2 \\
\cline{2-2}
\underline{9a^4 + 12a^3b + 21a^2b^2} & 3a^2 - 4ab + 8b^2 + \dfrac{-4ab^3 - 7b^4}{3a^2 + 4ab + 7b^2} \\
\quad -12a^3b + 8a^2b^2 & \\
\quad \underline{-12a^3b - 16a^2b^2 - 28ab^3} & \\
\qquad 24a^2b^2 + 28ab^3 + 49b^4 & \\
\qquad \underline{24a^2b^2 + 32ab^3 + 56b^4} & \\
\qquad \qquad -4ab^3 - 7b^4 &
\end{array}
$$

32.
$$
\begin{array}{l|l}
x^3 \qquad \qquad - y^3 & x - y \\
\cline{2-2}
\underline{x^3 - x^2y} & x^2 + xy + y^2 \\
\quad x^2y & \\
\quad \underline{x^2y - xy^2} & \\
\qquad xy^2 - y^3 & \\
\qquad \underline{xy^2 - y^3} &
\end{array}
$$

33.
$$
\begin{array}{l|l}
a^3 \qquad \qquad - 125b^3 & a - 5b \\
\cline{2-2}
\underline{a^3 - 5a^2b} & a^2 + 5ab + 25b^2 \\
\quad 5a^2b & \\
\quad \underline{5a^2b - 25ab^2} & \\
\qquad 25ab^2 - 125b^3 & \\
\qquad \underline{25ab^2 - 125b^3} &
\end{array}
$$

34.
$$
\begin{array}{l|l}
x^4 \qquad \qquad - 16 & x + 2 \\
\cline{2-2}
\underline{x^4 + 2x^3} & x^3 - 2x^2 + 4x - 8 \\
\quad -2x^3 & \\
\quad \underline{-2x^3 - 4x^2} & \\
\qquad 4x^2 & \\
\qquad \underline{4x^2 + 8x} & \\
\qquad \quad -8x - 16 & \\
\qquad \quad \underline{-8x - 16} &
\end{array}
$$

35.
$$
\begin{array}{l|l}
x^6 \qquad \qquad \qquad - 1 & x - 1 \\
\cline{2-2}
\underline{x^6 - x^5} & x^5 + x^4 + x^3 + x^2 + x + 1 \\
\quad x^5 & \\
\quad \underline{x^5 - x^4} & \\
\qquad x^4 & \\
\qquad \underline{x^4 - x^3} & \\
\qquad \quad x^3 & \\
\qquad \quad \underline{x^3 - x^2} & \\
\qquad \qquad x^2 & \\
\qquad \qquad \underline{x^2 - x} & \\
\qquad \qquad \quad x - 1 & \\
\qquad \qquad \quad \underline{x - 1} &
\end{array}
$$

36.

$$
\begin{array}{l|l}
a^6 \qquad\qquad\qquad + 343\,b^3 & a^2 + 7\,b \\ \cline{2-2}
\underline{a^6 + 7\,a^4b} & a^4 - 7\,a^2b + 49\,b^2 \\
\quad - 7\,a^4b & \\
\quad \underline{- 7\,a^4b - 49\,a^2b^2} & \\
\qquad\qquad 49\,a^2b^2 + 343\,b^3 & \\
\qquad\qquad \underline{49\,a^2b^2 + 343\,b^3} &
\end{array}
$$

37.

$$
\begin{array}{l|l}
8\,a^3 \qquad\qquad\qquad - 64\,x^9 & 2\,a - 4\,x^3 \\ \cline{2-2}
\underline{8\,a^3 - 16\,a^2x^3} & 4\,a^2 + 8\,ax^3 + 16\,x^6 \\
\qquad 16\,a^2x^3 & \\
\qquad \underline{16\,a^2x^3 - 32\,ax^6} & \\
\qquad\qquad 32\,ax^6 - 64\,x^9 & \\
\qquad\qquad \underline{32\,ax^6 - 64\,x^9} &
\end{array}
$$

38.

$$
\begin{array}{l|l}
x^5 \qquad\qquad\qquad\qquad + y^5 & x + y \\ \cline{2-2}
\underline{x^5 + x^4y} & x^4 - x^3y + x^2y^2 - xy^3 + y^4 \\
\quad - x^4y & \\
\quad \underline{- x^4y - x^3y^2} & \\
\qquad\quad x^3y^2 & \\
\qquad\quad \underline{x^3y^2 + x^2y^3} & \\
\qquad\qquad - x^2y^3 & \\
\qquad\qquad \underline{- x^2y^3 - xy^4} & \\
\qquad\qquad\qquad xy^4 + y^5 & \\
\qquad\qquad\qquad \underline{xy^4 + y^5} &
\end{array}
$$

39.

$$
\begin{array}{l|l}
x^5 \qquad\qquad\qquad\qquad + y^5 & x - y \\ \cline{2-2}
\underline{x^5 - x^4y} & x^4 + x^3y + x^2y^2 + xy^3 + y^4 + \dfrac{2\,y^5}{x - y} \\
\quad x^4y & \\
\quad \underline{x^4y - x^3y^2} & \\
\qquad\quad x^3y^2 & \\
\qquad\quad \underline{x^3y^2 - x^2y^3} & \\
\qquad\qquad x^2y^3 & \\
\qquad\qquad \underline{x^2y^3 - xy^4} & \\
\qquad\qquad\qquad xy^4 + y^5 & \\
\qquad\qquad\qquad \underline{xy^4 - y^5} & \\
\qquad\qquad\qquad\qquad 2\,y^5 &
\end{array}
$$

40.

$$
\begin{array}{l|l}
x^5 \qquad\qquad\qquad\qquad - y^5 & x + y \\
\hline
\underline{x^5 + x^4y} & x^4 - x^3y + x^2y^2 - xy^3 + y^4 + \dfrac{-2y^5}{x+y} \\
\quad - x^4y & \\
\quad \underline{- x^4y - x^3y^2} & \\
\qquad\qquad x^3y^2 & \\
\qquad\qquad \underline{x^3y^2 + x^2y^3} & \\
\qquad\qquad\qquad - x^2y^3 & \\
\qquad\qquad\qquad \underline{- x^2y^3 - xy^4} & \\
\qquad\qquad\qquad\qquad xy^4 - y^5 & \\
\qquad\qquad\qquad\qquad \underline{xy^4 + y^5} & \\
\qquad\qquad\qquad\qquad\quad - 2y^5 &
\end{array}
$$

41.

$$
\begin{array}{l|l}
x^5 \qquad\qquad\qquad\qquad - y^5 & x - y \\
\hline
\underline{x^5 - x^4y} & x^4 + x^3y + x^2y^2 + xy^3 + y^4 \\
\quad x^4y & \\
\quad \underline{x^4y - x^3y^2} & \\
\qquad\qquad x^3y^2 & \\
\qquad\qquad \underline{x^3y^2 - x^2y^3} & \\
\qquad\qquad\qquad x^2y^3 & \\
\qquad\qquad\qquad \underline{x^2y^3 - xy^4} & \\
\qquad\qquad\qquad\qquad xy^4 - y^5 & \\
\qquad\qquad\qquad\qquad \underline{xy^4 - y^5} &
\end{array}
$$

42.

$$
\begin{array}{l|l}
27x^6 \qquad\qquad\qquad + 8n^{12} & 3x^2 + 2n^4 \\
\hline
\underline{27x^6 + 18x^4n^4} & 9x^4 - 6x^2n^4 + 4n^8 \\
\quad - 18x^4n^4 & \\
\quad \underline{- 18x^4n^4 - 12x^2n^8} & \\
\qquad\qquad 12x^2n^8 + 8n^{12} & \\
\qquad\qquad \underline{12x^2n^8 + 8n^{12}} &
\end{array}
$$

43.

$$
\begin{array}{l|l}
y^5 \qquad\qquad - 5y^2 \qquad\qquad - 3000 & y - 5 \\
\hline
\underline{y^5 - 5y^4} & y^4 + 5y^3 + 25y^2 + 120y + 600 \\
\quad 5y^4 & \\
\quad \underline{5y^4 - 25y^3} & \\
\qquad 25y^3 & \\
\qquad \underline{25y^3 - 125y^2} & \\
\qquad\qquad 120y^2 & \\
\qquad\qquad \underline{120y^2 - 600y} & \\
\qquad\qquad\qquad 600y - 3000 & \\
\qquad\qquad\qquad \underline{600y - 3000} &
\end{array}
$$

Page 96

2. $bx + b = 4b.$
Transposing and combining,
$$bx = 3b.$$
Whence $x = 3.$

3. $x + a = a + b.$
Transposing and combining,
$$x = b.$$

4. $y - b = a - b.$
Transposing and combining,
$$y = a.$$

5. $3ax + 4a = 7a.$
Transposing and combining,
$$3ax = 3a.$$
Whence $x = 1.$

6. $cz + c^2 = 6c^2.$
Transposing and combining,
$$cz = 5c^2.$$
Whence $z = 5c.$

7. $ax + ab = ac.$
$$ax = ac - ab.$$
$$x = c - b.$$

8. $4bx - b^2 = 5b^2.$
$$4bx = 6b^2.$$
$$x = \tfrac{3}{2}b.$$

9. $a^2c^2 + 3cx = 7a^2c^2.$
$$3cx = 6a^2c^2.$$
$$x = 2a^2c.$$

10. $5a^2x + 6a^2 = a^2.$
$$5a^2x = -5a^2.$$
$$x = -1.$$

11. $bc - cx = 4bc.$
$$-cx = 3bc.$$
$$x = -3b.$$

12. $5(b + x) = 10b.$
$$5b + 5x = 10b.$$
$$5x = 5b.$$
$$x = b.$$

13. $6(c - x) + 18c = 0.$
$$6c - 6x + 18c = 0.$$
$$24c = 6x.$$
$$x = 4c.$$

14. $bx - (b + c) = 5b - c.$
$$bx - b - c = 5b - c.$$
$$bx = 6b.$$
$$x = 6.$$

15. $x + 2(b - c) = 4c + 2b.$
$$x + 2b - 2c = 4c + 2b.$$
$$x = 6c.$$

16. $3ax - ab = 2ax - ac.$
$$ax = ab - ac.$$
$$x = b - c.$$

17. $4cy - 3ac = 5ac + 2cy.$
$$2cy = 8ac.$$
$$y = 4a.$$

18. $3cy + 2bc = 6bc + 2cy - 3ac.$
$$cy = 4bc - 3ac.$$
$$y = 4b - 3a.$$

19. $4by - 7a^2b = 6ab^2 + 3by.$
$$by = 7a^2b + 6ab^2.$$
$$y = 7a^2 + 6ab.$$

21. $ax + bx = ac + bc.$
Writing with binomial coefficient,
$$(a + b)x = ac + bc.$$
Dividing, $x = c.$

22. $5ax + 4cx = 5ab + 4cb.$
Writing with binomial coefficient,
$$(5a + 4c)x = 5ab + 4cb.$$
Dividing, $x = b.$

23. $a^2x + 1 - a^4 - x = 0.$
Transposing, $a^2x - x = a^4 - 1.$
Writing with binomial coefficient,
$$(a^2 - 1)x = a^4 - 1.$$
Dividing, $x = a^2 + 1.$

24. $ax + 2ab = 2a^2 + bx.$
$ax - bx = 2a^2 - 2ab.$
$(a - b)x = 2a^2 - 2ab.$
$x = 2a.$

25. $ax - a^3 - 4 = 3a - x.$
$ax + x = a^3 + 3a + 4.$
$(a + 1)x = a^3 + 3a + 4.$
$x = a^2 - a + 4.$

26. $ax - ac + bc = 2ac - 5bc + 2bx.$
$ax - 2bx = 3ac - 6bc.$
$(a - 2b)x = 3ac - 6bc.$
$x = 3c.$

27. $4b^2c^2 + (a + bx)c = (a - bx)c.$
$4b^2c^2 + ac + bcx = ac - bcx.$
$2bcx = -4b^2c^2.$
$x = -2bc.$

28. $(x + a)(x + b) = x^2 + 2a^2 + 3ab.$
$x^2 + ax + bx + ab = x^2 + 2a^2 + 3ab.$
$ax + bx = 2a^2 + 2ab.$
$(a + b)x = 2a^2 + 2ab.$
$x = 2a.$

29. $15(x - a) - 6(x + a) = 3(5a - 3x).$
$15x - 15a - 6x - 6a = 15a - 9x.$
$18x = 36a.$
$x = 2a.$

Page 102

1. Let $x =$ number of hours.
Then $6x =$ distance by A in miles.
and $9x =$ distance by B in miles.
But $6x + 9x = 90.$
Whence $x = 6.$

2. Let $x =$ B's rate in miles per hour.
Then $3x =$ A's rate in miles per hour.
Also $3x \cdot 6 = 18x =$ distance by A in miles,
and $6x =$ distance by B in miles.
But $18x + 6x = 120.$
Whence $x = 5;\ 3x = 15.$

3. Let $x =$ B's rate in miles per hour.
Then $x + 4 =$ A's rate in miles per hour.
Also $8x =$ distance by B in miles,
and $8(x + 4) =$ distance by A in miles.
But $8x + 8(x + 4) = 144.$
Whence $x = 7;\ x + 4 = 11.$

4. Let $x =$ B's rate in miles per hour.
Then $x - 3 =$ A's rate in miles per hour.
Also $9x =$ distance by B in miles,
and $9(x - 3) =$ distance by A in miles.
But $9x + 9(x - 3) = 189.$
Whence $x = 12;\ x - 3 = 9.$

5. Let $x =$ A's rate in miles per hour.
Then $x - 4 =$ B's rate in miles per hour.
But $x - 4 = \frac{2}{3}x$.
Transposing, $\frac{1}{3}x = 4$.
Dividing by $\frac{1}{3}$, $x = 12$; $x - 4 = 8$.
Now let $y =$ number of hours.
Then $12y =$ distance by A in miles,
and $8y =$ distance by B in miles.
But $12y + 8y = 180$.
Whence $y = 9$.

6. Let $x =$ A's rate in miles per hour.
Then $2x =$ B's rate in miles per hour.
Also $2x =$ distance in miles by A in 2 hours,
and $10x =$ distance in miles by B in 5 hours.
But $2x + 10x = 144$.
Whence $x = 12$; $2x = 24$ (their respective rates),
and $2x = 24$; $10x = 120$ (their respective distances).

7. Let $x =$ A's rate in miles per hour.
Then $x - 4 =$ B's rate in miles per hour.
Also $6x =$ distance in miles by A in 6 hours,
and $9(x - 4) =$ distance in miles by B in 9 hours.
But $6x = 9(x - 4)$.
Whence $x = 12$; $x - 4 = 8$,
and $6x = 72$, A's distance,
$9(x - 4) = 72$, B's distance.

8. Let $x =$ common rate of A and B in miles per hour.
Then $8x =$ distance in miles by each.
But $8x + 8x = 192$.
Whence $x = 12$.

9. Let $x =$ B's rate in miles per hour.
Then $x - 2 =$ A's rate in miles per hour.
Also $12x =$ distance by B in miles,
and $12(x - 2) =$ distance by A in miles.
But $12x + 12(x - 2) = 192$.
Whence $x = 9$; $x - 2 = 7$.

10. Let $x =$ A's rate in miles per hour.
Then $3x =$ B's rate in miles per hour.
Also $12x =$ distance by A in miles,
and $36x =$ distance by B in miles.
But $12x + 36x = 192.$
Whence $x = 4;\ 3x = 12.$

11. Let $x =$ distance by B in miles.
Then $192 - x =$ distance by A in miles.
But $x - 42 = 192 - x.$
Whence $x = 117;\ 192 - x = 75.$
Also B's rate $= \frac{117}{9}$, or 13 miles per hour,
and A's rate $= \frac{75}{9}$, or $8\frac{1}{3}$ miles per hour.

12. Let $x =$ A's rate in miles per hour.
Then $x + 4 =$ B's rate in miles per hour.
Also $6x =$ distance by A in miles,
and $6(x + 4) =$ distance by B in miles.
But $6x + 6(x + 4) = 192.$
Whence $x = 14;\ x + 4 = 18.$

13. Let $x =$ B's rate in miles per hour.
Then $x + 6 =$ A's rate in miles per hour.
Also $12x =$ distance by B in miles,
and $12(x + 6) =$ distance by A in miles.
But $12x + 12(x + 6) = 192.$
Whence $x = 5;\ x + 6 = 11.$

14. Since B goes twice as far as A in the same time, B's rate is double A's rate.
Let $x =$ A's rate in miles per hour,
and $2x =$ B's rate in miles per hour.
Then $x + 4 = 2x.$
Whence $x = 4;\ 2x = 8.$ Thus they approached each other at the rate of 12 miles per hour;
hence $144 \div 12 = 12$, the number of hours they traveled.

15. Let $x =$ the number of hours A travels.
Then $x - 4 =$ the number of hours B travels.
Also $6x =$ rate $\times$ time $=$ distance by A in miles,
and $9(x - 4) =$ rate $\times$ time $=$ distance by B in miles.
Therefore $6x + 9(x - 4) = 144.$
Whence $x = 12$ hours, A's time and the total time elapsed.

16. Let $x =$ total time in hours.
Then $x - 3 =$ number of hours A travels,
and $x - 5 =$ number of hours B travels.
Then $16(x - 3) =$ distance by A in miles,
and $8(x - 5) =$ distance by B in miles.
But $16(x - 3) + 8(x - 5) = 144$.
Whence $x = 9\frac{2}{3}$.

17. Let $x =$ the number of hours.
Then $45x =$ distance in miles by passenger train,
and $12x =$ distance in miles by freight train.
But $45x + 12x = 285$.
Whence $x = 5$.

18. Let $x =$ A's time in hours.
Then $x - 6 =$ B's time in hours.
Also $4x =$ distance by A in miles,
and $6(x - 6) =$ distance by B in miles.
But $4x = 6(x - 6)$.
Whence $x = 18$.

19. Let $x =$ first cyclist's running time.
Then $x - 2 =$ second cyclist's running time.
Also $10x =$ distance in miles by first cyclist,
and $12(x - 2) =$ distance in miles by second cyclist.
But $10x + 12(x - 2) = 108$.
Whence $x = 6$; $10x = 60$; $12(x - 2) = 48$.

20. Let $x =$ running time of passenger train in hours.
Then $x + 2 =$ running time of freight train in hours.
Also $42x =$ distance by passenger train in miles,
and $24(x + 2) =$ distance by freight train in miles.
But $42x + 24(x + 2) = 246$.
Whence $x = 3$.

21. Let $x =$ second messenger's rate in miles per hour.
Then $8x =$ distance in miles by second messenger.

Since the first messenger will have traveled for 10 hours when overtaken, he will have gone $10 \times 8 = 80$ miles.

But both messengers must go the same distance.

Therefore $8x = 80$.
Whence $x = 10$.

22. Let $x =$ distance in miles on trolley.

Then $x - 1 =$ distance in miles on foot.

Also $\dfrac{d}{r} = \dfrac{x}{10} = \dfrac{1}{10}x$, number of hours on trolley,

and $\dfrac{d}{r} = \dfrac{x-1}{3} = \dfrac{x}{3} - \dfrac{1}{3} = \dfrac{1}{3}x - \dfrac{1}{3}$, number of hours walked.

But $\frac{1}{10}x + \frac{1}{3}x - \frac{1}{3} = 4.$

Combining and transposing,

$$\tfrac{13}{30}x = \tfrac{13}{3},$$

or $x = 10.$

Therefore $\dfrac{x}{10} = 1$, number of hours on trolley.

23. Let $x =$ average velocity of bullet in feet per second.

Then since the time necessary for the return of the sound is

$$\frac{d}{r} = \frac{1650}{1100} = \frac{3}{2} \text{ seconds,}$$

$$\tfrac{5}{2} - \tfrac{3}{2} = 1 \text{ second, time of bullet's flight.}$$

Therefore $x = \dfrac{d}{t} = \dfrac{1650}{1}$, or 1650 feet per second.

24. Let $x =$ average velocity of bullet in feet per second.

Then, as in Ex. 23,

$$\tfrac{825}{1100} = \tfrac{3}{4}, \text{ number of seconds for sound to return,}$$

and $\frac{7}{4} - \frac{3}{4} = 1$ second, time of bullet's flight.

Therefore $x = \dfrac{d}{t} = \dfrac{825}{1}$, or 825 feet per second.

Page 106

1. $(2c - 3d)^2 = 4c^2 - 12cd + 9d^2.$
2. $(5x - 2y)^2 = 25x^2 - 20xy + 4y^2.$
3. $(3a - 2b^2)^2 = 9a^2 - 12ab^2 + 4b^4.$
4. $(4x + 5y^3)^2 = 16x^2 + 40xy^3 + 25y^6.$
5. $(7x^2 + 2y)^2 = 49x^4 + 28x^2y + 4y^2.$
6. $(5x^2 - 3y)^2 = 25x^4 - 30x^2y + 9y^2.$
7. $(9a^2 + 2b^2)^2 = 81a^4 + 36a^2b^2 + 4b^4.$
8. $(11a^2b + 2)^2 = 121a^4b^2 + 44a^2b + 4.$
9. $(12ab^2 - 3c)^2 = 144a^2b^4 - 72ab^2c + 9c^2.$
10. $(5a^2b - 2a^2)^2 = 25a^4b^2 - 20a^4b + 4a^4.$
11. $(-2a^2c^5 + 3cd^2)^2 = 4a^4c^{10} - 12a^2c^6d^2 + 9c^2d^4.$

12. $(-4xy^2 - 3xz^2)^2 = 16x^2y^4 + 24x^2y^2z^2 + 9x^2z^4.$
14. Change $-mn$ to $-2mn$.
16. Change $+12x$ to $-12x$ or change -6 to $+6$.
18. Change $-x^2$ to $+x^2$.
19. Change -64 to $+64$.
20. Change -25 to $+25$.
21. Change $-6x$ to $+6x$ or change $+1$ in the denominator to -1.
22. Change $+12ab$ to $-12ab$ or change $-3b$ to $+3b$.

Page 110

1. $\dfrac{x^2+5x+6}{x+3} = x+2.$
2. $\dfrac{a^2+7a+10}{a+2} = a+5.$
3. $\dfrac{a^2+11a+10}{a+10} = a+1.$
4. $\dfrac{b^2+7b+6}{b+1} = b+6.$
5. $\dfrac{c^2-6c+8}{c-2} = c-4.$
6. $\dfrac{c^2-9c+14}{c-7} = c-2.$
7. $\dfrac{x^2-3x+2}{x-2} = x-1.$
8. $\dfrac{x^2-4x-12}{x+2} = x-6.$
9. $\dfrac{a^2-3a-28}{a-7} = a+4.$
10. $\dfrac{a^2-3ab+2b^2}{a-b} = a-2b.$
11. $\dfrac{x^2-xy-6y^2}{x-3y} = x+2y.$
12. $\dfrac{m^2-4mn-21n^2}{m+3n} = m-7n.$
13. $\dfrac{r^2-7rs-18s^2}{r-9s} = r+2s.$
14. $\dfrac{r^2-rs-90s^2}{r-10s} = r+9s.$

Page 111

1. $(2a+1)(a+3) = 2a^2+7a+3.$
2. $(3a+2)(a+5) = 3a^2+17a+10.$
3. $(x+4)(2x+3) = 2x^2+11x+12.$
4. $(2x+3)(x+7) = 2x^2+17x+21.$
5. $(2c+3)(c+8) = 2c^2+19c+24.$
6. $(4c+3)(4c+3) = 16c^2+24c+9.$
7. $(5m+2)(5m+2) = 25m^2+20m+4.$
8. $(4m-1)(2m+3) = 8m^2+10m-3.$
9. $(5n-3)(3n-5) = 15n^2-34n+15.$
10. $(2n+7)(3n-2) = 6n^2+17n-14.$
11. $(7h+3)(4h-5) = 28h^2-23h-15.$
12. $(5h-4)(5h-4) = 25h^2-40h+16.$

13. $(5k-6)(5k-6)=25k^2-60k+36$.
14. $(3+m)(4+2m)=12+10m+2m^2$.
15. $(7-n)(7-3n)=49-28n+3n^2$.
16. $(2a-b)(3a+2b)=6a^2+ab-2b^2$.
17. $(5a+9c)(2a-9c)=10a^2-27ac-81c^2$.
18. $(2x^2-7y)(3x^2-2y)=6x^4-25x^2y+14y^2$.
19. $(c^2-5d)(5c^2+2d)=5c^4-23c^2d-10d^2$.
20. $(4c+9d^2)(2c-5d^2)=8c^2-2cd^2-45d^4$.

Page 117

1. $3ax^2+18a=3a(x^2+6)$.
2. $8x^3+8x=8x(x^2+1)$.
3. $2x^4-6x^2=2x^2(x^2-3)$.
4. $a^3b^2+a^3b^3=a^3b^2(1+b)$.
5. $3r^2s-27r^2=3r^2(s-9)$.
6. $10x^3-4x^7=2x^3(5-2x^4)$.
7. $18x^2+27+9x=9(2x^2+3+x)$.
8. $x^3-x^2+x^4=x^2(x-1+x^2)$.
9. $2c^5-18c+2c^3=2c(c^4-9+c^2)$.
10. $4x^2-8ax+20x=4x(x-2a+5)$.
11. $5a^2+10a^4-25a^3=5a^2(1+2a^2-5a)$.
12. $3y^4+6y^3c-3y^3=3y^3(y+2c-1)$.
13. $a^2+a^3-a^4+2a=a(a+a^2-a^3+2)$.
14. $5r^2+10r+15r^3=5r(r+2+3r^2)$.
15. $6a^5-12a^4+6a-18a^6=6a(a^4-2a^3+1-3a^5)$.
16. $16m^3-32m^2n+24m^2n^2=8m^2(2m-4n+3n^2)$.
17. $-a^2b-2ab^2+a^2b^2=ab(ab-a-2b)$.
18. $m+n$. 19. $b+c$. 20. $r+s+t$. 21. $h-k-l$. 22. $2-b+a$.

Page 119

1. $2a(a-x)+3(x-a)=2a(a-x)-3(a-x)=(a-x)(2a-3)$.
2. $7m(m-2n)-2(2n-m)=7m(m-2n)+2(m-2n)$
$=(m-2n)(7m+2)$.
3. $3h(5h-k)-k(k-5h)-hk(5h-k)$
$=3h(5h-k)+k(5h-k)-hk(5h-k)$
$=(5h-k)(3h+k-hk)$.
4. $7x(2x-5y)-2y(2x-5y)+(5y-2x)$
$=7x(2x-5y)-2y(2x-5y)-(2x-5y)$
$=(2x-5y)(7x-2y-1)$.
5. $(3a-2x)+9a(-2x+3a)+x(2x-3a)$
$=(3a-2x)+9a(3a-2x)-x(3a-2x)$
$=(3a-2x)(1+9a-x)$.
6. $ah+bh+ak+bk=h(a+b)+k(a+b)=(a+b)(h+k)$.
7. $mr+nr+ms+ns=r(m+n)+s(m+n)=(m+n)(r+s)$.

8. $2ac + bc + 2ad + bd = c(2a + b) + d(2a + b) = (2a + b)(c + d)$.

9. $3ac + cx + 9ay + 3xy = c(3a + x) + 3y(3a + x)$
$= (3a + x)(c + 3y)$.

10. $6ax + 15bx + 4ay + 10by = 3x(2a + 5b) + 2y(2a + 5b)$
$= (2a + 5b)(3x + 2y)$.

11. $2a^3 + a^2x + 2ax + x^2 = a^2(2a + x) + x(2a + x) = (2a + x)(a^2 + x)$.

12. $6h^3 + 4h^2k + 3hk^2 + 2k^3 = 2h^2(3h + 2k) + k^2(3h + 2k)$
$= (3h + 2k)(2h^2 + k^2)$.

13. $mh - nh + mk - nk = h(m - n) + k(m - n) = (m - n)(h + k)$.

14. $2ax - 6ay + bx - 3by = 2a(x - 3y) + b(x - 3y)$
$= (x - 3y)(2a + b)$.

15. $ar - cr + as - cs = r(a - c) + s(a - c) = (a - c)(r + s)$.

16. $mr - nr + ms - ns = r(m - n) + s(m - n) = (m - n)(r + s)$.

17. $2hx - 2kx + hy - ky = 2x(h - k) + y(h - k) = (h - k)(2x + y)$.

18. $3x^3 - 3x^2y + xy - y^2 = 3x^2(x - y) + y(x - y) = (x - y)(3x^2 + y)$.

19. $2a^3 - 6a^2b^2 + ab^3 - 3b^5 = 2a^2(a - 3b^2) + b^3(a - 3b^2)$
$= (a - 3b^2)(2a^2 + b^3)$.

20. $5r^3 - 2rs^2 + 10r^2s - 4s^3 = r(5r^2 - 2s^2) + 2s(5r^2 - 2s^2)$
$= (5r^2 - 2s^2)(r + 2s)$.

21. $20x^3 - 5x^2y^2 + 4xy - y^3 = 5x^2(4x - y^2) + y(4x - y^2)$
$= (4x - y^2)(5x^2 + y)$.

22. $6ac - 3c + 2ad - d = 3c(2a - 1) + d(2a - 1) = (2a - 1)(3c + d)$.

24. $ac - bc - ad + bd = c(a - b) - d(a - b) = (a - b)(c - d)$.

25. $ac - ad - bc + bd = a(c - d) - b(c - d) = (c - d)(a - b)$.

26. $2am - 6bm - 3an + 9bn = 2m(a - 3b) - 3n(a - 3b)$
$= (a - 3b)(2m - 3n)$.

27. $3hx - 15kx - hy + 5ky = 3x(h - 5k) - y(h - 5k)$
$= (h - 5k)(3x - y)$.

28. $x^3 - 3xy - 2x^2y^3 + 6y^4 = x(x^2 - 3y) - 2y^3(x^2 - 3y)$
$= (x^2 - 3y)(x - 2y^3)$.

29. $2a^3 - 6a^2b^2 - ab^2 + 3b^4 = 2a^2(a - 3b^2) - b^2(a - 3b^2)$
$= (a - 3b^2)(2a^2 - b^2)$.

30. $10a^4 - 25a^3b^3 + 5b^4 - 2ab = 5a^3(2a - 5b^3) - b(2a - 5b^3)$
$= (2a - 5b^3)(5a^3 - b)$.

31. $14x^4 - 35ax^2 + 10a - 4x^2 = 7x^2(2x^2 - 5a) - 2(2x^2 - 5a)$
$= (2x^2 - 5a)(7x^2 - 2)$.

32. $30x^5 - 10x^2 + 1 - 3x^3 = 10x^2(3x^3 - 1) - (3x^3 - 1)$
$= (3x^3 - 1)(10x^2 - 1)$.

33. $ab^4 - 2b^3 - 5ab + 10 = b^3(ab - 2) - 5(ab - 2)$
$= (ab - 2)(b^3 - 5)$.

34. $$x(a+b) = r(a+b) + s(a+b).$$
$$x = r + s.$$

35. $$mx + nx = mr + nr + ms + ns.$$
$$x(m+n) = r(m+n) + s(m+n).$$
$$x = r + s.$$

36. $$ay + by = ac + bc - ab - b^2.$$
$$y(a+b) = c(a+b) - b(a+b).$$
$$y = c - b.$$

37. $$kz - lz = hk - hl - k^2 + lk.$$
$$z(k-l) = h(k-l) - k(k-l).$$
$$z = h - k.$$

38. $$cy + ad - ae = dc - ce + ay.$$
$$cy - ay = dc - ad - ce + ae.$$
$$y(c-a) = d(c-a) - e(c-a).$$
$$y = d - e.$$

39. $$2k + hl - mk = kl + 2h - mh.$$
$$mh - mk = 2h - 2k - hl + kl.$$
$$m(h-k) = 2(h-k) - l(h-k).$$
$$m = 2 - l.$$

Page 123 (First set)

1. $9r^2 + 6rs + s^2 = (3r+s)^2.$
2. $4a^2 + 4ax + x^2 = (2a+x)^2.$
3. $9x^2 - 6xy + y^2 = (3x-y)^2.$
4. $16x^2 - 8xm + m^2 = (4x-m)^2.$
5. $m^2 + 10mn + 25n^2 = (m+5n)^2.$
6. $25y^2 - 10yx + x^2 = (5y-x)^2.$
7. $81a^2 + 18ab + b^2 = (9a+b)^2.$
8. $m^2 - 26mn + 169n^2 = (m-13n)^2.$
9. $9h^2 - 60hk + 100k^2 = (3h-10k)^2.$
10. $121a^2 - 44ab + 4b^2 = (11a-2b)^2.$
11. $81x^2 + 126xy + 49y^2 = (9x+7y)^2.$
12. $36x^2 + 25y^2 - 60xy = (6x-5y)^2.$
13. $169a^2 + 9b^2 - 78ab = (13a-3b)^2.$
14. $49d^2 + 210cd + 225c^2 = (7d+15c)^2.$
15. $196a^2 - 140ab + 25b^2 = (14a-5b)^2.$
16. $9a^2b^2 - 12ab + 4 = (3ab-2)^2.$
17. $16c^2x^2 + 56cx + 49 = (4cx+7)^2.$
18. $9m^2n^2 - 24mnp + 16p^2 = (3mn-4p)^2.$

Page 123 (Second set)

1. $x^3 + 6x^2 + 9x = x(x^2 + 6x + 9) = x(x + 3)^2$.
2. $2x^2 + 4x + 2 = 2(x^2 + 2x + 1) = 2(x + 1)^2$.
3. $x^3 - 10x^2 + 25x = x(x^2 - 10x + 25) = x(x - 5)^2$.
4. $a^3 + 2a^2b + ab^2 = a(a^2 + 2ab + b^2) = a(a + b)^2$.
5. $a^3 + 2a^2x + ax^2 = a(a^2 + 2ax + x^2) = a(a + x)^2$.
6. $2c^3 - 20c^2 + 50c = 2c(c^2 - 10c + 25) = 2c(c - 5)^2$.
7. $100x - 80x^2 + 16x^3 = 4x(25 - 20x + 4x^2) = 4x(5 - 2x)^2$.
8. $98c + 28c^2 + 2c^3 = 2c(49 + 14c + c^2) = 2c(7 + c)^2$.
9. $80r - 40r^2 + 5r^3 = 5r(16 - 8r + r^2) = 5r(4 - r)^2$.
10. $245x^5 - 140x^4 + 20x^3 = 5x^3(49x^2 - 28x + 4) = 5x^3(7x - 2)^2$.
11. $45m^2 - 60mn + 20n^2 = 5(9m^2 - 12mn + 4n^2) = 5(3m - 2n)^2$.
12. $2ax + 4ax^2 + 2ax^3 = 2ax(1 + 2x + x^2) = 2ax(1 + x)^2$.
13. $126c^2d^2 + 147c^3d + 27cd^3 = 3cd(42cd + 49c^2 + 9d^2) = 3cd(7c + 3d)^2$.
14. $2ac + 4cx + 2ay + 4xy = 2[c(a + 2x) + y(a + 2x)]$
$= 2(a + 2x)(c + y)$.
15. $2ax - 6bx + 2ay - 6by = 2[x(a - 3b) + y(a - 3b)]$
$= 2(a - 3b)(x + y)$.
16. $3mr - 6nr + 15ms - 30ns = 3(mr - 2nr + 5ms - 10ns)$
$= 3[r(m - 2n) + 5s(m - 2n)]$
$= 3(m - 2n)(r + 5s)$.
17. $12ab - 6bc - 6ax + 3cx = 3(4ab - 2bc - 2ax + cx)$
$= 3[2b(2a - c) - x(2a - c)]$
$= 3(2a - c)(2b - x)$.

18. $ax + cx = m(a + c) - n(a + c)$.
$x = m - n$.

19. $(a + b)x = a^2 + 2ab + b^2$.
$x = a + b$.

20. $mh - mk = h^2 - 2hk + k^2$.
$m(h - k) = (h - k)^2$.
$m = h - k$.

21. $nr - 2ns = r^2 - 4rs + 4s^2$.
$n(r - 2s) = (r - 2s)^2$.
$n = r - 2s$.

22. $ky - 4kl - l^2 = 4k^2 - ky - ly$.
$2ky + ly = 4k^2 + 4kl + l^2$.
$y(2k + l) = (2k + l)^2$.
$y = 2k + l$.

23. $acy - ad^2 - ac^2 = ady - 2acd$.
$acy - ady = ac^2 - 2acd + ad^2$.
$cy - dy = c^2 - 2cd + d^2$.
$y(c - d) = (c - d)^2$.
$y = c - d$.

24. $2ade - ae^2 = ad^2 - adz + aez$.
$adz - aez = ad^2 - 2ade + ae^2$.
$dz - ez = d^2 - 2de + e^2$.
$z(d - e) = (d - e)^2$.
$z = d - e$.

Page 125

1. $a^4 - b^4 = (a^2 + b^2)(a^2 - b^2) = (a^2 + b^2)(a + b)(a - b)$.
2. $x^4 - 1 = (x^2 + 1)(x^2 - 1) = (x^2 + 1)(x + 1)(x - 1)$.
3. $a^4 - 16 = (a^2 + 4)(a^2 - 4) = (a^2 + 4)(a + 2)(a - 2)$.
4. $r^4 - 81 = (r^2 + 9)(r^2 - 9) = (r^2 + 9)(r + 3)(r - 3)$.
5. $16a^4 - 1 = (4a^2 + 1)(4a^2 - 1) = (4a^2 + 1)(2a + 1)(2a - 1)$.
6. $81 - c^4 = (9 + c^2)(9 - c^2) = (9 + c^2)(3 + c)(3 - c)$.
7. $625x^4 - 1 = (25x^2 + 1)(25x^2 - 1) = (25x^2 + 1)(5x + 1)(5x - 1)$.
8. $625 - a^4 = (25 + a^2)(25 - a^2) = (25 + a^2)(5 + a)(5 - a)$.
9. $625x^4 - 16y^4 = (25x^2 + 4y^2)(25x^2 - 4y^2)$
   $= (25x^2 + 4y^2)(5x + 2y)(5x - 2y)$.
10. $a^{12} - c^{16} = (a^6 + c^8)(a^6 - c^8) = (a^6 + c^8)(a^3 + c^4)(a^3 - c^4)$.
11. $c^4d^4 - 1 = (c^2d^2 + 1)(c^2d^2 - 1) = (c^2d^2 + 1)(cd + 1)(cd - 1)$.
12. $c^8 - 81 = (c^4 + 9)(c^4 - 9) = (c^4 + 9)(c^2 + 3)(c^2 - 3)$.
13. $c^8 - d^8 = (c^4 + d^4)(c^4 - d^4) = (c^4 + d^4)(c^2 + d^2)(c + d)(c - d)$.
14. $a^4b^8 - 16 = (a^2b^4 + 4)(a^2b^4 - 4) = (a^2b^4 + 4)(ab^2 + 2)(ab^2 - 2)$.
15. $x^4y^8 - z^4 = (x^2y^4 + z^2)(x^2y^4 - z^2) = (x^2y^4 + z^2)(xy^2 + z)(xy^2 - z)$.
16. $(x + y)^2 - z^2 = (x + y + z)(x + y - z)$.
17. $(x - y)^2 - 25 = (x - y + 5)(x - y - 5)$.
18. $(2x + 5)^2 - 4y^2 = (2x + 5 + 2y)(2x + 5 - 2y)$.
19. $(3a - 7)^2 - x^2 = (3a - 7 + x)(3a - 7 - x)$.
20. $4(x - 9)^2 - 9a^2 = (2x - 18 + 3a)(2x - 18 - 3a)$.
21. $9(a - b)^2 - 25c^2 = (3a - 3b + 5c)(3a - 3b - 5c)$.
22. $25x^2(c + d)^2 - 9y^2 = (5xc + 5xd + 3y)(5xc + 5xd - 3y)$.
23. $4a^2 - (2c - d)^2 = (2a + 2c - d)(2a - 2c + d)$.
24. $25x^2 - (a - 3b)^2 = (5x + a - 3b)(5x - a + 3b)$.
25. $49a^2 - (3x + 2)^2 = (7a + 3x + 2)(7a - 3x - 2)$.
26. $81a^2 - 4(c^4 + 2)^2 = (9a + 2c^4 + 4)(9a - 2c^4 - 4)$.
27. $100 - a^2(b + c)^2 = (10 + ab + ac)(10 - ab - ac)$.
28. $121a^2 - 4b^2(c - 3d^2)^2$
    $= (11a + 2bc - 6bd^2)(11a - 2bc + 6bd^2)$.
29. $y^2z^2 - (y + z)^2 = (yz + y + z)(yz - y - z)$.
30. $c^2e^2 - (c - e)^2 = (ce + c - e)(ce - c + e)$.
31. $c^2d^2 - (c^2 - d)^2 = (cd + c^2 - d)(cd - c^2 + d)$.
32. $9a^2b^2 - 4(a - b)^2 = (3ab + 2a - 2b)(3ab - 2a + 2b)$.
33. $25a^2b^2 - 9(a + b)^2 = (5ab + 3a + 3b)(5ab - 3a - 3b)$.
34. $4c^2d^2 - (c - 2d)^2 = (2cd + c - 2d)(2cd - c + 2d)$.
35. $16c^2d^2 - (a^2 + x)^2 = (4cd + a^2 + x)(4cd - a^2 - x)$.
36. $25c^2d^2 - 3^2(c + d)^2 = (5cd + 3c + 3d)(5cd - 3c - 3d)$.
37. $49r^2s^2 - 9(r^2 - s^2)^2 = (7rs + 3r^2 - 3s^2)(7rs - 3r^2 + 3s^2)$.

Page 126

2. $x^2 + 2x + 1 - y^2 = (x+1)^2 - y^2 = (x+1+y)(x+1-y).$

3. $a^2 - 6a + 9 - b^2 = (a-3)^2 - b^2 = (a-3+b)(a-3-b).$

4. $c^2 - 10c + 25 - 4d^2 = (c-5)^2 - 4d^2 = (c-5+2d)(c-5-2d).$

5. $4a^2 + 4a + 1 - 9b^2 = (2a+1)^2 - 9b^2 = (2a+1+3b)(2a+1-3b).$

6. $16 + 24a + 9a^2 - 25b^2c^2 = (4+3a)^2 - 25b^2c^2$
$= (4+3a+5bc)(4+3a-5bc).$

7. $4a^2 - 12ab + 9b^2 - 9b^4 = (2a-3b)^2 - 9b^4$
$= (2a-3b+3b^2)(2a-3b-3b^2).$

8. $25x^2 + 4y^2 - 20xy - 16z^2 = (5x-2y)^2 - 16z^2$
$= (5x-2y+4z)(5x-2y-4z).$

9. $9x^2 + 25y^4 - 81z^4 - 30xy^2 = (3x-5y^2)^2 - 81z^4$
$= (3x-5y^2+9z^2)(3x-5y^2-9z^2).$

10. $60ab - 25c^4 + 9b^2 + 100a^2 = (3b+10a)^2 - 25c^4$
$= (3b+10a+5c^2)(3b+10a-5c^2).$

11. $1 - 14ab^2 + 49a^2b^4 - b^4 = (1-7ab^2)^2 - b^4$
$= (1-7ab^2+b^2)(1-7ab^2-b^2).$

12. $4 - 20ab^2c^3 + 25a^2b^4c^6 - 4c^8 = (2-5ab^2c^3)^2 - 4c^8$
$= (2-5ab^2c^3+2c^4)(2-5ab^2c^3-2c^4).$

13. $9x^2 - 30xy + 25y^2 - 16z^4 = (3x-5y)^2 - 16z^4$
$= (3x-5y+4z^2)(3x-5y-4z^2).$

14. $16x^6 - 8x^3y^2 + y^4 - z^4 = (4x^3-y^2)^2 - z^4$
$= (4x^3-y^2+z^2)(4x^3-y^2-z^2).$

16. $x^2 - 4y^2 + 4yz - z^2 = x^2 - (4y^2 - 4yz + z^2)$
$= x^2 - (2y-z)^2 = (x+2y-z)(x-2y+z).$

17. $x^2 - y^2 + 10yz - 25z^2 = x^2 - (y^2 - 10yz + 25z^2)$
$= x^2 - (y-5z)^2 = (x+y-5z)(x-y+5z).$

18. $c^2 - 4a^2 - 12ab - 9b^2 = c^2 - (4a^2 + 12ab + 9b^2)$
$= c^2 - (2a+3b)^2 = (c+2a+3b)(c-2a-3b).$

19. $4c^2 - a^2 + 10ab - 25b^2 = 4c^2 - (a^2 - 10ab + 25b^2)$
$= 4c^2 - (a-5b)^2$
$= (2c+a-5b)(2c-a+5b).$

20. $9d^2 - a^2 + 6ab - 9b^2 = 9d^2 - (a^2 - 6ab + 9b^2)$
$= 9d^2 - (a-3b)^2$
$= (3d+a-3b)(3d-a+3b).$

21. $16e^2 - 25m^2 + 10mn - n^2 = 16e^2 - (25m^2 - 10mn + n^2)$
$= 16e^2 - (5m-n)^2$
$= (4e+5m-n)(4e-5m+n).$

Page 127

1. $ax^2 - ay^2 = a(x^2 - y^2) = a(x + y)(x - y)$.
2. $a^2c - 4b^2c = c(a^2 - 4b^2) = c(a + 2b)(a - 2b)$.
3. $a^3 + 6a^2b + 9ab^2 = a(a^2 + 6ab + 9b^2) = a(a + 3b)^2$.
4. $c^6 + 2c^4 + c^2 = c^2(c^4 + 2c^2 + 1) = c^2(c^2 + 1)^2$.
5. $dx^2 - 25d^3 = d(x^2 - 25d^2) = d(x + 5d)(x - 5d)$.
6. $25e^4 - 30e^3 + 9e^2 = e^2(25e^2 - 30e + 9) = e^2(5e - 3)^2$.
7. $2x^3 - 18xy^2 = 2x(x^2 - 9y^2) = 2x(x + 3y)(x - 3y)$.
8. $r^7 + 2r^5 + r^3 = r^3(r^4 + 2r^2 + 1) = r^3(r^2 + 1)^2$.
9. $a^3b - 6a^2b^3 + 9ab^5 = ab(a^2 - 6ab^2 + 9b^4) = ab(a - 3b^2)^2$.
10. $5a^4c^2 - 45c^4 = 5c^2(a^4 - 9c^2) = 5c^2(a^2 + 3c)(a^2 - 3c)$.
11. $a^3x^3 - 2a^2x^2 + ax = ax(a^2x^2 - 2ax + 1) = ax(ax - 1)^2$.
12. $ax^5 - a^5x = ax(x^4 - a^4) = ax(x^2 + a^2)(x + a)(x - a)$.
13. $2x^5 - 162x = 2x(x^4 - 81) = 2x(x^2 + 9)(x + 3)(x - 3)$.
14. $x^5y - 16xy^5 = xy(x^4 - 16y^4) = xy(x^2 + 4y^2)(x + 2y)(x - 2y)$.
15. $5a^4 - 5 = 5(a^4 - 1) = 5(a^2 + 1)(a + 1)(a - 1)$.
16. $x^5 - xy^4 = x(x^4 - y^4) = x(x^2 + y^2)(x + y)(x - y)$.
17. $32x^4 - 1250 = 2(16x^4 - 625) = 2(4x^2 + 25)(2x + 5)(2x - 5)$.
18. $3z(x + y)^2 - 12z^3 = 3z[(x + y)^2 - 4z^2]$
$= 3z(x + y + 2z)(x + y - 2z)$.
19. $20x^2 + 20x^3 + 5x^4 + 5x^5 = 5x^2(4 + 4x + x^2 + x^3)$
$= 5x^2[4(1 + x) + x^2(1 + x)]$
$= 5x^2(1 + x)(4 + x^2)$.
20. $8r^4 - 8r^2(3s + t)^2 = 8r^2[r^2 - (3s + t)^2]$
$= 8r^2(r + 3s + t)(r - 3s - t)$.
21. $8a^4 - 12a^2bc - 18a^2b^2 - 2a^2c^2 = 2a^2(4a^2 - 6bc - 9b^2 - c^2)$
$= 2a^2[4a^2 - (9b^2 + 6bc + c^2)]$
$= 2a^2(2a + 3b + c)(2a - 3b - c)$.

22. $x(a + 2) = a^2 - 4$.
$x = a - 2$.

23. $b^2y + 5y = b^4 - 25$.
$y(b^2 + 5) = b^4 - 25$.
$y = b^2 - 5$.

24. $z(c^2 + 25)(c - 5) = c^4 - 625$.
$z(c^2 + 25)(c - 5) = (c^2 + 25)(c + 5)(c - 5)$.
$z = c + 5$.

25. $mh^2 - mhk = h^3 - hk^2$.
$mh - mk = h^2 - k^2$.
$m(h - k) = h^2 - k^2$.
$m = h + k$.

26. $acn + 2c - a^2c = 2cn - 2c$.
$an + 2 - a^2 = 2n - 2$.
$an - 2n = a^2 - 4$.
$n(a - 2) = a^2 - 4$.
$n = a + 2$.

Page 129 (First set)

1. $a^2 + 5a + 6 = (a+3)(a+2)$.
2. $c^2 + 7c + 12 = (c+4)(c+3)$.
3. $x^2 + 4x + 4 = (x+2)(x+2)$.
4. $x^2 - 8x + 12 = (x-2)(x-6)$.
5. $m^2 - 13m + 30 = (m-3)(m-10)$.
6. $h^2 - 16h + 15 = (h-1)(h-15)$.
7. $n^2 - 8n + 16 = (n-4)(n-4)$.
8. $k^2 - 17k + 30 = (k-2)(k-15)$.
9. $m^2 - 10m + 25 = (m-5)(m-5)$.
10. $r^2 - 5r - 14 = (r+2)(r-7)$.
11. $s^2 + 2s - 35 = (s+7)(s-5)$.
12. $s^2 - 6s + 9 = (s-3)(s-3)$.
13. $t^2 + t - 42 = (t+7)(t-6)$.
14. $x^2 + 5x - 14 = (x+7)(x-2)$.
15. $10a - 39 + a^2 = (a+13)(a-3)$.
16. $a^4 - 4a^2 - 5 = (a^2-5)(a^2+1)$.
17. $a^4 - 14a^2 + 49 = (a^2-7)(a^2-7)$.
19. $-x^2 - x + 12 = -1(x^2 + x - 12) = -1(x+4)(x-3) = (x+4)(3-x)$.
20. $-m^2 - 8m + 9 = -1(m^2 + 8m - 9) = -1(m+9)(m-1) = (m+9)(1-m)$.
21. $-2a + 63 - a^2 = -1(a^2 + 2a - 63) = -1(a+9)(a-7) = (a+9)(7-a)$.
22. $20 - m - m^2 = -1(m^2 + m - 20) = -1(m+5)(m-4) = (m+5)(4-m)$.
23. $24 + 2n - n^2 = -1(n^2 - 2n - 24) = -1(n-6)(n+4) = (6-n)(n+4)$.
24. $80 - 2r - r^2 = -1(r^2 + 2r - 80) = -1(r+10)(r-8) = (r+10)(8-r)$.
25. $x^2 - x - 72 = (x-9)(x+8)$.
26. $a^2 - a - 110 = (a-11)(a+10)$.
27. $a^2 + 12a + 36 = (a+6)(a+6)$.
28. $c^2 - 50 + 5c = c^2 + 5c - 50 = (c+10)(c-5)$.
29. $h^2 - 55 + 6h = h^2 + 6h - 55 = (h+11)(h-5)$.
30. $-4s - 21 + s^2 = s^2 - 4s - 21 = (s-7)(s+3)$.
31. $27 + 6x - x^2 = -1(x^2 - 6x - 27) = -1(x-9)(x+3) = (x+3)(9-x)$.
32. $x^2 + 2rx - 3r^2 = (x+3r)(x-r)$.
33. $m^2 + 2mn - 99n^2 = (m+11n)(m-9n)$.
34. $h^2 - 3hk - 130k^2 = (h-13k)(h+10k)$.
35. $k^2 - 4kl + 4l^2 = (k-2l)(k-2l)$.
36. $r^2 + 15rs - 100s^2 = (r+20s)(r-5s)$.
37. $5hk + k^2 - 36h^2 = k^2 + 5hk - 36h^2 = (k+9h)(k-4h)$.
38. $r^2s^2 - 3rst - 40t^2 = (rs-8t)(rs+5t)$.
39. $a^2b^2 + 8abc + 7c^2 = (ab+7c)(ab+c)$.
40. $33p^2 - m^2n^2 - 8mnp = -1(m^2n^2 + 8mnp - 33p^2)$
$= -1(mn+11p)(mn-3p)$
$= (mn+11p)(3p-mn)$.

Page 129 (Second set)

1. $x^4 + 7x^3 + 12x^2 = x^2(x^2 + 7x + 12) = x^2(x + 3)(x + 4).$
2. $a^4 - 12a^3 + 36a^2 = a^2(a^2 - 12a + 36) = a^2(a - 6)(a - 6).$
3. $2x^2 + 10x - 48 = 2(x^2 + 5x - 24) = 2(x + 8)(x - 3).$
4. $5c^3 + 15c^2 - 140c = 5c(c^2 + 3c - 28) = 5c(c + 7)(c - 4).$
5. $3m^4 + 66m^3 + 363m^2 = 3m^2(m^2 + 22m + 121) = 3m^2(m + 11)^2.$
6. $16a^3 - 4ab^2 = 4a(4a^2 - b^2) = 4a(2a + b)(2a - b).$
7. $20c^5 - 5cd^4 = 5c(4c^4 - d^4) = 5c(2c^2 + d^2)(2c^2 - d^2).$
8. $5x^3 - 20xy^2 = 5x(x^2 - 4y^2) = 5x(x + 2y)(x - 2y).$
9. $r^3s + 10r^2s^2 + 21rs^3 = rs(r^2 + 10rs + 21s^2) = rs(r + 7s)(r + 3s).$
10. $2h^3k^2 + 2h^2k^3 - 12hk^4 = 2hk^2(h^2 + hk - 6k^2) = 2hk^2(h + 3k)(h - 2k).$
11. $2r^5 - 162r = 2r(r^4 - 81) = 2r(r^2 + 9)(r + 3)(r - 3).$
12. $45m^3n^2 - 20mn^4 = 5mn^2(9m^2 - 4n^2) = 5mn^2(3m + 2n)(3m - 2n).$
13. $2x^3y + 10x^2y^2 + 12xy^3 = 2xy(x^2 + 5xy + 6y^2)$
 $= 2xy(x + 2y)(x + 3y).$
14. $4c^4d + 4c^3d^2 - 24c^2d^3 = 4c^2d(c^2 + cd - 6d^2) = 4c^2d(c + 3d)(c - 2d).$
15. $5r^4s - 40r^3s^2 + 60r^2s^3 = 5r^2s(r^2 - 8rs + 12s^2)$
 $= 5r^2s(r - 2s)(r - 6s).$
16. $39d^4 - 10cd^3 - c^2d^2 = -d^2(c^2 + 10cd - 39d^2)$
 $= -d^2(c + 13d)(c - 3d)$
 $= d^2(c + 13d)(3d - c).$
17. $18x^5 + 7kx^4 - k^2x^3 = -x^3(k^2 - 7kx - 18x^2)$
 $= -x^3(k - 9x)(k + 2x)$
 $= x^3(9x - k)(k + 2x).$
18. $8abc^5 + a^2b^2c^4 - 65c^6 = c^4(a^2b^2 + 8abc - 65c^2)$
 $= c^4(ab - 5c)(ab + 13c).$
19. $6ah^2 + 3h^2c + 18ahk + 9chk = 3h(2ah + hc + 6ak + 3ck)$
 $= 3h[h(2a + c) + 3k(2a + c)]$
 $= 3h(2a + c)(h + 3k).$
20. $30m^2nr - 10mr + 45m^2n - 15m = 5m(6mnr - 2r + 9mn - 3)$
 $= 5m[2r(3mn - 1) + 3(3mn - 1)]$
 $= 5m(3mn - 1)(2r + 3).$
21. $2a^3 + 4a^2b + 2ab^2 - 2ac^2 = 2a(a^2 + 2ab + b^2 - c^2)$
 $= 2a[(a + b)^2 - c^2]$
 $= 2a(a + b + c)(a + b - c).$
22. $3c^3 + 6c^2d + 3cd^2 - 12c = 3c(c^2 + 2cd + d^2 - 4)$
 $= 3c(c + d + 2)(c + d - 2).$
23. $4x^3 - 4xy^2 - 8xyz - 4xz^2 = 4x(x^2 - y^2 - 2yz - z^2)$
 $= 4x[x^2 - (y + z)^2]$
 $= 4x(x + y + z)(x - y - z).$

24. $x(a+2)=a^2+5a+6.$
$x(a+2)=(a+2)(a+3).$
$x=a+3.$

25. $ym-y=m^2-4m+3.$
$y(m-1)=(m-1)(m-3).$
$y=m-3.$

26. $rz+r=r^2+5z-20.$
$rz-5z=r^2-r-20.$
$z(r-5)=(r-5)(r+4).$
$z=r+4.$

27. $ar+3ac-a^2=2cr+2c^2.$
$ar-2cr=a^2-3ac+2c^2.$
$r(a-2c)=(a-2c)(a-c).$
$r=a-c.$

28. $2as+a^3+a^4=6a^2+a^2s.$
$2s+a^2+a^3=6a+as.$
$2s-as=-a^3-a^2+6a.$
$s(2-a)=-a(a^2+a-6).$
$s(a-2)=a(a+3)(a-2).$
$s=a(a+3).$

Page 131

1. $3x^2+7x+2=(3x+1)(x+2).$
2. $2x^2+7x+6=(2x+3)(x+2).$
3. $3x^2+8x+5=(3x+5)(x+1).$
4. $2a^2-9a+10=(2a-5)(a-2).$
5. $5a^2-2a-3=(5a+3)(a-1).$
6. $6a^2+7a-5=(3a+5)(2a-1).$
7. $10x^2+13x-3=(2x+3)(5x-1).$
8. $6c^2+7c-20=(3c-4)(2c+5).$
9. $10x^2+9x+2=(5x+2)(2x+1).$
10. $6x^2+5x-6=(3x-2)(2x+3).$
11. $2x^2+13x+18=(2x+9)(x+2).$
12. $3d^2-10d-25=(3d+5)(d-5).$
13. $5x^2-38x-16=(5x+2)(x-8).$
14. $10x^2+7x-6=(5x+6)(2x-1).$
15. $4k^2+20k+21=(2k+7)(2k+3).$
16. $9x^2+3x-2=(3x+2)(3x-1).$
17. $16c^2-8c-3=(4c-3)(4c+1).$
18. $10l^2-7l-12=(5l+4)(2l-3).$
19. $12x^2-8x-15=(6x+5)(2x-3).$
20. $14r^2-39+10=(7r-2)(2r-5).$
21. $21x^2-61x-30=(7x+3)(3x-10).$
22. $25s^2-15s+2=(5s-2)(5s-1).$
23. $36x^2-36x+5=(6x-5)(6x-1).$
24. $36a^2+23a-3=(9a-1)(4a+3).$
25. $49x^2-21x+2=(7x-1)(7x-2).$
26. $2+x-15x^2=(2-5x)(1+3x).$
27. $12x^4+x^2-20=(4x^2-5)(3x^2+4).$
28. $50x^4+5x^2-3=(10x^2+3)(5x^2-1).$
29. $6+7x^2-5x^4=(3+5x^2)(2-x^2).$

30. $12 + 17x^3 + 6x^6 = (3 + 2x^3)(4 + 3x^3)$.
31. $a^2 - 3ab + 2b^2 = (a - 2b)(a - b)$.
32. $a^2 + 2ab - 8b^2 = (a + 4b)(a - 2b)$.
33. $c^2 - cd - 12d^2 = (c - 4d)(c + 3d)$.
34. $2x^2 + 5xy + 2y^2 = (2x + y)(x + 2y)$.
35. $2a^2 - 5ab + 2b^2 = (2a - b)(a - 2b)$.
36. $3x^2 - 10xy + 3y^2 = (3x - y)(x - 3y)$.
37. $10x^2 - 27xy + 5y^2 = (2x - 5y)(5x - y)$.
38. $12x^2 + 23xy - 2y^2 = (12x - y)(x + 2y)$.
39. $30x^2 - 13xy - y^2 = (15x + y)(2x - y)$.
40. $30x^2 + 109xy + 30y^2 = (10x + 3y)(3x + 10y)$.

Page 132

1. $4x^2 + 10x + 4 = 2(2x^2 + 5x + 2) = 2(2x + 1)(x + 2)$.
2. $3x^3 + 18x^2 + 27x = 3x(x^2 + 6x + 9) = 3x(x + 3)^2$.
3. $20x^4 - 60x^3 + 45x^2 = 5x^2(4x^2 - 12x + 9) = 5x^2(2x - 3)^2$.
4. $50ax^2 - 140ax + 98a = 2a(25x^2 - 70x + 49) = 2a(5x - 7)^2$.
5. $27x^3 - 36x^4 + 12x^5 = 3x^3(9 - 12x + 4x^2) = 3x^3(3 - 2x)^2$.
6. $8x^4 + 2x^2 - 1 = (2x^2 + 1)(2x + 1)(2x - 1)$.
7. $60x^3 - 35x^2 - 60x = 5x(12x^2 - 7x - 12) = 5x(4x + 3)(3x - 4)$.
8. $3x^3y - 4x^2y^2 + xy^3 = xy(3x^2 - 4xy + y^2) = xy(3x - y)(x - y)$.
9. $12x^5y + 21x^3y^3 - 6xy^5 = 3xy(4x^4 + 7x^2y^2 - 2y^4)$
$= 3xy(x^2 + 2y^2)(2x + y)(2x - y)$.
10. $5x^3y + 10x^2y^2 - 75xy^3 = 5xy(x^2 + 2xy - 15y^2)$
$= 5xy(x + 5y)(x - 3y)$.
11. $6x^2r^2 + 12ar^2x + 4rx^2 + 8axr = 2xr(3xr + 6ar + 2x + 4a)$
$= 2xr[3r(x + 2a) + 2(x + 2a)]$
$= 2xr(x + 2a)(3r + 2)$.
12. $-18ax^2y - 45a^2xy - 6ax^2 - 15a^2x = -3ax(6xy + 15ay + 2x + 5a)$
$= -3ax[3y(2x + 5a) + (2x + 5a)]$
$= -3ax(2x + 5a)(3y + 1)$.
13. $45x^4y^2 + 21x^3y - 6x^2 = 3x^2(15x^2y^2 + 7xy - 2)$
$= 3x^2(5xy - 1)(3xy + 2)$.
14. $r(a + 1) = 2a^2 + 3a + 1$.
$r(a + 1) = (a + 1)(2a + 1)$.
$r = 2a + 1$.
15. $sb + sc = b^2 - bc - 2c^2$.
$s(b + c) = (b - 2c)(b + c)$.
$s = b - 2c$.
16. $2hx = 6h^2 + h - 2 + x$.
$2hx - x = 6h^2 + h - 2$.
$x(2h - 1) = (2h - 1)(3h + 2)$.
$x = 3h + 2$.
17. $cy - cd - 2c^2 = dy - 3d^2$.
$cy - dy = 2c^2 + cd - 3d^2$.
$y(c - d) = (c - d)(2c + 3d)$.
$y = 2c + 3d$.

Page 134 (First set)

1. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$.
2. $m^3 - n^3 = (m - n)(m^2 + mn + n^2)$.
3. $a^3 + 8 = a^3 + 2^3 = (a + 2)(a^2 - 2a + 4)$.
4. $b^3 - 64 = b^3 - 4^3 = (b - 4)(b^2 + 4b + 16)$.
5. $c^3 + 8d^3 = c^3 + (2d)^3 = (c + 2d)(c^2 - 2cd + 4d^2)$.
6. $m^3 - 125n^3 = m^3 - (5n)^3 = (m - 5n)(m^2 + 5mn + 25n^2)$.
7. $8 - x^3 = 2^3 - x^3 = (2 - x)(4 + 2x + x^2)$.
8. $64 + y^3 = 4^3 + y^3 = (4 + y)(16 - 4y + y^2)$.
9. $27 + 8a^3 = 3^3 + (2a)^3 = (3 + 2a)(9 - 6a + 4a^2)$.
10. $m^9 - n^3 = (m^3)^3 - n^3 = (m^3 - n)(m^6 + m^3n + n^2)$.
11. $125x^3 - y^3 = (5x)^3 - y^3 = (25x^2 + 5xy + y^2)(5x - y)$.
12. $1 - 27m^3n^9 = 1^3 - (3mn^3)^3 = (1 - 3mn^3)(1 + 3mn^3 + 9m^2n^6)$.
13. $8x^3y^6 + 1 = (2xy^2)^3 + 1 = (2xy^2 + 1)(4x^2y^4 - 2xy^2 + 1)$.
14. $216 - x^3 = 6^3 - x^3 = (6 - x)(36 + 6x + x^2)$.
15. $m^6 + n^6 = (m^2)^3 + (n^2)^3 = (m^2 + n^2)(m^4 - m^2n^2 + n^4)$.
16. $x^6 + y^6 = (x^2)^3 + (y^2)^3 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$.
17. $x^{12} + y^{12} = (x^4)^3 + (y^4)^3 = (x^4 + y^4)(x^8 - x^4y^4 + y^8)$.
18. $64x^6 + y^6 = (4x^2)^3 + (y^2)^3 = (4x^2 + y^2)(16x^4 - 4x^2y^2 + y^4)$.
19. $a^6 + 64 = (a^2)^3 + 4^3 = (a^2 + 4)(a^4 - 4a^2 + 16)$.
20. $m^6 - n^6 = (m^3 + n^3)(m^3 - n^3)$
    $= (m + n)(m - n)(m^2 - mn + n^2)(m^2 + mn + n^2)$.
21. $x^6 - y^6 = (x^3 + y^3)(x^3 - y^3) = (x + y)(x - y)(x^2 - xy + y^2)(x^2 + xy + y^2)$.
22. $a^6 - 64 = (a^3 + 8)(a^3 - 8) = (a + 2)(a - 2)(a^2 - 2a + 4)(a^2 + 2a + 4)$.
23. $x^6 - 64y^6 = (x^3 + 8y^3)(x^3 - 8y^3)$
    $= (x + 2y)(x - 2y)(x^2 - 2xy + 4y^2)(x^2 + 2xy + 4y^2)$.
24. $1 - a^6 = (1 + a^3)(1 - a^3) = (1 + a)(1 - a)(1 - a + a^2)(1 + a + a^2)$.
25. $64x^6 - 1 = (8x^3 + 1)(8x^3 - 1)$
    $= (2x + 1)(2x - 1)(4x^2 - 2x + 1)(4x^2 + 2x + 1)$.
26. $a^{12} - b^6 = (a^6 + b^3)(a^6 - b^3)$
    $= (a^2 + b)(a^4 - a^2b + b^2)(a^2 - b)(a^4 + a^2b + b^2)$.
27. $x^{12} - 1 = (x^6 + 1)(x^6 - 1)$
    $= (x^2 + 1)(x^4 - x^2 + 1)(x + 1)(x - 1)(x^2 - x + 1)(x^2 + x + 1)$.
28. $y^{12} - x^6 = (y^6 + x^3)(y^6 - x^3)$
    $= (y^2 + x)(y^4 - y^2x + x^2)(y^2 - x)(y^4 + y^2x + x^2)$.

Page 134 (Second set)

1. $x^4 + xy^3 = x(x^3 + y^3) = x(x + y)(x^2 - xy + y^2)$.
2. $2x^4y - 2xy^4 = 2xy(x^3 - y^3) = 2xy(x - y)(x^2 + xy + y^2)$.
3. $2a^4 - 54a = 2a(a^3 - 27) = 2a(a - 3)(a^2 + 3a + 9)$.
4. $5x^6 - 40x^3 = 5x^3(x^3 - 8) = 5x^3(x - 2)(x^2 + 2x + 4)$.

5. $x^4 - 2x^3 + 27x - 54 = x^3(x-2) + 27(x-2)$
$= (x-2)(x^3+27)$
$= (x-2)(x+3)(x^2-3x+9).$

6. $x^6 - 7x^3 - 8 = (x^3-8)(x^3+1)$
$= (x-2)(x^2+2x+4)(x+1)(x^2-x+1).$

7. $x^{10} - 64x^4 = x^4(x^6-64) = x^4(x^3+8)(x^3-8)$
$= x^4(x+2)(x^2-2x+4)(x-2)(x^2+2x+4).$

8. $3x^4 + 2x^3 - 24x - 16 = x^3(3x+2) - 8(3x+2)$
$= (3x+2)(x^3-8)$
$= (3x+2)(x-2)(x^2+2x+4).$

9. $3x - ax = 27 - a^3.$
$x(3-a) = (3-a)(9+3a+a^2).$
$x = 9 + 3a + a^2.$

10. $bx - 8 = b^3 - 2x.$
$bx + 2x = b^3 + 8.$
$x(b+2) = (b+2)(b^2-2b+4).$
$x = b^2 - 2b + 4.$

11. $c^2x - 27 = c^3 + 3cx - 9x.$
$c^2x - 3cx + 9x = c^3 + 27.$
$x(c^2-3c+9) = (c+3)(c^2-3c+9).$
$x = c + 3.$

12. $a^3 + a^2r = 8 - 2ar - 4r.$
$a^2r + 2ar + 4r = 8 - a^3.$
$r(a^2+2a+4) = (2-a)(4+2a+a^2).$
$r = 2 - a.$

13. $abs + as + a = ab^3 - ab^2s.$
$abs + as + ab^2s = ab^3 - a.$
$bs + s + b^2s = b^3 - 1.$
$s(b+1+b^2) = (b-1)(b^2+b+1).$
$s = b - 1.$

Page 136

1. $x^3 - 4x = x(x^2-4) = x(x+2)(x-2).$
2. $2a^3 + 4a^2b + 2ab^2 = 2a(a^2+2ab+b^2) = 2a(a+b)^2.$
3. $ax^3 + ax^2y + a^2x^2 + a^2xy = ax(x^2+xy+ax+ay)$
$= ax[x(x+y) + a(x+y)]$
$= ax(x+y)(x+a).$
4. $4x^4 + 400x^8 - 116x^6 = 4x^4(1+100x^4-29x^2)$
$= 4x^4(1-25x^2)(1-4x^2)$
$= 4x^4(1+5x)(1-5x)(1+2x)(1-2x).$
5. $2cx^3 + 2cx^2 - 12cx = 2cx(x^2+x-6) = 2cx(x+3)(x-2).$
6. $5a^3 - 10a^2b - 75ab^2 = 5a(a^2-2ab-15b^2) = 5a(a-5b)(a+3b).$
7. $10x^4 - 5x^3 - 30x^2 = 5x^2(2x^2-x-6) = 5x^2(2x+3)(x-2).$

8. $5x^{10} + 40x^7 = 5x^7(x^3 + 8) = 5x^7(x + 2)(x^2 - 2x + 4).$

9. $x^3y + 3x^3 + 8y + 24 = x^3(y + 3) + 8(y + 3)$
$= (y + 3)(x^3 + 8)$
$= (y + 3)(x + 2)(x^2 - 2x + 4).$

10. $x^5 - 4x^3 - 8x^2 + 32 = x^3(x^2 - 4) - 8(x^2 - 4)$
$= (x^2 - 4)(x^3 - 8)$
$= (x - 2)^2(x + 2)(x^2 + 2x + 4).$

11. $16x^4 - 8x^3 - 2x + 1 = 8x^3(2x - 1) - 1(2x - 1)$
$= (2x - 1)(8x^3 - 1)$
$= (2x - 1)^2(4x^2 + 2x + 1).$

12. $a^5 - 25a^3 - a^2 + 25 = a^3(a^2 - 25) - 1(a^2 - 25)$
$= (a^2 - 25)(a^3 - 1)$
$= (a + 5)(a - 5)(a - 1)(a^2 + a + 1).$

13. $-45a^3x^3 - 3a^4x^2 + 18a^5x = -3a^3x(15x^2 + ax - 6a^2)$
$= 3a^3x(2a + 3x)(3a - 5x).$

14. $9x^2 - 30axy^2 + 25a^2y^4 = (3x - 5ay^2)^2.$

15. $x^{12} - 63x^6 - 64 = (x^6 - 64)(x^6 + 1)$
$= (x^3 + 8)(x^3 - 8)(x^6 + 1)$
$= (x + 2)(x^2 - 2x + 4)(x - 2)(x^2 + 2x + 4)(x^2 + 1)(x^4 - x^2 + 1).$

16. $x^6 - 64y^{12} = (x^3 + 8y^6)(x^3 - 8y^6)$
$= (x + 2y^2)(x^2 - 2xy^2 + 4y^4)(x - 2y^2)(x^2 + 2xy^2 + 4y^4).$

17. $6x^4 - 7x^2y - 5y^2 = (2x^2 + y)(3x^2 - 5y).$

18. $3 + 15x - 2ax - 10ax^2 = 3(1 + 5x) - 2ax(1 + 5x)$
$= (1 + 5x)(3 - 2ax).$

19. $x^7 - 27x^4 - x^3 + 27 = x^4(x^3 - 27) - (x^3 - 27)$
$= (x^3 - 27)(x^4 - 1)$
$= (x - 3)(x^2 + 3x + 9)(x^2 + 1)(x + 1)(x - 1).$

20. $a^3 + 3a^2b - ab^2 - 3b^3 = a^2(a + 3b) - b^2(a + 3b)$
$= (a + 3b)(a^2 - b^2)$
$= (a + 3b)(a + b)(a - b).$

21. $20x^4y + 38x^3y^2 - 30x^2y^3 = 2x^2y(10x^2 + 19xy - 15y^2)$
$= 2x^2y(5x - 3y)(2x + 5y).$

22. $8ax^2 - 2a - 20x^2 + 5 = 2a(4x^2 - 1) - 5(4x^2 - 1)$
$= (4x^2 - 1)(2a - 5) = (2x + 1)(2x - 1)(2a - 5).$

23. $9a^2 + 4b^2 - 12ab - 4x^2y^2 = (3a - 2b)^2 - 4x^2y^2$
$= (3a - 2b + 2xy)(3a - 2b - 2xy).$

24. $4x^2 - x^4y^4 - 4xy + y^2 = (2x - y)^2 - x^4y^4$
$= (2x - y + x^2y^2)(2x - y - x^2y^2).$

25. $x^2 + y^2 - 4a^2 - 2xy = (x - y)^2 - 4a^2$
$= (x - y + 2a)(x - y - 2a).$

26. $x^5 - 4x^3y^2 + x^2y^3 - 4y^5 = x^3(x^2 - 4y^2) + y^3(x^2 - 4y^2)$
$= (x^2 - 4y^2)(x^3 + y^3)$
$= (x + 2y)(x - 2y)(x + y)(x^2 - xy + y^2).$

27. $x^2 + x^3 - 6x^4 = x^2(1 + x - 6x^2) = x^2(1 + 3x)(1 - 2x).$

28. $1 - x^2 + x^3 - x^5 = (1 - x^2) + x^3(1 - x^2)$
$= (1 - x^2)(1 + x^3)$
$= (1 + x)^2(1 - x)(1 - x + x^2).$

29. $1 - 4x^2 + 8x^3 - 32x^5 = (1 - 4x^2) + 8x^3(1 - 4x^2)$
$= (1 - 4x^2)(1 + 8x^3)$
$= (1 + 2x)^2(1 - 2x)(1 - 2x + 4x^2).$

30. $x^6 - 8x^4 + 16x^2 = x^2(x^4 - 8x^2 + 16) = x^2(x^2 - 4)^2 = x^2(x + 2)^2(x - 2)^2.$

31. $2x^3y + 4x^2y + 2xy - 2xy^3 = 2xy(x^2 + 2x + 1 - y^2)$
$= 2xy(x + 1 + y)(x + 1 - y).$

32. $8a^2y - 8ay^2 + 2y^3 - 2y = 2y(4a^2 - 4ay + y^2 - 1)$
$= 2y(2a - y + 1)(2a - y - 1).$

33. $4x^4 + 4x^3 - 4x^6 + x^2 = x^2(4x^2 + 4x + 1 - 4x^4)$
$= x^2(2x + 1 + 2x^2)(2x + 1 - 2x^2).$

34. $a^6 - 13a^4 + 36a^2 = a^2(a^4 - 13a^2 + 36)$
$= a^2(a^2 - 9)(a^2 - 4)$
$= a^2(a + 3)(a - 3)(a + 2)(a - 2).$

35. $x^8y^2 - 4x^6y^2 + 3x^4y^6 - 12x^2y^6 = x^2y^2(x^6 - 4x^4 + 3x^2y^4 - 12y^4)$
$= x^2y^2[x^4(x^2 - 4) + 3y^4(x^2 - 4)]$
$= x^2y^2(x^2 - 4)(x^4 + 3y^4)$
$= x^2y^2(x + 2)(x - 2)(x^4 + 3y^4).$

36.
$$\begin{aligned} ax + ad + bd &= ac + bc - bx. \\ ax + bx &= ac + bc - ad - bd. \\ x(a + b) &= c(a + b) - d(a + b). \\ x &= c - d. \end{aligned}$$

37.
$$\begin{aligned} ax + bx + 3 &= a + b + 3x. \\ ax + bx - 3x &= a + b - 3. \\ x(a + b - 3) &= a + b - 3. \\ x &= 1. \end{aligned}$$

38.
$$\begin{aligned} 10r - 25 - 5x &= r^2 - rx. \\ rx - 5x &= r^2 - 10r + 25. \\ x(r - 5) &= (r - 5)^2. \\ x &= r - 5. \end{aligned}$$

39.
$$\begin{aligned} 4d^2 + 2dx - c^2 &= - cx. \\ cx + 2dx &= c^2 - 4d^2. \\ x(c + 2d) &= (c + 2d)(c - 2d). \\ x &= c - 2d. \end{aligned}$$

40. $4r^2 + 9 + 3z = 2rz + 12r.$

$$3z - 2rz = -(9 - 12r + 4r^2).$$
$$z(3 - 2r) = -(3 - 2r)^2.$$
$$z = -(3 - 2r) = 2r - 3.$$

41. $m + 42 - 7x = m^2 - mx.$

$$mx - 7x = m^2 - m - 42.$$
$$x(m - 7) = (m - 7)(m + 6).$$
$$x = m + 6.$$

42. $a + 15 + 5x = 2a^2 - 2ax.$

$$5x + 2ax = 2a^2 - a - 15.$$
$$x(2a + 5) = (2a + 5)(a - 3).$$
$$x = a - 3.$$

43. $-3c - x = 2c^2 - 2cx - 2.$

$$2cx - x = 2c^2 + 3c - 2.$$
$$x(2c - 1) = (2c - 1)(c + 2).$$
$$x = c + 2.$$

44. $6r^2 - 12r + 4x = r^3 - r^2x + 4rx - 8.$

$$r^2x - 4rx + 4x = r^3 - 6r^2 + 12r - 8.$$
$$x(r^2 - 4r + 4) = (r^3 - 8) - 6r(r - 2).$$
$$x(r - 2) = r^2 + 2r + 4 - 6r.$$
$$x(r - 2) = r^2 - 4r + 4.$$
$$x = r - 2.$$

45. $a^3 - c^3 = ay - cy.$

$$y(a - c) = (a - c)(a^2 + ac + c^2).$$
$$y = a^2 + ac + c^2.$$

Page 141

1. $x^2 - 9 = 0.$

Factoring,

$$(x - 3)(x + 3) = 0.$$
$$x - 3 = 0,$$

or $x + 3 = 0.$

$$x = 3, -3.$$

2. $x^2 - 25 = 0.$

Factoring,

$$(x - 5)(x + 5) = 0.$$

Whence $x = 5, -5.$

3. $x^2 = 16.$

Transposing,

$$x^2 - 16 = 0.$$
$$(x - 4)(x + 4) = 0.$$

Whence $x = 4, -4.$

4. $x^2 = 49.$

$$x^2 - 49 = 0.$$
$$(x - 7)(x + 7) = 0.$$
$$x = 7, -7.$$

5.
$$x^2 - 2x = 0.$$
$$x(x - 2) = 0.$$
$$x = 0,\ 2.$$

6.
$$2x^2 + 6x = 0.$$
$$2x(x + 3) = 0.$$
$$x = 0,\ -3.$$

7
$$x^2 = 5x.$$
$$x^2 - 5x = 0.$$
$$x(x - 5) = 0.$$
$$x = 0,\ 5.$$

8.
$$3x^2 = 9x.$$
$$3x^2 - 9x = 0.$$
$$3x(x - 3) = 0.$$
$$x = 0,\ 3.$$

9.
$$x^2 - 4x + 3 = 0.$$
$$(x - 3)(x - 1) = 0.$$
$$x = 3,\ 1.$$

10.
$$x^2 - 6x + 9 = 0.$$
$$(x - 3)^2 = 0.$$
$$x = 3 \text{ (for both factors).}$$

11.
$$x^2 - 7x = -12.$$
$$x^2 - 7x + 12 = 0.$$
$$(x - 3)(x - 4) = 0.$$
$$x = 3,\ 4.$$

12.
$$x^2 + x = 20.$$
$$x^2 + x - 20 = 0.$$
$$(x - 4)(x + 5) = 0.$$
$$x = 4,\ -5.$$

13.
$$x^2 + 8 = -6x.$$
$$x^2 + 6x + 8 = 0.$$
$$(x + 2)(x + 4) = 0.$$
$$x = -2,\ -4.$$

14.
$$x^2 = 3x + 10.$$
$$x^2 - 3x - 10 = 0.$$
$$(x - 5)(x + 2) = 0.$$
$$x = 5,\ -2.$$

15.
$$4x^2 - 16x = 0.$$
$$4x(x - 4) = 0.$$
$$x = 0,\ 4.$$

16.
$$5x^2 + 35x = 0.$$
$$5x(x + 7) = 0.$$
$$x = 0,\ -7.$$

17.
$$8 - 9x = -x^2.$$
$$x^2 - 9x + 8 = 0.$$
$$(x - 1)(x - 8) = 0.$$
$$x = 1,\ 8.$$

18.
$$12x - 28 = -x^2.$$
$$x^2 + 12x - 28 = 0.$$
$$(x - 2)(x + 14) = 0.$$
$$x = 2,\ -14.$$

19.
$$x^2 - 16x + 64 = 0.$$
$$(x - 8)^2 = 0.$$
$$x = 8 \text{ (for both factors).}$$

20.
$$x^2 - 54 = 15x.$$
$$x^2 - 15x - 54 = 0.$$
$$(x - 18)(x + 3) = 0.$$
$$x = 18,\ -3.$$

21.
$$12 - 25x + 12x^2 = 0.$$
$$(4 - 3x)(3 - 4x) = 0.$$
$$x = \tfrac{4}{3},\ \tfrac{3}{4}.$$

22.
$$3x^2 + x = 4.$$
$$3x^2 + x - 4 = 0.$$
$$(x - 1)(3x + 4) = 0.$$
$$x = 1,\ -\tfrac{4}{3}.$$

23.
$$18x^2 = 9x + 20.$$
$$18x^2 - 9x - 20 = 0.$$
$$(3x - 4)(6x + 5) = 0.$$
$$x = \tfrac{4}{3},\ -\tfrac{5}{6}.$$

24. $(x+8)(x+1)=-12.$
$x^2+9x+8=-12.$
$x^2+9x+20=0.$
$(x+4)(x+5)=0.$
$x=-4,\ -5.$

25. $x^2+9x-12=3x+15.$
$x^2+6x-27=0.$
$(x-3)(x+9)=0.$
$x=3,\ -9.$

26. $x+12=x^2.$
$0=x^2-x-12.$
$(x-4)(x+3)=0.$
$x=4,\ -3.$

27. $50x+24=25x^2.$
$0=25x^2-50x-24.$
$(5x-12)(5x+2)=0.$
$x=\frac{12}{5},\ -\frac{2}{5}.$

28. $(x-11)(x+3)=2x+42.$
$x^2-8x-33=2x+42.$
$x^2-10x-75=0.$
$(x-15)(x+5)=0.$
$x=15,\ -5.$

29. $x^2-a^2=0.$
$(x-a)(x+a)=0.$
$x=a,\ -a.$

30. $x^2=4b^2.$
$x^2-4b^2=0.$
$(x-2b)(x+2b)=0.$
$x=2b,\ -2b.$

31. $x^2=16k^2.$
$x^2-16k^2=0.$
$(x-4k)(x+4k)=0.$
$x=4k,\ -4k.$

32. $x^2-2bx+b^2=0.$
$(x-b)^2=0.$
$x=b$ (for both factors).

33. $x^2+4a^2=4ax.$
$x^2-4ax+4a^2=0.$
$(x-2a)^2=0.$
$x=2a$ (for both factors).

34. $x^2-ax=0.$
$x(x-a)=0.$
$x=0,\ a.$

35. $x^2=7ax.$
$x^2-7ax=0.$
$x(x-7a)=0.$
$x=0,\ 7a.$

36. $2x^2-7x=21-6x.$
$2x^2-x-21=0.$
$(2x-7)(x+3)=0.$
$x=\frac{7}{2},\ -3.$

37. $x^2-bx-ax=0.$
$x(x-b-a)=0.$
$x=0,\ b+a.$

38. $x^2-5x=2ax.$
$x^2-2ax-5x=0.$
$x(x-2a-5)=0.$
$x=0,\ 2a+5.$

39. $x^2-ax-bx+ab=0.$
$x^2-(a+b)x+ab=0.$
$(x-a)(x-b)=0.$
$x=a,\ b.$

40. $x^2+bx=4b+4x.$
$x^2-4x+bx-4b=0.$
$x^2+(b-4)x-4b=0.$
$(x-4)(x+b)=0.$
$x=4,\ -b.$

Page 142 (First set)

1. $x^3 - 9x = 0.$
$x(x-3)(x+3) = 0.$
$x = 0, 3, -3.$

2. $3x^3 = 12x.$
$3x^3 - 12x = 0.$
$3x(x-2)(x+2) = 0.$
$x = 0, 2, -2.$

3. $x^3 + 4x^2 - 12x = 0.$
$x(x^2 + 4x - 12) = 0.$
$x(x-2)(x+6) = 0.$
$x = 0, 2, -6.$

4. $x^3 - 14x = 5x^2.$
$x^3 - 5x^2 - 14x = 0.$
$x(x^2 - 5x - 14) = 0.$
$x(x-7)(x+2) = 0.$
$x = 0, 7, -2.$

5. $5x^2 = 2x - 3x^3.$
$3x^3 + 5x^2 - 2x = 0.$
$x(3x^2 + 5x - 2) = 0.$
$x(3x-1)(x+2) = 0.$
$x = 0, \frac{1}{3}, -2.$

6. $x^3 - x^2 - 4x + 4 = 0.$
$(x-1)(x^2-4) = 0.$
$(x-1)(x-2)(x+2) = 0.$
$x = 1, 2, -2.$

7. $2x^3 - x^2 = 8x - 4.$
$2x^3 - x^2 - 8x + 4 = 0.$
$(2x-1)(x^2-4) = 0.$
$(2x-1)(x-2)(x+2) = 0.$
$x = \frac{1}{2}, 2, -2.$

8. $3x^3 - 2x^2 - 12x + 8 = 0.$
$(3x-2)(x^2-4) = 0.$
$(3x-2)(x-2)(x+2) = 0.$
$x = \frac{2}{3}, 2, -2.$

9. $x^3 - 25 = 25x - x^2.$
$x^3 + x^2 - 25x - 25 = 0.$
$(x^2-25)(x+1) = 0.$
$(x-5)(x+5)(x+1) = 0.$
$x = 5, -5, -1.$

10. $2(x^3 - x) = 3(1 - x^2).$
$2x^3 + 3x^2 - 2x - 3 = 0.$
$(x^2-1)(2x+3) = 0.$
$(x-1)(x+1)(2x+3) = 0.$
$x = 1, -1, -\frac{3}{2}.$

11. $x^4 - 5x^2 + 4 = 0.$
$(x^2-4)(x^2-1) = 0.$
$(x-2)(x+2)(x-1)(x+1) = 0.$
$x = 2, -2, 1, -1.$

12. $x^4 - 13x^2 + 36 = 0.$
$(x^2-4)(x^2-9) = 0.$
$(x-2)(x+2)(x-3)(x+3) = 0.$
$x = 2, -2, 3, -3.$

13. $$x^4 = 4x^2.$$
$$x^4 - 4x^2 = 0.$$
$$x^2(x-2)(x+2) = 0.$$
$$x = 0,\ 0,\ 2,\ -2.$$

14. $$x^4 = 26x^2 - 25.$$
$$x^4 - 26x^2 + 25 = 0.$$
$$(x^2 - 25)(x^2 - 1) = 0.$$
$$(x-5)(x+5)(x-1)(x+1) = 0.$$
$$x = 5,\ -5,\ 1,\ -1.$$

Page 142 (Second set)

1. Let $n =$ the number.
Then $n^2 + n = 42.$
Whence $n = 6$ or $-7.$

2. Let $n =$ the number.
Then $n^2 - 3n = 54.$
Whence $n = 9$ or $-6.$

3. Let $n =$ the number.
Then $n^2 + 2n + 9 = 129.$
Whence $n = 10$ or $-12.$

4. Let $n =$ the number.
Then $3n^2 = 4n.$
Whence $n = 0$ or $\frac{4}{3}.$

5. Let $n =$ the number.
Then
$(n+16)(n+21) = 546.$
Whence $n = 5$ or $-42.$

6. Let $n =$ the number.
Then
$(15-n)(25-n) = 119.$
Whence $n = 8$ or $32.$

7. Let $n =$ the number.
Then
$(30-n)(18+n) = 560.$
Whence $n = 2$ or $10.$

8. Let $n =$ the number.
Then
$(n+15)(22-n) = 50n + 36.$
Whence $n = 6$ or $-49.$

9. Let $n =$ the number.
Then $(4n)^2 - 5n = 15n^2.$
Whence $n = 0$ or $5.$

10. Let $x =$ the frontage in feet.
Then $4x =$ the depth in feet,
and $(x)(4x) = 1600.$
Whence $x = 20$ or $-20.$
But the answer -20 is absurd and must be rejected.
Therefore $x = 20;\ 4x = 80.$

11. Let $x =$ the breadth in yards.
Then $x + 3 =$ the length in yards,
and $x(x+3) = 54.$
Whence $x = 6$, root -9 rejected;
$x + 3 = 9.$

12. Let $x =$ the width in rods.

Then $x + 6 =$ the length in rods,

and $x(x + 6) = 216.$

Whence $x = 12$, root -18 rejected;

$x + 6 = 18.$

13. Let x, $x + 1$, be the integers.

Then $x^2 + (x + 1)^2 = 313.$

Whence $x = 12,\ x + 1 = 13;$

or $x = -13,\ x + 1 = -12.$

14. Let x, $x + 2$, be the integers.

Then $x^2 + (x + 2)^2 = 514.$

Whence $x = 15,\ x + 2 = 17;$

or $x = -17,\ x + 2 = -15.$

15. Let x, $x + 2$, $x + 4$, be the integers.

Then $x^2 + (x + 2)^2 + (x + 4)^2 = 251.$

Whence $x = 7,\ x + 2 = 9,\ x + 4 = 11;$

or $x = -11,\ x + 2 = -9,\ x + 4 = -7.$

16. Let $x =$ the length and breadth in inches.

Then $x^2 =$ the area of the bottom in square inches,

and $6x =$ the area of one side in square inches.

Therefore $x^2 + 24x = 112.$

Whence $x = 4$, root -28 rejected.

17. Let $x =$ the edge in inches.

Then $x^2 =$ the area of a single face in square inches.

Therefore $6x^2 = 216.$

Whence $x = 6$, root -6 rejected.

18. Let $x =$ the depth in inches.

Then $2x =$ the width, and $3x =$ the length, in inches.

There will be two sides $x \cdot 2x$ each in area, two sides $x \cdot 3x$ each, and top and bottom $2x \cdot 3x$ each.

Therefore $4x^2 + 6x^2 + 12x^2 = 550.$

Whence $x = 5$, root -5 rejected.

Then $2x = 10;\ 3x = 15.$

19. Let $x =$ the depth in inches.

Then $x + 1 =$ the width, and $x + 3$ the length, in inches.

As in Ex. 18, the total surface $=$

$$2x(x + 1) + 2x(x + 3) + 2(x + 1)(x + 3) = 62.$$

Whence $(3x + 14)(x - 2) = 0,$

$x = 2$, root $-\frac{14}{3}$ rejected;

$x + 1 = 3,\ x + 3 = 5.$

20. Let $b =$ the base in feet.

Then $\frac{8b}{2} = 40$, or $4b = 40$.

Whence $b = 10$.

21. Let $b =$ the base in feet.

Then $2b =$ the altitude in feet,

and $\frac{b \times 2b}{2} = 36$, or $b^2 = 36$.

Whence $b = 6$, root -6 rejected;

$2b = 12$.

22. Let $a =$ the altitude in feet.

Then $5a =$ the base in feet,

and $\frac{a \times 5a}{2} = 40$, or $\frac{5}{2}a^2 = 40$,

or $a^2 = 16$.

Whence $a = 4$, root -4 rejected;

$5a = 20$.

23. Let $a =$ the altitude in inches.

Then $6a =$ the base in inches,

and $\frac{a \times 6a}{2} = 48$, or $3a^2 = 48$.

Whence $a = 4$, root -4 rejected;

$6a = 24$.

25. Let $x =$ the base in feet.

Then $x + 4 =$ the altitude in feet,

and $\frac{x(x+4)}{2} = 6$.

Multiplying by 2, $x(x+4) = 12$.

Whence $x = 2$, root -6 rejected;

$x + 4 = 6$.

26. Let $x =$ the shorter leg in yards.

Then $x + 4 =$ the longer leg in yards,

and $\frac{x(x+4)}{2} = 30$.

Multiplying by 2, $x(x+4) = 60$.

Whence $x = 6$, root -10 rejected;

$x + 4 = 10$.

27. Let $x =$ the shorter leg in feet.
Then $x + 8 =$ the longer leg in feet,
and
$$\frac{x(x+8)}{2} = 9 \times 26 = 234.$$
Multiplying by 2, $x(x+8) = 468.$
Whence $x = 18$, root -26 rejected;
$$x + 8 = 26.$$

28. Let $x =$ the altitude in inches.
Then $2x + 6 =$ the base in inches,
and
$$\frac{x(2x+6)}{2} = \frac{21}{8} \times 144 = 378.$$
Performing the indicated multiplication and division,
$$x^2 + 3x = 378.$$
Whence $x = 18$, root -21 rejected;
$$2x + 6 = 42.$$

29. Let $x =$ the base in feet.
Then $2x - 6 =$ the altitude in feet,
and
$$\frac{x(2x-6)}{2} = 12 \times 9.$$
Performing the indicated operations,
$$x^2 - 3x = 108.$$
Whence $x = 12$, root -9 rejected;
$$2x - 6 = 18.$$

30. Let $A =$ the area.
Then
$$A = \frac{12(10+18)}{2} = \frac{12 \times 28}{2} = 168.$$

32. Let $x =$ the altitude in feet.
Then $3x =$ the second base in feet,
and
$$\text{area} = \frac{x(3x+12)}{2} = 90.$$
Multiplying by 2, $x(3x+12) = 180.$
Whence $x = 6$, root -10 rejected;
$$3x = 18.$$

33. Let $x =$ the longer base in feet.
Then $\frac{2}{3}x =$ the shorter base in feet,
and $\frac{1}{2} \cdot \frac{2}{3}x = \frac{1}{3}x =$ the altitude in feet.

Therefore $\dfrac{\frac{1}{3}x(x+\frac{2}{3}x)}{2} = \dfrac{\frac{1}{3}x \cdot \frac{5}{3}x}{2} = \dfrac{5}{18}x^2 = 250,$

or $x^2 = 900.$

Whence $x = 30$, root -30 rejected;

$\frac{2}{3}x = 20$; $\frac{1}{3}x = 10.$

34. Let $x =$ the altitude in feet.

Then $x + 12$ and $x + 6 =$ the bases in feet,

and $\dfrac{x(x+12+x+6)}{2} = 112.$

Performing the indicated operations,

$$x^2 + 9x = 112.$$

Whence $x = 7$, root -16 rejected;

$x + 6 = 13$; $x + 12 = 19.$

35. Let $x =$ the altitude in feet.

Then $x + 1$ and $x + 5 =$ the bases in feet,

and $\dfrac{x(x+1+x+5)}{2} = 9 \times 30.$

Performing the indicated operations,

$$x^2 + 3x = 270.$$

Whence $x = 15$, root -18 rejected;

$x + 1 = 16$; $x + 5 = 20.$

36. Let $x =$ the shorter base in feet.

Then $x + 6 =$ the longer base in feet,

and $\frac{1}{2}(x + x + 6) = x + 3 =$ the altitude in feet.

Therefore $\dfrac{(x+3)(2x+6)}{2} = 9 \times 9.$

Performing the indicated operations,

$$x^2 + 6x + 9 = 81.$$

Whence $x = 6$, root -12 rejected;

$x + 6 = 12$; $x + 3 = 9.$

37. Let $x =$ the altitude and the shorter base in feet.

Then $x + 2 =$ the longer base in feet,

and $\dfrac{x(x+x+2)}{2} = 8 \times 9.$

Performing the indicated operations,

$$x^2 + x = 72.$$

Whence $x = 8$, root -9 rejected;

$x + 2 = 10.$

38. Let $x =$ the shorter base in feet.
Then $x + 10 =$ the longer base in feet,
and $5x + 2 =$ the altitude in feet.

Therefore $\dfrac{(5x+2)(x+x+10)}{2} = 9 \times 22.$

Performing the indicated operations,

$$5x^2 + 27x + 10 = 198.$$

Whence $x = 4$, root $-\frac{47}{5}$ rejected;

$$x + 10 = 14;$$
$$5x + 2 = 22.$$

Page 151

1. $\dfrac{36a}{54a^2} = \dfrac{\overset{2}{\cancel{2^2}}\cdot\cancel{3^2}\cdot\cancel{a}}{\cancel{2}\cdot\underset{3}{\cancel{3^3}}\cdot\underset{a}{\cancel{a^2}}} = \dfrac{2}{3a}.$

2. $\dfrac{52x^2}{130xy} = \dfrac{\overset{2}{\cancel{2^2}}\cdot\cancel{13}\overset{x}{\cancel{x^2}}}{\cancel{2}\cdot 5\cdot\cancel{13}\cancel{x}y} = \dfrac{2x}{5y}.$

3. $\dfrac{84m^2}{126mn} = \dfrac{\overset{2}{\cancel{2^2}}\cdot\cancel{3}\cdot\cancel{7}\overset{m}{\cancel{m^2}}}{\cancel{2}\cdot\underset{3}{\cancel{3^2}}\cdot\cancel{7}\cancel{m}n} = \dfrac{2m}{3n}.$

4. $\dfrac{x^3y}{xy^3} = \dfrac{x^2}{y^2}.$

5. $\dfrac{xy^2}{2x^2y} = \dfrac{y}{2x}.$

6. $\dfrac{5ab^2c}{3ac^2} = \dfrac{5b^2}{3c}.$

7. $\dfrac{10ax^2}{25ax^3} = \dfrac{2}{5x}.$

8. $\dfrac{27cd^2e}{45c^2e^3} = \dfrac{3d^2}{5ce^2}.$

9. $\dfrac{108m^2n^3}{252md^2} = \dfrac{3mn^3}{7d^2}.$

10. $\dfrac{2a}{a^2+ab} = \dfrac{2}{a+b}.$

11. $\dfrac{4c^2 - 8cd}{12c} = \dfrac{4c(c-2d)}{12c} = \dfrac{c-2d}{3}.$

12. $\dfrac{10x^2y^2 - 5xy^3}{15xy^3} = \dfrac{5xy^2(2x-y)}{15xy^3} = \dfrac{2x-y}{3y}.$

13. $\dfrac{9a^2 - 6a}{3a-2} = \dfrac{3a(3a-2)}{3a-2} = 3a.$

14. $\dfrac{4x^2-25}{4x^2-20x+25} = \dfrac{(2x+5)(2x-5)}{(2x-5)(2x-5)} = \dfrac{2x+5}{2x-5}.$

15. $\dfrac{9-6x+x^2}{9-x^2} = \dfrac{(3-x)(3-x)}{(3+x)(3-x)} = \dfrac{3-x}{3+x}.$

16. $\dfrac{9x^2-1}{9x^2-9x+2} = \dfrac{(3x+1)(3x-1)}{(3x-1)(3x-2)} = \dfrac{3x+1}{3x-2}.$

17. $\dfrac{10a^2-2a}{15a^2+7a-2} = \dfrac{2a(5a-1)}{(5a-1)(3a+2)} = \dfrac{2a}{3a+2}.$

18. $\dfrac{c^3-4c}{c^5-8c^2} = \dfrac{c(c+2)(c-2)}{c^2(c-2)(c^2+2c+4)} = \dfrac{c+2}{c(c^2+2c+4)}.$

19. $\dfrac{c^2-6c+8}{c^2+c-6}=\dfrac{(c-4)(c-2)}{(c+3)(c-2)}=\dfrac{c-4}{c+3}.$

20. $\dfrac{2x^3+x^2-3x}{3x^4-3x^2}=\dfrac{x(2x+3)(x-1)}{3x^2(x+1)(x-1)}=\dfrac{2x+3}{3x(x+1)}.$

21. $\dfrac{50x-2x^3}{x^3+8x^2+15x}=\dfrac{2x(5+x)(5-x)}{x(x+5)(x+3)}=\dfrac{2(5-x)}{x+3}.$

22. $\dfrac{x^3-49x}{42x^2-27x^3+3x^4}=\dfrac{x(x+7)(x-7)}{3x^2(7-x)(2-x)}=\dfrac{x+7}{3x(x-2)}.$

23. $\dfrac{a^3-a^2+2a-2}{3a^3-a^2+6a-2}=\dfrac{(a-1)(a^2+2)}{(3a-1)(a^2+2)}=\dfrac{a-1}{3a-1}.$

24. $\dfrac{2a^3y+2a^2xy+2acy+2cxy}{2ax+2a^2-ay-xy}=\dfrac{2y(a+x)(a^2+c)}{(x+a)(2a-y)}=\dfrac{2y(a^2+c)}{2a-y}.$

25. $\dfrac{m^2-n^2}{m^3-n^3}=\dfrac{(m+n)(m-n)}{(m-n)(m^2+mn+n^2)}=\dfrac{m+n}{m^2+mn+n^2}.$

26. $\dfrac{x^4-8x}{x^3-2x^2}=\dfrac{x(x-2)(x^2+2x+4)}{x^2(x-2)}=\dfrac{x^2+2x+4}{x}.$

27. $\dfrac{a^2-4}{(a-2)^2}=\dfrac{(a+2)(a-2)}{(a-2)^2}=\dfrac{a+2}{a-2}.$

28. $\dfrac{a^6-1}{a^2-1}=\dfrac{(a+1)(a^2-a+1)(a-1)(a^2+a+1)}{(a+1)(a-1)}$
$=(a^2-a+1)(a^2+a+1).$

29. $\dfrac{a^4-x^4}{a^4+3a^2x^2+2x^4}=\dfrac{(a^2+x^2)(a+x)(a-x)}{(a^2+x^2)(a^2+2x^2)}=\dfrac{a^2-x^2}{a^2+2x^2}.$

30. $\dfrac{27x^3+a^3}{9x^2-a^2}=\dfrac{(3x+a)(9x^2-3ax+a^2)}{(3x+a)(3x-a)}=\dfrac{9x^2-3ax+a^2}{3x-a}.$

Page 153

1. $96=2^5\cdot 3.$
$36=2^2\cdot 3^2.$
$40=2^3\cdot 5.$
L.C.M. $=2^5\cdot 3^2\cdot 5.$

2. $45=3^2\cdot 5.$
$105=3\cdot 5\cdot 7.$
$175=5^2\cdot 7.$
L.C.M. $=3^2\cdot 5^2\cdot 7.$

3. $50=2\cdot 5^2.$
$44=2^2\cdot 11.$
$110=2\cdot 5\cdot 11.$
$275=11\cdot 5^2.$
L.C.M. $=2^2\cdot 5^2\cdot 11.$

4. $a^2x.$
$ax^3.$
$a^2x^2.$
L.C.M. $=a^2\cdot x^3.$

5. $mn.$
$m^2.$
$m^5n^2.$
L.C.M. $=m^5\cdot n^2.$

6. $2x=2x.$
$6x^2=2\cdot 3x^2.$
$4x^3=2^2\cdot x^3.$
L.C.M. $=2^2\cdot 3\cdot x^3$
$=12x^3.$

7.
$$3r = 3r.$$
$$12 = 2^2 \cdot 3.$$
$$15r^2 = 3 \cdot 5 \cdot r^2.$$
$$\text{L.C.M.} = 2^2 \cdot 3 \cdot 5r^2 = 60r^2.$$

8.
$$4a = 2^2 \cdot a.$$
$$6a^2 = 2 \cdot 3 \cdot a^2.$$
$$8a^5 = 2^3 \cdot a^5.$$
$$\text{L.C.M.} = 2^3 \cdot 3a^5 = 24a^5.$$

9. $2ax^2$.
$3a^2x$.
a^2cx^3.
$$\text{L.C.M.} = 2 \cdot 3a^2cx^3 = 6a^2cx^3.$$

10.
$$4x^2y = 2^2 \cdot x^2y.$$
$$6x^3yz^2 = 2 \cdot 3 \cdot x^3yz^2.$$
$$10xz^2 = 2 \cdot 5xz^2.$$
$$\text{L.C.M.} = 2^2 \cdot 3 \cdot 5x^3yz^2 = 60x^3yz^2.$$

11.
$$6a^2b = 2 \cdot 3a^2b.$$
$$9ab^2c = 3^2ab^2c.$$
$$30b^3c^3 = 2 \cdot 3 \cdot 5b^3c^3.$$
$$\text{L.C.M.} = 2 \cdot 3^2 \cdot 5a^2b^3c^3 = 90a^2b^3c^3.$$

12.
$$4c^2f = 2^2c^2f.$$
$$10df^2 = 2 \cdot 5df^2.$$
$$24cd^2e^3 = 2^3 \cdot 3cd^2e^3.$$
$$\text{L.C.M.} = 2^3 \cdot 3 \cdot 5c^2d^2e^3f^2 = 120c^2d^2e^3f^2.$$

13.
$$2a^2c = 2a^2c.$$
$$18ax^2k^3 = 2 \cdot 3^2 \cdot ax^2k^3.$$
$$45x^3k = 3^2 \cdot 5x^3k.$$
$$\text{L.C.M.} = 2 \cdot 3^2 \cdot 5a^2ck^3x^3 = 90a^2ck^3x^3.$$

14.
$$x^2 - xy = x(x-y).$$
$$2x^2y^3 = 2x^2y^3.$$
$$\text{L.C.M.} = 2x^2y^3(x-y).$$

15.
$$a^2 + ab = a(a+b).$$
$$2ab = 2ab.$$
$$\text{L.C.M.} = 2ab(a+b).$$

16.
$$5c - 10 = 5(c-2).$$
$$10c = 2 \cdot 5c.$$
$$\text{L.C.M.} = 10c(c-2).$$

17.
$$b^2 - bk = b(b-k).$$
$$b^2k = b^2k.$$
$$\text{L.C.M.} = b^2k(b-k).$$

18.
$$ac^2 = ac^2.$$
$$3a^2c = 3a^2c.$$
$$6a^2c - 9ac^2 = 3ac(2a - 3c).$$
$$\text{L.C.M.} = 3a^2c^2(2a - 3c).$$

19.
$$a^2 + ab = a(a+b).$$
$$ab + b^2 = b(a+b).$$
$$\text{L.C.M.} = ab(a+b).$$

21.
$$a^2 - ab = a(a-b).$$
$$2a - 2b = 2(a-b).$$
$$2a^2 - 2ab = 2a(a-b).$$
$$\text{L.C.M.} = 2a(a-b).$$

22.
$$ax - ay = a(x-y).$$
$$x^2 + xy = x(x+y).$$
$$x^2 - y^2 = (x+y)(x-y).$$
$$\text{L.C.M.} = ax(x+y)(x-y).$$

23.
$$a^2 - 4 = (a+2)(a-2).$$
$$a^2 + 3a + 2 = (a+2)(a+1).$$
$$a^2 - a - 2 = (a-2)(a+1).$$
$$\text{L.C.M.} = (a+2)(a-2)(a+1).$$

24.
$$4x^2 - 9 = (2x+3)(2x-3).$$
$$2x^2 + 7x + 6 = (2x+3)(x+2).$$
$$2x^2 + x - 6 = (2x-3)(x+2).$$
$$\text{L.C.M.} = (2x+3)(2x-3)(x+2).$$

25.
$$x^2 - 1 = (x+1)(x-1).$$
$$x^3 - 1 = (x-1)(x^2+x+1).$$
$$x^2 + 2x + 1 = (x+1)^2.$$
$$\text{L.C.M.} = (x+1)^2(x-1)(x^2+x+1).$$

26.
$$a^2 - 9 = (a+3)(a-3).$$
$$2a^3 + 6a^2 + 18a = 2a(a^2+3a+9).$$
$$2a^4 - 54a = 2a(a-3)(a^2+3a+9).$$
$$\text{L.C.M.} = 2a(a+3)(a-3)(a^2+3a+9).$$

27.
$$x^3 - 4x = x(x+2)(x-2).$$
$$4x^2 + 2x = 2x(2x+1).$$
$$2x^2 + 4x = 2x(x+2).$$
$$2x^2 - 3x - 2 = (2x+1)(x-2).$$
$$\text{L.C.M.} = 2x(x+2)(x-2)(2x+1).$$

Page 155

1. $\frac{x}{3}, \frac{y}{4}, \frac{z}{5}$. L.C.D. $= 60$.

$$\frac{x}{3} = \frac{20x}{60},\ \frac{y}{4} = \frac{15y}{60},\ \frac{z}{5} = \frac{12z}{60}.$$

2. $\frac{3x}{8}, \frac{5x^2}{12}, \frac{7x^3}{24}$. L.C.D. $= 24$.

$$\frac{3x}{8} = \frac{9x}{24},\ \frac{5x^2}{12} = \frac{10x^2}{24},\ \frac{7x^3}{24} = \frac{7x^3}{24}.$$

3. $\frac{5a}{7}, \frac{3ax}{14}, \frac{2ax^2}{21}$. L.C.D. $= 42$.

$$\frac{5a}{7} = \frac{30a}{42},\ \frac{3ax}{14} = \frac{9ax}{42},\ \frac{2ax^2}{21} = \frac{4ax^2}{42}.$$

4. $\frac{3a^2}{4}, \frac{6ab}{10}, \frac{15a^2b}{24}$. L.C.D. $= 40$.

$$\frac{3a^2}{4} = \frac{30a^2}{40},\ \frac{6ab}{10} = \frac{24ab}{40},\ \frac{15a^2b}{24} \text{ or } \frac{5a^2b}{8} = \frac{25a^2b}{40}.$$

5. $\frac{3}{5a}, \frac{2c}{3d}, \frac{5a}{10d}$. L.C.D. $= 30ad$.

$$\frac{3}{5a} = \frac{18d}{30ad},\ \frac{2c}{3d} = \frac{20ac}{30ad},\ \frac{5a}{10d} = \frac{15a^2}{30ad}.$$

6. $\frac{3x}{4y}, \frac{5x}{6y}, \frac{3}{10y}$. L.C.D. $= 60y$.

$$\frac{3x}{4y} = \frac{45x}{60y},\ \frac{5x}{6y} = \frac{50x}{60y},\ \frac{3}{10y} = \frac{18}{60y}.$$

7. $\dfrac{2a+b}{3a}, \dfrac{a}{2a^2}, \dfrac{5a^2}{4a}$. L.C.D. $= 12a$.

$$\frac{2a+b}{3a} = \frac{8a+4b}{12a},\ \frac{a}{2a^2} \text{ or } \frac{1}{2a} = \frac{6}{12a},\ \frac{5a^2}{4a} = \frac{15a^2}{12a}.$$

8. $\dfrac{c+d}{12c}, \dfrac{3c-d}{15c^2}, \dfrac{4a}{24c^3}$. L.C.D. $= 60c^3$.

$$\frac{c+d}{12c} = \frac{5c^3+5c^2d}{60c^3},\ \frac{3c-d}{15c^2} = \frac{12c^2-4cd}{60c^3},\ \frac{4a}{24c^3} \text{ or } \frac{a}{6c^3} = \frac{10a}{60c^3}.$$

9. $\dfrac{7a}{cde^2}, \dfrac{5e}{cd^2e}, \dfrac{4c}{c^2d^2e^2}$.

Reducing to lowest terms,

$\dfrac{7a}{cde^2}, \dfrac{5}{cd^2}, \dfrac{4}{cd^2e^2}$. L.C.D. $= cd^2e^2$.

$$\frac{7a}{cde^2} = \frac{7ad}{cd^2e^2},\ \frac{5}{cd^2} = \frac{5e^2}{cd^2e^2},\ \frac{4}{cd^2e^2} = \frac{4}{cd^2e^2}.$$

10. $\dfrac{7x}{3xy^2}, \dfrac{4y}{5xz}, \dfrac{2z}{10yz^2}$.

Reducing to lowest terms,

$\dfrac{7}{3y^2}, \dfrac{4y}{5xz}, \dfrac{1}{5yz}$. L.C.D. $= 15xy^2z$.

$$\frac{7}{3y^2} = \frac{35xz}{15xy^2z},\ \frac{4y}{5xz} = \frac{12y^3}{15xy^2z},\ \frac{1}{5yz} = \frac{3xy}{15xy^2z}.$$

12. $\dfrac{5}{a-2}, \dfrac{7}{a+2}$. L.C.D. $= (a-2)(a+2)$.

$$\frac{5}{a-2} = \frac{5(a+2)}{a^2-4},\ \frac{7}{a+2} = \frac{7(a-2)}{a^2-4}.$$

13. $\dfrac{a}{a-b}, \dfrac{b}{a+b}$. L.C.D. $= (a-b)(a+b)$.

$$\frac{a}{a-b} = \frac{a(a+b)}{a^2-b^2},\ \frac{b}{a+b} = \frac{b(a-b)}{a^2-b^2}.$$

14. $\dfrac{x}{2x-5}, \dfrac{3x}{2x+5}$. L.C.D. $= (2x-5)(2x+5)$.

$$\frac{x}{2x-5} = \frac{x(2x+5)}{4x^2-25},\ \frac{3x}{2x+5} = \frac{3x(2x-5)}{4x^2-25}.$$

15. $\dfrac{-2c}{c-3}, \dfrac{3c}{c+3}$. L.C.D. $= (c-3)(c+3)$.

$$\frac{-2c}{c-3} = \frac{-2c(c+3)}{c^2-9},\ \frac{3c}{c+3} = \frac{3c(c-3)}{c^2-9}.$$

17. $\dfrac{x}{x+y}, \dfrac{2x}{x-y}, \dfrac{2}{x}$. L.C.D. $= (x+y)(x-y)x$.

$$\frac{x}{x+y} = \frac{x^2(x-y)}{x(x+y)(x-y)}, \quad \frac{2x}{x-y} = \frac{2x^2(x+y)}{x(x+y)(x-y)}, \quad \frac{2}{x} = \frac{2(x+y)(x-y)}{x(x+y)(x-y)}.$$

18. $\dfrac{2a}{a-3}, \dfrac{3}{a+3}, \dfrac{-a}{2}$. L.C.D. $= 2(a-3)(a+3)$.

$$\frac{2a}{a-3} = \frac{4a(a+3)}{2(a+3)(a-3)}, \quad \frac{3}{a+3} = \frac{6(a-3)}{2(a+3)(a-3)},$$

$$\frac{-a}{2} = \frac{-a(a+3)(a-3)}{2(a-3)(a+3)}.$$

19. $\dfrac{5c}{c-5}, \dfrac{-2c}{c-5}, \dfrac{3}{c}$. L.C.D. $= c(c-5)$.

$$\frac{5c}{c-5} = \frac{5c^2}{c(c-5)}, \quad \frac{-2c}{c-5} = \frac{-2c^2}{c(c-5)}, \quad \frac{3}{c} = \frac{3(c-5)}{c(c-5)}.$$

20. $\dfrac{m}{m-n}, \dfrac{-2m}{m-n}, \dfrac{2}{m}$. L.C.D. $= (m-n)m$.

$$\frac{m}{m-n} = \frac{m^2}{m(m-n)}, \quad \frac{-2m}{m-n} = \frac{-2m^2}{m(m-n)}, \quad \frac{2}{m} = \frac{2(m-n)}{m(m-n)}.$$

21. $\dfrac{5}{a-2}, \dfrac{2}{3a-6}$.

$\dfrac{5}{a-2}, \dfrac{2}{3(a-2)}$. L.C.D. $= 3(a-2)$.

$$\frac{5}{a-2} = \frac{15}{3(a-2)}, \quad \frac{2}{3(a-2)} = \frac{2}{3(a-2)}.$$

22. $\dfrac{a}{x^2-xy}, \dfrac{5}{x-y}$.

$\dfrac{a}{x(x-y)}, \dfrac{5}{x-y}$. L.C.D. $= x(x-y)$.

$$\frac{a}{x(x-y)} = \frac{a}{x(x-y)}, \quad \frac{5}{x-y} = \frac{5x}{x(x-y)}.$$

23. $\dfrac{3a}{a^2-b^2}, \dfrac{2a}{a+b}$.

$\dfrac{3a}{(a+b)(a-b)}, \dfrac{2a}{a+b}$. L.C.D. $= (a+b)(a-b)$.

$$\frac{3a}{(a+b)(a-b)}, \quad \frac{2a}{(a+b)} = \frac{2a(a-b)}{(a+b)(a-b)}.$$

24. $\dfrac{5a}{a^2-b^2}, \dfrac{-4a}{a-b}.$

$\dfrac{5a}{(a+b)(a-b)}, \dfrac{-4a}{a-b}.$ L.C.D. $=(a+b)(a-b).$

$\dfrac{5a}{(a+b)(a-b)}, \dfrac{-4a}{a-b}=\dfrac{-4a(a+b)}{(a+b)(a-b)}.$

25. $\dfrac{-3x}{x^2-y^2}, \dfrac{5x}{x-y}.$

$\dfrac{-3x}{(x+y)(x-y)}, \dfrac{5x}{x-y}.$ L.C.D. $=(x+y)(x-y).$

$\dfrac{-3x}{(x+y)(x-y)}, \dfrac{5x(x+y)}{(x+y)(x-y)}.$

26. $\dfrac{2x+3}{9-x^2}, \dfrac{3-2x}{9+3x}, \dfrac{x}{3-x}.$

$\dfrac{2x+3}{(3+x)(3-x)}, \dfrac{3-2x}{3(3+x)}, \dfrac{x}{3-x}.$ L.C.D. $=3(3+x)(3-x).$

$\dfrac{2x+3}{(3+x)(3-x)}=\dfrac{3(2x+3)}{3(3+x)(3-x)}, \dfrac{3-2x}{3(3+x)}=\dfrac{(3-2x)(3-x)}{3(3+x)(3-x)},$

$\dfrac{x}{3-x}=\dfrac{3x(3+x)}{3(3-x)(3+x)}.$

27. $\dfrac{-2x}{x-2}, \dfrac{3}{x^2-4}, \dfrac{5x}{x^2-5x+6}.$

$\dfrac{-2x}{x-2}, \dfrac{3}{(x+2)(x-2)}, \dfrac{5x}{(x-2)(x-3)}.$ L.C.D. $=(x+2)(x-2)(x-3).$

$\dfrac{-2x}{x-2}=\dfrac{-2x(x+2)(x-3)}{(x+2)(x-2)(x-3)}, \dfrac{3}{(x+2)(x-2)}=\dfrac{3(x-3)}{(x+2)(x-2)(x-3)},$

$\dfrac{5x}{(x-2)(x-3)}=\dfrac{5x(x+2)}{(x+2)(x-2)(x-3)}.$

28. $\dfrac{2a-7}{a^3-9a}, \dfrac{3-5a}{a^3-5a^2+6a}, \dfrac{2}{a-3}.$

$\dfrac{2a-7}{a(a+3)(a-3)}, \dfrac{3-5a}{a(a-2)(a-3)}, \dfrac{2}{a-3}.$ L.C.D. $=a(a+3)(a-3)(a-2).$

$\dfrac{2a-7}{a(a+3)(a-3)}=\dfrac{(2a-7)(a-2)}{a(a+3)(a-3)(a-2)},$

$\dfrac{3-5a}{a(a-2)(a-3)}=\dfrac{(3-5a)(a+3)}{a(a-2)(a-3)(a+3)},$

$\dfrac{2}{a-3}=\dfrac{2a(a-2)(a+3)}{a(a-2)(a+3)(a-3)}.$

29. $\dfrac{a}{a^3-8}, \dfrac{2a}{a^3+2a^2+4a}.$

$\dfrac{a}{(a-2)(a^2+2a+4)}, \dfrac{2}{a^2+2a+4}.$ L.C.D. $=(a-2)(a^2+2a+4).$

$\dfrac{a}{(a-2)(a^2+2a+4)}, \dfrac{2}{a^2+2a+4}=\dfrac{2(a-2)}{(a-2)(a^2+2a+4)}.$

30. $\dfrac{5x}{2x^2+3x}, \dfrac{3x}{2x^2-x-6}.$

$\dfrac{5}{(2x+3)}, \dfrac{3x}{(2x+3)(x-2)}.$ L.C.D. $=(2x+3)(x-2).$

$\dfrac{5}{2x+3}=\dfrac{5(x-2)}{(2x+3)(x-2)}, \dfrac{3x}{(2x+3)(x-2)}.$

31. $\dfrac{-4x}{3-x}, \dfrac{5x}{3+8x-3x^2}.$

$\dfrac{-4x}{3-x}, \dfrac{5x}{(3-x)(1+3x)}.$ L.C.D. $=(3-x)(1+3x).$

$\dfrac{-4x}{3-x}=\dfrac{-4x(1+3x)}{(3-x)(1+3x)}; \dfrac{5x}{(3-x)(1+3x)}.$

32. $\dfrac{-1}{x+y-3}, \dfrac{5x+y}{x^2+y^2-9+2xy}.$

$\dfrac{-1}{x+y-3}, \dfrac{5x+y}{(x+y+3)(x+y-3)}.$ L.C.D. $=(x+y+3)(x+y-3).$

$\dfrac{-1}{x+y-3}=\dfrac{-1(x+y+3)}{(x+y-3)(x+y+3)}; \dfrac{5x+y}{(x+y+3)(x+y-3)}.$

Page 160

1. $\frac{3}{4}+\frac{5}{6}$. L.C.D. $=12$.
$\frac{9}{12}+\frac{10}{12}=\frac{19}{12}$.

2. $\frac{3}{10}+\frac{8}{15}$. L.C.D. $=30$.
$\frac{9}{30}+\frac{16}{30}=\frac{25}{30}$, or $\frac{5}{6}$.

3. $\frac{9}{25}-\frac{3}{10}$. L.C.D. $=50$.
$\frac{18}{50}-\frac{15}{50}=\frac{3}{50}$.

4. $\frac{5}{12}-\frac{7}{18}$. L.C.D. $=36$.
$\frac{15}{36}-\frac{14}{36}=\frac{1}{36}$.

5. $\frac{3}{14}+\frac{2}{21}$. L.C.D. $=42$.
$\frac{9}{42}+\frac{4}{42}=\frac{13}{42}$.

6. $\frac{5}{6}-\frac{3}{16}$. L.C.D. $=48$.
$\frac{40}{48}-\frac{9}{48}=\frac{31}{48}$.

7. $\frac{7}{24}-\frac{15}{32}$. L.C.D. $=96$.
$\dfrac{28}{96}-\dfrac{45}{96}=\dfrac{-17}{96}.$

8. $\frac{17}{40}+\frac{19}{48}$. L.C.D. $=240$.
$\frac{102}{240}+\frac{95}{240}=\frac{197}{240}$.

9. $\frac{15}{28}-\frac{87}{40}$. L.C.D. $=280$.
$\dfrac{150}{280}-\dfrac{609}{280}=\dfrac{-459}{280}.$

10. $\frac{43}{25}+2\frac{29}{45}$. L.C.D. $=225$.
$\frac{387}{225}+\frac{595}{225}=\frac{982}{225}$.

11. $\frac{19}{24}-\frac{89}{54}$. L.C.D. $=216$.
$\dfrac{171}{216}-\dfrac{356}{216}=\dfrac{-185}{216}.$

12. $\frac{55}{38}+\frac{43}{57}$. L.C.D. $=114$.

$\frac{165}{114}+\frac{86}{114}=\frac{251}{114}$.

13. $\frac{7}{6}-1\frac{4}{9}+3\frac{7}{12}$. L.C.D. $=36$.

$$\frac{42}{36}-\frac{52}{36}+\frac{129}{36}=\frac{42-52+129}{36}=\frac{119}{36}.$$

14. $\frac{11}{10}-\frac{13}{12}+2\frac{5}{16}$. L.C.D. $=240$.

$$\frac{264}{240}-\frac{260}{240}+\frac{555}{240}=\frac{264-260+555}{240}=\frac{559}{240}.$$

15. $\frac{13}{32}+1-3\frac{9}{48}$. L.C.D. $=96$.

$$\frac{39}{96}+\frac{96}{96}-\frac{306}{96}=\frac{39+96-306}{96}=-\frac{171}{96}.$$

16. $\frac{31}{36}-\frac{41}{32}+\frac{11}{42}$. L.C.D. $=2016$.

$$\frac{1736}{2016}-\frac{2583}{2016}+\frac{528}{2016}=\frac{-319}{2016}.$$

17. $\frac{15}{26}-\frac{11}{39}-\frac{25}{52}$. L.C.D. $=156$.

$$\frac{90}{156}-\frac{44}{156}-\frac{75}{156}=\frac{-29}{156}.$$

18. $\frac{23}{35}-\frac{19}{45}+\frac{27}{50}$. L.C.D. $=3150$.

$\frac{2070}{3150}-\frac{1330}{3150}+\frac{1701}{3150}=\frac{2441}{3150}$.

19. $\frac{19}{48}-\frac{13}{72}+\frac{43}{36}$. L.C.D. $=144$.

$\frac{57}{144}-\frac{26}{144}+\frac{172}{144}=\frac{203}{144}$.

20. $\frac{11}{64}+2-\frac{17}{24}$. L.C.D. $=192$.

$\frac{33}{192}+\frac{384}{192}-\frac{136}{192}=\frac{281}{192}$.

21. $\frac{14}{69}-\frac{15}{46}+\frac{19}{115}$. L.C.D. $=690$.

$\frac{140}{690}-\frac{225}{690}+\frac{114}{690}=\frac{29}{690}$.

22. $\frac{3a}{2}+\frac{4a}{3}$. L.C.D. $=6$.

$$\frac{9a}{6}+\frac{8a}{6}=\frac{17a}{6}.$$

23. $\frac{2a}{5}+\frac{3a}{2}+\frac{11a}{10}$. L.C.D. $=10$.

$$\frac{4a}{10}+\frac{15a}{10}+\frac{11a}{10}=3a.$$

24. $\frac{7a}{10}-\frac{6a}{15}+\frac{3a}{5}$. L.C.D. $=30$.

$$\frac{21a}{30}-\frac{12a}{30}+\frac{18a}{30}=\frac{9a}{10}.$$

25. $\frac{4x}{7}+\frac{2x}{21}-\frac{3x}{14}$. L.C.D. $=42$.

$$\frac{24x}{42}+\frac{4x}{42}-\frac{9x}{42}=\frac{19x}{42}.$$

26. $\frac{a+2}{8}-\frac{3a-2}{10}$. L.C.D. $=40$.

$$\frac{5a+10}{40}-\frac{12a-8}{40}=\frac{5a+10-12a+8}{40}=\frac{18-7a}{40}.$$

27. $\dfrac{5x-3}{12}-\dfrac{2x-7}{14}$. L.C.D. $=84$.

$$\frac{35x-21}{84}-\frac{12x-42}{84}=\frac{35x-21-12x+42}{84}=\frac{23x+21}{84}.$$

28. $\dfrac{3a-5}{4}-\dfrac{2-a}{6}+\dfrac{7a}{8}$. L.C.D. $=24$.

$$\frac{18a-30}{24}-\frac{8-4a}{24}+\frac{21a}{24}=\frac{18a-30-8+4a+21a}{24}=\frac{43a-38}{24}.$$

29. $\dfrac{x-5a}{10}-\dfrac{3x+2a}{18}-\dfrac{a-7x}{20}$. L.C.D. $=180$.

$$\frac{18x-90a}{180}-\frac{30x+20a}{180}-\frac{9a-63x}{180}$$

$$=\frac{18x-90a-30x-20a-9a+63x}{180}$$

$$=\frac{51x-119a}{180}.$$

30. $\dfrac{4m-1}{6}-\dfrac{a-m}{8}-\dfrac{m+3a-5}{30}$. L.C.D. $=120$.

$$\frac{80m-20}{120}-\frac{15a-15m}{120}-\frac{4m+12a-20}{120}$$

$$=\frac{80m-20-15a+15m-4m-12a+20}{120}$$

$$=\frac{91m-27a}{120}.$$

31. $\dfrac{3}{x}+\dfrac{2}{3x^2}$. L.C.D. $=3x^2$.

$$\frac{9x}{3x^2}+\frac{2}{3x^2}=\frac{9x+2}{3x^2}.$$

32. $\dfrac{2}{3a}+\dfrac{5}{2a^2}-\dfrac{10a}{4a^3}$. L.C.D. $=12a^2$.

$$\frac{8a}{12a^2}+\frac{30}{12a^2}-\frac{30}{12a^2}=\frac{2}{3a}.$$

33. $\dfrac{2a}{5a^3}-\dfrac{3}{10a}-\dfrac{5}{6a^2}$. L.C.D. $=30a^2$.

$$\frac{12}{30a^2}-\frac{9a}{30a^2}-\frac{25}{30a^2}=\frac{-9a-13}{30a^2}.$$

34. $\dfrac{a}{x}+\dfrac{x}{a}-\dfrac{2}{ax}$. L.C.D. $=ax$.

$$\frac{a^2}{ax}+\frac{x^2}{ax}-\frac{2}{ax}=\frac{a^2+x^2-2}{ax}.$$

35. $\dfrac{a}{c}-\dfrac{1}{ac}-\dfrac{3}{2a}$. L.C.D. $=2ac$.

$$\frac{2a^2}{2ac}-\frac{2}{2ac}-\frac{3c}{2ac}=\frac{2a^2-3c-2}{2ac}.$$

36. $\dfrac{7}{3a^2}-\dfrac{2}{ax}+\dfrac{3}{x^2}$. L.C.D. $=3a^2x^2$.

$$\frac{7x^2}{3a^2x^2}-\frac{6ax}{3a^2x^2}+\frac{9a^2}{3a^2x^2}=\frac{7x^2-6ax+9a^2}{3a^2x^2}.$$

37. $\dfrac{2a}{3x}+\dfrac{11}{2ax^2}-\dfrac{5x}{a}$. L.C.D. $=6ax^2$.

$$\frac{4a^2x}{6ax^2}+\frac{33}{6ax^2}-\frac{30x^3}{6ax^2}=\frac{4a^2x+33-30x^3}{6ax^2}.$$

38. $\dfrac{3a+1}{5a}-\dfrac{4a+3}{3a^2}-\dfrac{5-a}{6a}$. L.C.D. $=30a^2$.

$$\frac{18a^2+6a}{30a^2}-\frac{40a+30}{30a^2}-\frac{25a-5a^2}{30a^2}$$

$$=\frac{18a^2+6a-40a-30-25a+5a^2}{30a^2}$$

$$=\frac{23a^2-59a-30}{30a^2}.$$

39. $\dfrac{4x^2-5}{3x^2}-\dfrac{2-3x}{2x}+\dfrac{3x-7}{5x^3}$. L.C.D. $=30x^3$.

$$\frac{40x^3-50x}{30x^3}-\frac{30x^2-45x^3}{30x^3}+\frac{18x-42}{30x^3}$$

$$=\frac{40x^3-50x-30x^2+45x^3+18x-42}{30x^3}$$

$$=\frac{85x^3-30x^2-32x-42}{30x^3}.$$

40. $\dfrac{2x}{5x^2y}-\dfrac{3y-1}{10xy^2}-\dfrac{4xy+3}{15x^2y^2}$. L.C.D. $=30x^2y^2$.

$$\frac{12xy}{30x^2y^2}-\frac{9xy-3x}{30x^2y^2}-\frac{8xy+6}{30x^2y^2}=\frac{12xy-9xy+3x-8xy-6}{30x^2y^2}$$

$$=\frac{3x-6-5xy}{30x^2y^2}.$$

42. $\dfrac{12}{x^2-9}-\dfrac{2}{x^2-5x+6}$.

$$\frac{12}{(x+3)(x-3)}-\frac{2}{(x-2)(x-3)}.\quad \text{L.C.D.}=(x+3)(x-3)(x-2).$$

$$\frac{12(x-2)-2(x+3)}{(x+3)(x-3)(x-2)}=\frac{12x-24-2x-6}{(x+3)(x-3)(x-2)}$$

$$=\frac{10x-30}{(x+3)(x-3)(x-2)}$$

$$=\frac{10}{(x+3)(x-2)}.$$

43. $\dfrac{7}{a^2-49}-\dfrac{2}{a^2-6a-7}.$

$$\frac{7}{(a+7)(a-7)}-\frac{2}{(a-7)(a+1)}.\quad \text{L.C.D.}=(a+7)(a-7)(a+1).$$

$$\frac{7(a+1)-2(a+7)}{(a+7)(a-7)(a+1)}=\frac{7a+7-2a-14}{(a+7)(a-7)(a+1)}=\frac{5a-7}{(a+7)(a-7)(a+1)}.$$

44. $\dfrac{5}{4x^2-1}-\dfrac{3x}{8x^3-1}.$

$$\frac{5}{(2x+1)(2x-1)}-\frac{3x}{(2x-1)(4x^2+2x+1)}.$$

L.C.D. $=(2x+1)(2x-1)(4x^2+2x+1).$

$$\frac{5(4x^2+2x+1)-3x(2x+1)}{(2x+1)(2x-1)(4x^2+2x+1)}=\frac{20x^2+10x+5-6x^2-3x}{(2x+1)(2x-1)(4x^2+2x+1)}$$

$$=\frac{14x^2+7x+5}{(2x+1)(2x-1)(4x^2+2x+1)}.$$

45. $\dfrac{4c+3}{1-c^2}+\dfrac{2c}{c+c^2}.$

$$\frac{4c+3}{(1+c)(1-c)}+\frac{2c}{c(1+c)}.\quad \text{L.C.D.}=(1+c)(1-c)c.$$

$$\frac{(4c+3)c+2c(1-c)}{(1+c)(1-c)c}=\frac{4c^2+3c+2c-2c^2}{c(1+c)(1-c)}$$

$$=\frac{2c^2+5c}{c(1+c)(1-c)}$$

$$=\frac{2c+5}{(1+c)(1-c)}.$$

46. $\dfrac{2y+7}{3+y}-\dfrac{2y^2-35}{y^2-11y-42}.$

$$\frac{2y+7}{(3+y)}-\frac{2y^2-35}{(y-14)(y+3)}.\quad \text{L.C.D.}=(y+3)(y-14).$$

$$\frac{(2y+7)(y-14)-(2y^2-35)}{(y+3)(y-14)}=\frac{2y^2-21y-98-2y^2+35}{(y+3)(y-14)}$$

$$=\frac{-21y-63}{(y+3)(y-14)}$$

$$=\frac{-21\cancel{(y+3)}}{\cancel{(y+3)}(y-14)}$$

$$=\frac{-21}{y-14}.$$

47. $\frac{5a+1}{a+3} - \frac{7}{2a} + \frac{2}{3}$. L.C.D. $= 6a(a+3)$.

$$\frac{6a(5a+1)-21(a+3)+4a(a+3)}{6a(a+3)} = \frac{30a^2+6a-21a-63+4a^2+12a}{6a(a+3)}$$

$$= \frac{34a^2-3a-63}{6a(a+3)}.$$

48. $\frac{2}{x^2-7x} - \frac{3}{x} + \frac{3}{x-7}$. L.C.D. $= x(x-7)$.

$$\frac{2-3(x-7)+3x}{x(x-7)} = \frac{2-3x+21+3x}{(x-7)x} = \frac{23}{x(x-7)}.$$

49. $\frac{7a+b}{a^2-b^2} + \frac{2}{a-b} - \frac{3}{a+b}$. L.C.D. $= a^2-b^2$.

$$\frac{7a+b+2(a+b)-3(a-b)}{a^2-b^2} = \frac{7a+b+2a+2b-3a+3b}{a^2-b^2}$$

$$= \frac{6a+6b}{a^2-b^2} = \frac{6}{a-b}.$$

50. $\frac{5c^2}{4c^2-1} - \frac{3c+2}{2c+1} + \frac{5c}{2c-1}$. L.C.D. $= 4c^2-1$.

$$\frac{5c^2-(3c+2)(2c-1)+5c(2c+1)}{4c^2-1} = \frac{5c^2-6c^2-c+2+10c^2+5c}{4c^2-1}$$

$$= \frac{9c^2+4c+2}{4c^2-1}.$$

51. $\frac{m^3+m}{m^3-8} - \frac{3m+7}{m^3+2m^2+4m} - \frac{m-3}{m-2}$.

$$\frac{m(m^2+1)}{(m-2)(m^2+2m+4)} - \frac{3m+7}{m(m^2+2m+4)} - \frac{m-3}{m-2}.$$

L.C.D. $= m(m-2)(m^2+2m+4)$.

$$\frac{m^2(m^2+1)-(3m+7)(m-2)-m(m-3)(m^2+2m+4)}{m(m-2)(m^2+2m+4)}$$

$$= \frac{m^4+m^2-3m^2-m+14-m^4+m^3+2m^2+12m}{m(m-2)(m^2+2m+4)}$$

$$= \frac{m^3+11m+14}{m(m^3-8)}.$$

52. $\dfrac{a^2+2\,ab+b^2}{a^2-b^2}+\dfrac{ab}{a^2+ab}-\dfrac{2\,b}{ab-b^2}.$

$$\frac{\cancel{(a+b)}(a+b)}{\cancel{(a+b)}(a-b)}+\frac{\cancel{a}b}{(a+b)\cancel{a}}-\frac{2\cancel{b}}{\cancel{b}(a-b)}.\quad \text{L.C.D.}=(a+b)(a-b).$$

$$\frac{(a+b)^2+b(a-b)-2(a+b)}{(a+b)(a-b)}=\frac{a^2+2\,ab+b^2+ab-b^2-2\,a-2\,b}{(a+b)(a-b)}$$

$$=\frac{a^2+3\,ab-2\,a-2\,b}{(a+b)(a-b)}.$$

53. $\dfrac{x^2+3}{x^2-x-2}-\dfrac{x}{x-2}+\dfrac{x}{x^2+x}.$

$$\frac{x^2+3}{(x-2)(x+1)}-\frac{x}{x-2}+\frac{1}{x+1}.\quad \text{L.C.D.}=(x-2)(x+1).$$

$$\frac{x^2+3-x(x+1)+x-2}{(x-2)(x+1)}=\frac{x^2+3-x^2-x+x-2}{(x-2)(x+1)}=\frac{1}{(x-2)(x+1)}.$$

54. $\dfrac{a}{a^2-ab}+\dfrac{b+a}{a^2-2\,ab+b^2}+\dfrac{2\,b}{-ab+a^2}.$

$$\frac{1}{a-b}+\frac{a+b}{(a-b)^2}+\frac{2\,b}{a(a-b)}.\quad \text{L.C.D.}=a(a-b)^2.$$

$$\frac{a(a-b)+a(a+b)+2\,b(a-b)}{a(a-b)^2}=\frac{a^2-ab+a^2+ab+2\,ab-2\,b^2}{a(a-b)^2}$$

$$=\frac{2\,a^2+2\,ab-2\,b^2}{a(a-b)^2}.$$

55. $\dfrac{cx^2}{c^2-x^2}-\dfrac{2\,cx}{c^2-cx}-\dfrac{cx^2(c-2)}{c^3-cx^2}.$

$$\frac{cx^2}{(c+x)(c-x)}-\frac{2\cancel{c}x}{\cancel{c}(c-x)}-\frac{\cancel{c}x^2(c-2)}{\cancel{c}(c+x)(c-x)}.\quad \text{L.C.D.}=(c+x)(c-x).$$

$$\frac{cx^2-2\,x(c+x)-x^2(c-2)}{(c+x)(c-x)}=\frac{cx^2-2\,cx-2\,x^2-cx^2+2\,x^2}{(c+x)(c-x)}$$

$$=\frac{-2\,cx}{c^2-x^2}.$$

56. $\dfrac{a+c}{a-c}-\dfrac{a^2-ac+c^2}{a^2+ac+c^2}-\dfrac{4\,a^2c}{a^3-c^3}.\quad \text{L.C.D.}=a^3-c^3.$

$$\frac{(a+c)(a^2+ac+c^2)-(a-c)(a^2-ac+c^2)-4\,a^2c}{a^3-c^3}$$

$$=\frac{a^3+2\,a^2c+2\,ac^2+c^3-a^3+2\,a^2c-2\,ac^2+c^3-4\,a^2c}{a^3-c^3}$$

$$=\frac{2\,c^3}{a^3-c^3}.$$

57. $\dfrac{x^2+3x+9}{x^2-3x+9}-\dfrac{x-3}{x+3}-\dfrac{54}{x^3+27}$. L.C.D. $= x^3+27$.

$$\frac{(x^2+3x+9)(x+3)-(x-3)(x^2-3x+9)-54}{x^3+27}$$

$$=\frac{x^3+6x^2+18x+27-x^3+6x^2-18x+27-54}{x^3+27}$$

$$=\frac{12x^2}{x^3+27}.$$

58. $\dfrac{x-5}{2x+5}+\dfrac{-5x+x^2}{2x^2+15x+25}-\dfrac{2x-5}{x+5}$.

$\dfrac{x-5}{2x+5}+\dfrac{x^2-5x}{(2x+5)(x+5)}-\dfrac{2x-5}{x+5}$. L.C.D. $=(2x+5)(x+5)$.

$$\frac{(x-5)(x+5)+x^2-5x-(2x-5)(2x+5)}{(2x+5)(x+5)}$$

$$=\frac{x^2-25+x^2-5x-4x^2+25}{(2x+5)(x+5)}=\frac{-2x^2-5x}{(2x+5)(x+5)}.$$

$$=\frac{-x}{x+5}.$$

Page 163

1. $\dfrac{a}{-h}=\dfrac{-a}{h}=-\dfrac{-a}{-h}=-\dfrac{a}{h}.$

2. $\dfrac{-c}{2d}=\dfrac{c}{-2d}=-\dfrac{c}{2d}=-\dfrac{-c}{-2d}.$

3. $-\dfrac{n}{d}=-\dfrac{-n}{-d}=\dfrac{-n}{d}=\dfrac{n}{-d}.$

4. $-\dfrac{2}{a-c}=-\dfrac{-2}{c-a}=\dfrac{-2}{a-c}=\dfrac{2}{c-a}.$

5. $\dfrac{-3}{x-y}=\dfrac{3}{y-x}=-\dfrac{3}{x-y}=-\dfrac{-3}{y-x}.$

6. $\dfrac{x-2}{2x-3}=\dfrac{2-x}{3-2x}=-\dfrac{2-x}{2x-3}=-\dfrac{x-2}{3-2x}.$

7. $-\dfrac{2a-b}{b^2-a^2}=-\dfrac{b-2a}{a^2-b^2}=\dfrac{b-2a}{b^2-a^2}=\dfrac{2a-b}{a^2-b^2}.$

8. $\dfrac{c-3d}{d^2-c^2}=\dfrac{3d-c}{c^2-d^2}=-\dfrac{3d-c}{d^2-c^2}=-\dfrac{c-3d}{c^2-d^2}.$

9. $\dfrac{x-5}{5x-6-x^2}=\dfrac{5-x}{x^2-5x+6}=-\dfrac{5-x}{5x-6-x^2}=-\dfrac{x-5}{x^2-5x+6}.$

10. $-\dfrac{2-x}{1-x^3}=-\dfrac{x-2}{x^3-1}=\dfrac{x-2}{1-x^3}=\dfrac{2-x}{x^3-1}.$

Page 164

1. $\dfrac{7}{x-5}+\dfrac{3}{5-x}=\dfrac{7}{x-5}-\dfrac{3}{x-5}=\dfrac{4}{x-5}.$

2. $\dfrac{a}{a-b}-\dfrac{b}{b-a}=\dfrac{a}{a-b}+\dfrac{b}{a-b}=\dfrac{a+b}{a-b}.$

3. $\dfrac{x^2-2a}{x-a}+\dfrac{a+x^2}{a-x}=\dfrac{x^2-2a}{x-a}-\dfrac{a+x^2}{x-a}=\dfrac{-3a}{x-a}=\dfrac{3a}{a-x}.$

4. $\dfrac{x-2}{3-x}-\dfrac{2+x}{x-3}+\dfrac{3x}{x-3}=\dfrac{2-x}{x-3}-\dfrac{2+x}{x-3}+\dfrac{3x}{x-3}=\dfrac{x}{x-3}.$

5. $\dfrac{a}{5-a}+\dfrac{a^2+5a}{a^2-25}=\dfrac{-a(a+5)}{a^2-25}+\dfrac{a^2+5a}{a^2-25}=0.$

6. $\dfrac{2}{c^2-9}-\dfrac{9}{3-c}=\dfrac{2}{c^2-9}+\dfrac{9}{c-3}=\dfrac{2}{c^2-9}+\dfrac{9(c+3)}{c^2-9}=\dfrac{9c+29}{c^2-9}.$

7. $\dfrac{3}{7-x}-\dfrac{4}{x^2-49}=\dfrac{3}{7-x}+\dfrac{4}{49-x^2}=\dfrac{3(7+x)+4}{49-x^2}=\dfrac{25+3x}{49-x^2}.$

8. $\dfrac{2a+5}{5a+1}+\dfrac{2a-5}{1-5a}-\dfrac{46a+1}{25a^2-1}$

$$=\frac{(5a-1)(2a+5)-(2a-5)(5a+1)-(46a+1)}{25a^2-1}$$

$$=\frac{10a^2+23a-5-10a^2+23a+5-46a-1}{25a^2-1}$$

$$=\frac{-1}{25a^2-1}.$$

9. $\dfrac{a-b}{3a+2b}-\dfrac{10ab}{4b^2-9a^2}-\dfrac{a+b}{3a-2b}=\dfrac{a-b}{3a+2b}+\dfrac{10ab}{9a^2-4b^2}-\dfrac{a+b}{3a-2b}$

$$=\frac{(a-b)(3a-2b)+10ab-(a+b)(3a+2b)}{9a^2-4b^2}$$

$$=\frac{3a^2-5ab+2b^2+10ab-3a^2-5ab-2b^2}{9a^2-4b^2}$$

$$=0.$$

10. $\dfrac{2x+1}{2x-1}-\dfrac{-6x+1}{1-4x^2}-\dfrac{2x-1}{2x+1}=\dfrac{2x+1}{2x-1}+\dfrac{1-6x}{4x^2-1}-\dfrac{2x-1}{2x+1}$

$$=\frac{(2x+1)^2+1-6x-(2x-1)^2}{4x^2-1}$$

$$=\frac{4x^2+4x+1+1-6x-4x^2+4x-1}{4x^2-1}$$

$$=\frac{1}{2x-1}.$$

11. $$\frac{-11a+46-6a^2}{a^2+5a-14}+\frac{2a-5}{a+7}+\frac{7-4a}{2-a}$$
$$=\frac{-11a+46-6a^2}{a^2+5a-14}+\frac{(2a-5)(a-2)}{(a+7)(a-2)}+\frac{(4a-7)(a+7)}{(a-2)(a+7)}$$
$$=\frac{-11a+46-6a^2+2a^2-9a+10+4a^2+21a-49}{a^2+5a-14}$$
$$=\frac{a+7}{a^2+5a-14}$$
$$=\frac{1}{a-2}.$$

Page 166

1. $a+\frac{2}{x}=\frac{ax+2}{x}.$

2. $b-\frac{c}{d}=\frac{bd-c}{d}.$

3. $2c+\frac{c}{a}=\frac{2ac+c}{a}.$

4. $5x-\frac{x}{d}=\frac{5dx-x}{d}.$

5. $\frac{m}{n}-2=\frac{m-2n}{n}.$

6. $\frac{c}{5d}+3=\frac{c+15d}{5d}.$

7. $s+\frac{s}{2}=\frac{2s+s}{2}=\frac{3s}{2}.$

8. $x^2-\frac{3x^2}{5}=\frac{5x^2-3x^2}{5}=\frac{2x^2}{5}.$

9. $s^2-\left(\frac{s}{2}\right)^2=s^2-\frac{s^2}{4}=\frac{4s^2-s^2}{4}=\frac{3s^2}{4}.$

10. $x^2-\left(\frac{2x}{3}\right)^2=x^2-\frac{4x^2}{9}=\frac{9x^2-4x^2}{9}=\frac{5x^2}{9}.$

11. $2a^2-\frac{2a^2}{3}=\frac{6a^2-2a^2}{3}=\frac{4a^2}{3}.$

12. $a+2+\frac{a-2}{2}=\frac{2a+4+a-2}{2}=\frac{3a+2}{2}.$

13. $c-d+\frac{c^2-d^2}{c+d}=\frac{c^2-d^2+c^2-d^2}{c+d}=\frac{2c^2-2d^2}{c+d}=\frac{2(c-d)}{1}.$

14. $x-2y-\frac{x^2+y^2}{x+2y}=\frac{x^2-4y^2-x^2-y^2}{x+2y}=\frac{-5y^2}{x+2y}.$

15. $a^2+b^2-\frac{(a+b)^2}{2}=\frac{2a^2+2b^2-a^2-2ab-b^2}{2}=\frac{a^2-2ab+b^2}{2}.$

16. $5a-7-\frac{25a^2}{5a+7}=\frac{25a^2-49-25a^2}{5a+7}=\frac{-49}{5a+7}.$

17. $m^2+mn+n^2-\frac{2m^3-n^3}{m-n}=\frac{m^3-n^3-2m^3+n^3}{m-n}=\frac{m^3}{n-m}.$

18. $x^2-xy-\frac{x^3-y^3}{x+y}+y^2=\frac{x^3+y^3-x^3+y^3}{x+y}=\frac{2y^3}{x+y}.$

19. $5-\frac{125-2x^3}{25+5x+x^2}-x=\frac{125-x^3-125+2x^3}{25+5x+x^2}=\frac{x^3}{25+5x+x^2}.$

20. $\dfrac{x+2}{x-2}-3+\dfrac{x-3}{2x+3}=\dfrac{(x+2)(2x+3)-3(x-2)(2x+3)+(x-3)(x-2)}{(2x+3)(x-2)}$

$$=\frac{2x^2+7x+6-6x^2+3x+18+x^2-5x+6}{(2x+3)(x-2)}$$

$$=\frac{-3x^2+5x+30}{(2x+3)(x-2)}.$$

21. $x^3+4x-\dfrac{x^4-18}{x-2}+8+2x^2=\dfrac{x^4-16-x^4+18}{x-2}=\dfrac{2}{x-2}.$

22. $b+2-\dfrac{4b}{2b-1}-\left(2b-1-\dfrac{b^2+9b}{b+5}\right)$

$$=b+2-\frac{14b}{2b-1}-2b+1+\frac{b^2+9b}{b+5}$$

$$=3-b-\frac{14b}{2b-1}+\frac{b^2+9b}{b+5}$$

$$=\frac{(3-b)(2b-1)(b+5)-14b(b+5)+(b^2+9b)(2b-1)}{(2b-1)(b+5)}$$

$$=\frac{-2b^3-3b^2+32b-15-14b^2-70b+2b^3+17b^2-9b}{(2b-1)(b+5)}$$

$$=\frac{-47b-15}{(2b-1)(b+5)}.$$

23. $\left(2+\dfrac{6a}{b}\right)-\left(3-\dfrac{4a}{3b}\right)=2+\dfrac{6a}{b}-3+\dfrac{4a}{3b}=\dfrac{22a}{3b}-1=\dfrac{22a-3b}{3b}.$

24. $\left(3-\dfrac{7}{2ac}\right)-\left(2-\dfrac{5}{3c^2}\right)=3-\dfrac{7}{2ac}-2+\dfrac{5}{3c^2}=1-\dfrac{7}{2ac}+\dfrac{5}{3c^2}$

$$=\frac{6ac^2-21c+10a}{6ac^2}.$$

25. $\left(x+\dfrac{4y^2}{x-2y}\right)-\left(\dfrac{x^2}{x+2y}+2y\right)=x+\dfrac{4y^2}{x-2y}-\dfrac{x^2}{x+2y}-2y$

$$=\frac{(x-2y)^2(x+2y)+4y^2(x+2y)-x^2(x-2y)}{(x+2y)(x-2y)}$$

$$=\frac{x^3-2x^2y-4xy^2+8y^3+4xy^2+8y^3-x^3+2x^2y}{(x+2y)(x-2y)}$$

$$=\frac{16y^3}{(x+2y)(x-2y)}.$$

Page 168

1. $\dfrac{\overset{\overset{2}{\cancel{6}}}{\cancel{12}}}{\underset{5}{\cancel{15}}}\cdot\dfrac{\cancel{10}}{\underset{\cancel{2}}{\cancel{20}}}=\dfrac{2}{5}.$ **2.** $\dfrac{\overset{2}{\cancel{4}}}{\underset{7}{\cancel{21}}}\cdot\dfrac{\overset{4}{\cancel{12}}}{\underset{5}{\cancel{10}}}=\dfrac{8}{35}.$ **3.** $\dfrac{\overset{2}{\cancel{4}}}{\underset{\cancel{3}}{\cancel{9}}}\cdot\dfrac{\overset{5}{\cancel{15}}}{\underset{11}{\cancel{22}}}\cdot\cancel{3}=\dfrac{10}{11}.$

4. $1\frac{1}{6}\cdot\frac{8}{21}=\frac{\cancel{7}}{\underset{3}{\cancel{6}}}\cdot\frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{21}}}=\frac{4}{9}.$

5. $-\frac{\cancel{14}}{\cancel{11}}\cdot\frac{\overset{\cancel{11}}{\cancel{33}}}{\underset{\cancel{3}}{\cancel{42}}}=-1.$

6. $2\frac{4}{7}\cdot-1\frac{17}{18}=\frac{\cancel{18}}{\cancel{7}}\cdot-\frac{\overset{5}{\cancel{35}}}{\cancel{18}}=-5.$

7. $2\frac{1}{4}\cdot\frac{1}{21}\cdot1\frac{7}{8}=\frac{9}{4}\cdot\frac{1}{\underset{7}{\cancel{21}}}\cdot\frac{\overset{5}{\cancel{15}}}{8}=\frac{45}{224}.$

8. $-1\frac{1}{11}\cdot1\frac{11}{21}=-\frac{\overset{4}{\cancel{12}}}{11}\cdot\frac{32}{\underset{7}{\cancel{21}}}=-\frac{128}{77}.$

9. $\frac{\cancel{14}}{\underset{3}{\cancel{24}}}\cdot\overset{2}{\cancel{16}}\cdot\frac{32}{\underset{5}{\cancel{70}}}=\frac{64}{15}.$

10. $\frac{22}{7}\cdot6^2\cdot\frac{1}{144}=\frac{\overset{11}{\cancel{22}}}{7}\cdot\cancel{36}\cdot\frac{1}{\underset{\underset{2}{\cancel{4}}}{\cancel{144}}}=\frac{11}{14}.$

11. $\frac{4}{3}\cdot\frac{22}{7}\cdot(2\frac{1}{2})^3=\frac{\cancel{4}}{3}\cdot\frac{\overset{11}{\cancel{22}}}{7}\cdot\frac{125}{\underset{\cancel{2}}{\cancel{8}}}=\frac{1375}{21}.$

12. $\frac{\overset{a}{\cancel{5a^2b}}}{\underset{c^3}{\cancel{3c^5}}}\cdot\frac{\overset{\cancel{2}}{\cancel{6c^2}}}{\underset{\cancel{5}\ b^2}{\cancel{10ab^3}}}=\frac{a}{b^2c^3}.$

13. $\frac{\overset{2}{\cancel{10x}y^2}}{\underset{3}{\cancel{18z^2}}}\cdot\frac{\overset{4z}{\cancel{24z^3}}}{\underset{3x^3}{\cancel{15x^4}}}=\frac{8y^2z}{9x^3}.$

14. $\frac{\overset{\cancel{5}\ x^2}{\cancel{15ax^3}}}{\cancel{4x}}\cdot\frac{\overset{\cancel{3}}{\cancel{12a^5}}}{\underset{\cancel{8}}{\cancel{40x^2}}}\cdot\frac{\overset{2}{\cancel{16x^2}}}{\underset{\cancel{3}}{\cancel{9a^5}}}=2ax^2.$

15. $\frac{\cancel{8cd^2}}{\cancel{3x}}\cdot\frac{\overset{6}{\cancel{18}}}{\underset{\underset{d^2}{\cancel{8}\ c^2\cancel{d^3}}}{\cancel{40}\,\cancel{c^3}\cancel{d^5}}}\cdot\overset{x}{\cancel{5x^2d}}=\frac{6x}{c^2d^2}.$

16. $\left(\frac{5x}{2ac}\right)^2\cdot\frac{6a^2}{10x^3}\cdot\frac{5c^5}{3a^4}=\frac{25\cancel{x^2}}{4\cancel{a^2c^2}}\cdot\frac{\overset{\cancel{3}}{\cancel{6a^2}}}{\underset{\cancel{2}x}{\cancel{10x^3}}}\cdot\frac{\overset{c^3}{\cancel{5c^5}}}{\cancel{3}a^4}=\frac{25c^3}{4a^4x}.$

17. $21x^2y\cdot\frac{9ay^2}{14x^3}\left(\frac{2x}{3y^2}\right)^3=\overset{\cancel{3}}{\cancel{21x^2y}}\cdot\frac{\cancel{9}a\cancel{y^2}}{\underset{\underset{x^3}{\cancel{2x^5}}}{\cancel{14x^3}}}\cdot\frac{\overset{4}{\cancel{8x^3}}}{\underset{\underset{y^3}{\cancel{3y^4}}}{\cancel{27y^6}}}=\frac{4a}{x^3y^3}.$

18. $\left(\frac{a}{5c^2}\right)^2\cdot\left(\frac{-c}{2d}\right)^3\cdot\frac{15d}{ac}=\frac{\overset{a}{\cancel{a^2}}}{\underset{5c}{\cancel{25c^4}}}\cdot-\frac{\cancel{c^3}}{8\underset{d^2}{\cancel{d^3}}}\cdot\frac{\overset{3}{\cancel{15d}}}{\cancel{ac}}=-\frac{3a}{40c^2d^2}.$

19. $\frac{10m}{n^7}\cdot\left(\frac{4n^2}{15m}\right)^2\cdot\left(\frac{5m^3}{6m^2n}\right)^2=\frac{10m}{\underset{n^5}{\cancel{n^7}}}\cdot\frac{\overset{4\cancel{n^2}}{\cancel{16n^4}}}{\underset{9}{\cancel{225m^2}}}\cdot\frac{\overset{\cancel{m^2}}{\cancel{25m^6}}}{\underset{9}{\cancel{36m^4n^2}}}=\frac{40m}{81n^5}.$

20. $\dfrac{(2x)^2}{3y}\cdot\dfrac{(3y)^2}{2x^2}=\dfrac{\overset{2}{\cancel{4x^2}}}{\cancel{3y}}\cdot\dfrac{\overset{3y}{\cancel{9y^2}}}{\cancel{2x^2}}=6y.$

21. $\dfrac{(6a)^2}{(10b^2)^3}\cdot\dfrac{125b^4}{2(a^3)^2}=\dfrac{\overset{9}{\cancel{36}}\cancel{a^2}}{\underset{\underset{2}{\cancel{8}b^2}}{\cancel{1000}\cancel{b^6}}}\cdot\dfrac{\cancel{125}\cancel{b^4}}{2\underset{a^4}{\cancel{a^6}}}=\dfrac{9}{4a^4b^2}.$

22. $\left(\dfrac{8m^2}{10mn^2}\right)^2\cdot\left(\dfrac{15n^3}{4m}\right)^3=\dfrac{\cancel{64}\overset{m}{\cancel{m^4}}}{\underset{4}{\cancel{100}}\underset{m}{\cancel{m^2}}\cancel{n^4}}\cdot\dfrac{\overset{5}{\cancel{5^3}}\cdot3^3\overset{n^5}{\cancel{n^9}}}{\cancel{64}\cancel{m^3}}=\dfrac{135n^5}{4m}.$

24. $\dfrac{2x^2+6}{5y^3}\cdot\dfrac{10y^4}{3x^2+9}=\dfrac{2\cancel{(x^2+3)}}{\cancel{5y^3}}\cdot\dfrac{\overset{2y}{\cancel{10y^4}}}{3\cancel{(x^2+3)}}=\dfrac{4y}{3}.$

25. $\dfrac{c^2-4}{x-y}\cdot\dfrac{3x-3y}{c^2+6c+8}=\dfrac{\cancel{(c+2)}(c-2)}{\cancel{x-y}}\cdot\dfrac{3\cancel{(x-y)}}{\cancel{(c+2)}(c+4)}=\dfrac{3c-6}{c+4}.$

26. $\dfrac{x^2-2x+4}{x^2-9}\cdot\dfrac{x^2-x-6}{x^3+8}=\dfrac{\cancel{x^2-2x+4}}{(x+3)\cancel{(x-3)}}\cdot\dfrac{\cancel{(x-3)}\cancel{(x+2)}}{\cancel{(x+2)}\cancel{(x^2-2x+4)}}$

$=\dfrac{1}{x+3}.$

27. $\dfrac{x^2-3x+9}{2x-6}\cdot\dfrac{x^2-9}{x^3+27}=\dfrac{\cancel{x^2-3x+9}}{2\cancel{(x-3)}}\cdot\dfrac{\cancel{(x+3)}\cancel{(x-3)}}{\cancel{(x+3)}\cancel{(x^2-3x+9)}}=\dfrac{1}{2}.$

28. $\dfrac{x^2-7x+10}{x^3-125}\cdot\dfrac{2x^3+10x^2+50x}{x^2-4}$

$=\dfrac{\cancel{(x-2)}\cancel{(x-5)}}{\cancel{(x-5)}\cancel{(x^2+5x+25)}}\cdot\dfrac{2x\cancel{(x^2+5x+25)}}{(x+2)\cancel{(x-2)}}$

$=\dfrac{2x}{x+2}.$

29. $\dfrac{x^4-16}{(x^2-4)^2}\cdot\dfrac{x^2-4}{4+x^2}=\dfrac{\cancel{(x^2+4)}\cancel{(x+2)}\cancel{(x-2)}}{\cancel{(x+2)^2}\cancel{(x-2)^2}}\cdot\dfrac{\cancel{(x+2)}\cancel{(x-2)}}{\cancel{x^2+4}}=1.$

30. $\dfrac{c^2-9}{a^2-4x^2}\cdot\dfrac{a^2+4ax+4x^2}{ac+2cx-3a-6x}=\dfrac{(c+3)\cancel{(c-3)}}{\cancel{(a+2x)}(a-2x)}\cdot\dfrac{\cancel{(a+2x)}\cancel{(a+2x)}}{\cancel{(c-3)}\cancel{(a+2x)}}$

$=\dfrac{c+3}{a-2x}.$

31. $\dfrac{2a^2-5ac-3c^2}{c^2-9a^2}\cdot\dfrac{3c+9a}{10a^2+5ac}=\dfrac{(a-3c)\cancel{(2a+c)}}{\cancel{(c+3a)}(c-3a)}\cdot\dfrac{3\cancel{(c+3a)}}{5a\cancel{(2a+c)}}$

$=\dfrac{3(a-3c)}{5a(c-3a)}.$

32. $\dfrac{2x^2+10x+50}{3x^3-12x}\cdot\dfrac{x^2-3x-10}{4x^3-500}\cdot\dfrac{2-x}{2}$

$= \dfrac{\cancel{2}\cancel{(x^2+5x+25)}}{3x\cancel{(x+2)}\cancel{(x-2)}}\cdot\dfrac{\cancel{(x-5)}\cancel{(x+2)}}{4\cancel{(x-5)}\cancel{(x^2+5x+25)}}\cdot\dfrac{\overset{-1}{\cancel{2-x}}}{\cancel{2}}$

$= \dfrac{-1}{12x}.$

33. $\dfrac{(x^2-4)(x+3)}{2x^2+12x+18}\cdot\dfrac{x^2-x-6}{(x^2-4)^2}\cdot\dfrac{(x-2)}{x+3}$

$= \dfrac{\cancel{(x+2)}\cancel{(x-2)}\cancel{(x+3)}}{2(x+3)(x+3)}\cdot\dfrac{(x-3)\cancel{(x+2)}}{\cancel{(x+2)^2}\cancel{(x-2)^2}}\cdot\dfrac{\cancel{x-2}}{\cancel{x+3}}$

$= \dfrac{x-3}{2(x+3)^2}.$

34. $\dfrac{4e^2-12ex+9x^2}{(e^2+ex+x^2)(3x-2e)}\cdot(e^3-x^3)\cdot\dfrac{x-e}{2e^2-5ex+3x^2}$

$= \dfrac{\cancel{(2e-3x)}\cancel{(2e-3x)}}{\cancel{(e^2+ex+x^2)}\underset{-1}{\cancel{(3x-2e)}}}\cdot\cancel{(e-x)}\cancel{(e^2+ex+x^2)}\cdot\dfrac{x-e}{\cancel{(2e-3x)}\cancel{(e-x)}}$

$= e-x.$

35. $\left(4+\dfrac{3}{a^2-1}\right)\left(\dfrac{3a}{2a-1}-1\right) = \dfrac{\overset{2a+1}{\cancel{4a^2-1}}}{\underset{a-1}{\cancel{a^2-1}}}\cdot\dfrac{\cancel{a+1}}{\cancel{2a-1}} = \dfrac{2a+1}{a-1}.$

36. $\left(\dfrac{x^2}{y^2}-4\right)\left(\dfrac{2x+4y}{x^3-8y^3}\right)\left(\dfrac{x^2+2xy+4y^2}{x^2+4xy+4y^2}\right)$

$= \dfrac{\cancel{(x+2y)}\cancel{(x-2y)}}{y^2}\cdot\dfrac{2\cancel{(x+2y)}}{\cancel{(x^2+2xy+4y^2)}\cancel{(x-2y)}}\cdot\dfrac{\cancel{x^2+2xy+4y^2}}{\cancel{(x+2y)^2}}$

$= \dfrac{2}{y^2}.$

37. $\left(3x-5-\dfrac{168}{9x+15}\right)\left(3-\dfrac{4}{3+x}\right) = \left(3x-5-\dfrac{56}{3x+5}\right)\left(3-\dfrac{4}{3+x}\right)$

$= \dfrac{9x^2-81}{3x+5}\cdot\dfrac{3x+5}{3+x}$

$= \dfrac{9\cancel{(x+3)}(x-3)}{\cancel{3x+5}}\cdot\dfrac{\cancel{3x+5}}{\cancel{x+3}}$

$= 9x-27.$

38. $\left(3x-4+\dfrac{21}{4+x}\right)\left(\dfrac{1}{x+1}\right)^2\left(x+\dfrac{x-3}{3x+5}+3\right)$

$= \dfrac{3x^2+8x+5}{4+x}\cdot\dfrac{1}{(x+1)^2}\cdot\dfrac{3x^2+15x+12}{3x+5}$

$= \dfrac{\cancel{(3x+5)}\cancel{(x+1)}}{\cancel{x+4}}\cdot\dfrac{1}{\cancel{(x+1)^2}}\cdot\dfrac{3\cancel{(x+1)}\cancel{(x+4)}}{\cancel{3x+5}}$

$= 3.$

39. $\left(2x-\frac{1}{2x}\right)\left(6x+\frac{3}{2x+1}\right)\left(\frac{1}{8}\right)\frac{1}{8x^3-1}$.

$$=\frac{\cancel{(2x+1)}\cancel{(2x-1)}}{2x}\cdot\frac{3\cancel{(4x^2+2x+1)}}{\cancel{2x+1}}\cdot\frac{1}{8}\cdot\frac{1}{\cancel{(2x-1)}\cancel{(4x^2+2x+1)}}$$

$$=\frac{3}{16x}.$$

40. $\left(\frac{4}{x^2}-\frac{5}{x}+1\right)\left(\frac{5x^4+5x^3}{x^2-11x+28}\right)\left(1-\frac{7x-1}{x^2-1}\right)$

$$=\frac{4-5x+x^2}{x^2}\cdot\frac{5x^3(x+1)}{(x-7)(x-4)}\cdot\frac{x^2-7x}{x^2-1}$$

$$=\frac{\cancel{(x-1)}\cancel{(x-4)}}{\cancel{x^2}}\cdot\frac{5\overset{x}{\cancel{x^3}}\cancel{(x+1)}}{\cancel{(x-7)}\cancel{(x-4)}}\cdot\frac{x\cancel{(x-7)}}{\cancel{(x+1)}\cancel{(x-1)}}$$

$$=5x^2.$$

41. $\left(2b+\frac{a^2}{2b}-2a\right)\left(a+\frac{12b^2}{a-2b}+4b\right)\cdot\frac{2b^2}{a^3-8b^3}$

$$=\frac{4b^2-4ab+a^2}{2b}\cdot\frac{a^2+2ab+4b^2}{a-2b}\cdot\frac{2b^2}{a^3-8b^3}$$

$$=\frac{\cancel{(a-2b)}\cancel{(a-2b)}}{\cancel{2b}}\cdot\frac{\cancel{a^2+2ab+4b^2}}{\cancel{a-2b}}\cdot\frac{\overset{b}{\cancel{2b^2}}}{\cancel{(a-2b)}\cancel{(a^2+2ab+4b^2)}}=b.$$

Page 171

1. $\frac{2}{3}\div\frac{5}{3}=\frac{2}{\cancel{3}}\cdot\frac{\cancel{3}}{5}=\frac{2}{5}$.

2. $\frac{4}{5}\div\frac{9}{5}=\frac{4}{\cancel{5}}\cdot\frac{\cancel{5}}{9}=\frac{4}{9}$.

3. $\frac{10}{3}\div\frac{5}{2}=\frac{\overset{2}{\cancel{10}}}{3}\cdot\frac{2}{\cancel{5}}=\frac{4}{3}$.

4. $\frac{12}{15}\div\left(-\frac{4}{5}\right)=\frac{\overset{\cancel{3}}{\cancel{12}}}{\underset{\cancel{3}}{\cancel{15}}}\cdot-\frac{\cancel{5}}{\cancel{4}}=-1$.

5. $\frac{15}{21}\div-\frac{3}{7}=\frac{\overset{5}{\cancel{15}}}{\underset{3}{\cancel{21}}}\cdot-\frac{\cancel{7}}{\cancel{3}}=-\frac{5}{3}$.

6. $\frac{18}{20}\div-\frac{45}{10}=\frac{\overset{\cancel{9}}{\cancel{18}}}{\underset{\cancel{2}}{\cancel{20}}}\cdot-\frac{\cancel{10}}{\underset{5}{\cancel{45}}}=-\frac{1}{5}$.

7. $\frac{a}{2x}\div\frac{c}{x}=\frac{a}{2\cancel{x}}\cdot\frac{\cancel{x}}{c}=\frac{a}{2c}$.

8. $\frac{2a}{b}\div\frac{-3c}{2b}=\frac{2a}{\cancel{b}}\cdot\frac{2\cancel{b}}{-3c}=-\frac{4a}{3c}$.

9. $\frac{x^2}{y}\div\frac{2x}{y^2}=\frac{\overset{x}{\cancel{x^2}}}{\cancel{y}}\cdot\frac{\overset{y}{\cancel{y^2}}}{2\cancel{x}}=\frac{xy}{2}$.

10. $\frac{2m}{n}\div\frac{-m}{3n}=\frac{2\cancel{m}}{\cancel{n}}\cdot\frac{3\cancel{n}}{\underset{-1}{-\cancel{m}}}=-6$.

11. $\frac{10x^2}{14y^2}\div\frac{15x^3}{21y^4}=\frac{\overset{\cancel{2}}{\cancel{10}}\cancel{x^2}}{\underset{\cancel{2}}{\cancel{14}}\cancel{y^2}}\cdot\frac{\overset{\cancel{3}}{\cancel{21}}\overset{y^2}{\cancel{y^4}}}{\underset{\cancel{5}}{\cancel{15}}\underset{x}{\cancel{x^3}}}=\frac{y^2}{x}$.

12. $\frac{12ab^2}{5c} \div \frac{18a^2b}{15c^3} = \frac{\overset{2\ \ b}{\cancel{12}\cancel{a}\cancel{b^2}}}{\cancel{5}\cancel{c}} \cdot \frac{\overset{\cancel{3}\ c^2}{\cancel{15}\cancel{c^3}}}{\underset{\cancel{6}\ a}{\cancel{18}\cancel{a^2}\cancel{b}}} = \frac{2bc^2}{a}.$

13. $\frac{4a^2x}{51c} \div \frac{2ax}{17c^3} = \frac{\overset{2a}{\cancel{4}\cancel{a^2}\cancel{x}}}{\underset{3}{\cancel{51}\cancel{c}}} \cdot \frac{\overset{c^2}{\cancel{17}\cancel{c^3}}}{\cancel{2}\cancel{a}\cancel{x}} = \frac{2ac^2}{3}.$

14. $\left(\frac{3ab}{5c}\right)^2 \div \frac{18a^3b}{100c^3} = \frac{\overset{b}{\cancel{9}\cancel{a^2}\cancel{b^2}}}{\cancel{25}\cancel{c^2}} \cdot \frac{\overset{\overset{2}{\cancel{4}}\ c}{\cancel{100}\cancel{c^3}}}{\underset{\cancel{2}\ a}{\cancel{18}\cancel{a^3}\cancel{b}}} = \frac{2bc}{a}.$

15. $\frac{a}{b} \div \frac{c}{b} \div \frac{a^2}{c^2} = \frac{\cancel{a}}{\cancel{b}} \cdot \frac{\cancel{b}}{\cancel{c}} \cdot \frac{\overset{c}{\cancel{c^2}}}{\underset{a}{\cancel{a^2}}} = \frac{c}{a}.$

16. $\frac{2x^6}{5a} \div \frac{4x^2}{15} \div \frac{(3x^2)^2}{12a^2} = \frac{\overset{x^4}{\cancel{2}\cancel{x^6}}}{\cancel{5}\cancel{a}} \cdot \frac{\overset{\cancel{5}}{\cancel{15}}}{\cancel{4}\cancel{x^2}} \cdot \frac{\overset{\cancel{3}\ a}{\cancel{12}\cancel{a^2}}}{\underset{\cancel{3}}{\cancel{9}\cancel{x^4}}} = 2a.$

17. $\frac{12c^3d}{35e^4} \div \frac{8c^2}{7e^2d} \cdot \frac{10de}{3c^2d^3} = \frac{\overset{\cancel{3}\ \cancel{c}}{\cancel{12}\cancel{c^3}\cancel{d}}}{\underset{\underset{e}{\cancel{5}\cancel{e^2}}}{\cancel{35}\cancel{e^4}}} \cdot \frac{\cancel{7}\cancel{e^2}\cancel{d}}{\underset{\cancel{4}}{\cancel{8}\cancel{c^2}}} \cdot \frac{\overset{\cancel{2}}{\cancel{10}\cancel{d}\cancel{e}}}{\underset{\underset{\cancel{d}}{c\ \cancel{d^2}}}{\cancel{3}\cancel{c^2}\cancel{d^3}}} = \frac{1}{ec}.$

18. $\frac{18x^3}{(7ab^2)^2} \div 12x^2 \cdot \frac{(14ab^4)^2}{6x^2} = \frac{\overset{\cancel{6}\ \cancel{x}}{\cancel{18}\cancel{x^3}}}{\cancel{49}\cancel{a^2}\cancel{b^4}} \cdot \frac{1}{\underset{\cancel{3}}{\cancel{12}\cancel{x^2}}} \cdot \frac{\overset{\cancel{4}\ \ b^4}{\cancel{196}\cancel{a^2}\cancel{b^8}}}{\underset{x}{\cancel{6}\cancel{x^2}}} = \frac{b^4}{x}.$

19. $\left(\frac{3}{5x^2}\right)^2 \div \frac{256}{(10x^2)^2} \div \frac{5x^3}{(8x^4)^2} = \frac{9}{\cancel{25}\cancel{x^4}} \cdot \frac{\overset{\cancel{25}}{\cancel{100}\cancel{x^4}}}{\underset{\cancel{4}}{\cancel{256}}} \cdot \frac{\overset{x^5}{\cancel{64}\cancel{x^8}}}{5\cancel{x^3}} = \frac{9x^5}{5}.$

20. $\frac{2a}{3b} \div \frac{(2a)^3}{12b^3} \div \frac{(7a^4)^2}{15b^5} \cdot \left(\frac{7}{5}\right)^2 = \frac{\cancel{2}\cancel{a}}{\cancel{3}\cancel{b}} \cdot \frac{\overset{\cancel{3}\ b^2}{\cancel{12}\cancel{b^3}}}{\underset{\cancel{4}\ a^2}{\cancel{8}\cancel{a^3}}} \cdot \frac{\overset{3}{\cancel{15}b^5}}{\cancel{49}a^8} \cdot \frac{\cancel{49}}{\underset{5}{\cancel{25}}} = \frac{3b^7}{5a^{10}}.$

21. $\frac{10-4a}{12x^2} \div \frac{5a-2a^2}{9x} = \frac{\cancel{2}\cancel{(5-2a)}}{\underset{\underset{2}{\cancel{4}\ x}}{\cancel{12}\cancel{x^2}}} \cdot \frac{\overset{3}{\cancel{9}\cancel{x}}}{a\cancel{(5-2a)}} = \frac{3}{2ax}.$

22. $\frac{(a-3)^2}{a^2-4a+3} \div \frac{a^2-9}{a^2-a} = \frac{\cancel{(a-3)}\cancel{(a-3)}}{\cancel{(a-3)}\cancel{(a-1)}} \cdot \frac{a\cancel{(a-1)}}{\cancel{(a-3)}(a+3)} = \frac{a}{a+3}.$

23. $\dfrac{x^2+x-30}{x^2+5x-6} \div \dfrac{x^2-25}{x^2-6x+5} = \dfrac{\cancel{(x+6)(x-5)}}{\cancel{(x+6)(x-1)}}\cdot\dfrac{(x-5)\cancel{(x-1)}}{(x+5)\cancel{(x-5)}} = \dfrac{x-5}{x+5}.$

24. $\dfrac{a^3-1}{a^3-a}\div\dfrac{a^4+a^3+a^2}{a^2+3a+2} = \dfrac{\cancel{(a-1)(a^2+a+1)}}{a\cancel{(a+1)(a-1)}}\cdot\dfrac{(a+2)\cancel{(a+1)}}{a^2\cancel{(a^2+a+1)}}=\dfrac{a+2}{a^3}.$

25. $(2m^2-4m+8)\div\dfrac{m^4+8m}{3m} = \dfrac{2\cancel{(m^2-2m+4)}}{1}\cdot\dfrac{3\cancel{m}}{\cancel{m}(m+2)\cancel{(m^2-2m+4)}}$
$= \dfrac{6}{m+2}.$

26. $\left(x+1-\dfrac{3}{4x}\right)\left(3+\dfrac{3}{4x^2-1}\right)\div\dfrac{2x+3}{2x+1}$

$=\dfrac{4x^2+4x-3}{4x}\cdot\dfrac{12x^2}{4x^2-1}\div\dfrac{2x+3}{2x+1}$

$=\dfrac{\cancel{(2x+3)(2x-1)}}{\cancel{4x}}\cdot\dfrac{\overset{3x}{\cancel{12x^2}}}{\cancel{(2x+1)(2x-1)}}\cdot\dfrac{\cancel{2x+1}}{\cancel{2x+3}}$

$=3x.$

27. $\left(\dfrac{-c}{2d^2}\right)^2\div\left(\dfrac{-d}{c^3}\right)^2\cdot\left(\dfrac{-d^3}{c^2}\right)^3\div\left(\dfrac{d}{-2c}\right)^2 = \dfrac{c^2}{\cancel{4}\cancel{d^4}}\cdot\dfrac{\cancel{c^6}}{\cancel{d^2}}\cdot\dfrac{\overset{-d}{\cancel{-d^9}}}{\cancel{c^6}}\cdot\dfrac{\cancel{4}c^2}{\cancel{d^2}} = -c^4d.$

28. $\left(\dfrac{x^2}{y}-\dfrac{y^2}{x}\right)\div\left(\dfrac{x^2+xy+y^2}{2x^2-2xy}\right)\cdot\dfrac{1}{2(x-y)^2}$

$=\dfrac{x^3-y^3}{xy}\cdot\dfrac{2x^2-2xy}{x^2+xy+y^2}\cdot\dfrac{1}{2(x-y)^2}$

$=\dfrac{\cancel{(x-y)(x^2+xy+y^2)}}{\cancel{x}y}\cdot\dfrac{\cancel{2}\cancel{x}\cancel{(x-y)}}{\cancel{x^2+xy+y^2}}\cdot\dfrac{1}{\cancel{2}\cancel{(x-y)^2}}=\dfrac{1}{y}.$

29. $\dfrac{x^2-5xy+6y^2}{9x^2-6xy+y^2}\cdot\left(3x+5y+\dfrac{10y^2}{x-2y}\right)\div\dfrac{x^2-9y^2}{3x^2+8xy-3y^2}$

$=\dfrac{\cancel{(x-2y)(x-3y)}}{\cancel{(3x-y)(3x-y)}}\cdot\dfrac{x\cancel{(3x-y)}}{\cancel{x-2y}}\cdot\dfrac{\cancel{(3x-y)(x+3y)}}{\cancel{(x+3y)(x-3y)}}=x.$

30. $\dfrac{(2m)^2}{m^6+64}\cdot\left(m^2-4+\dfrac{16}{m^2}\right)\div\dfrac{1}{2m}\div\dfrac{4m}{4-m^2}$

$=\dfrac{\cancel{4}\cancel{m^2}}{(m^2+4)\cancel{(m^4-4m^2+16)}}\cdot\dfrac{\cancel{m^4-4m^2+16}}{\cancel{m^2}}\cdot\dfrac{2\cancel{m}}{1}\cdot\dfrac{(2+m)(2-m)}{\cancel{4}\cancel{m}}$

$=\dfrac{2(4-m^2)}{4+m^2}.$

31. $\dfrac{3x-2}{x+1}\cdot\left(3x+4+\dfrac{4}{3x}\right)\div\dfrac{9x^2-4}{9x^2+9x}$

$=\dfrac{\cancel{3x-2}}{\cancel{x+1}}\cdot\dfrac{(3x+2)\cancel{(3x+2)}}{\cancel{3}\cancel{x}}\cdot\dfrac{\overset{3}{\cancel{9}}\cancel{x}\cancel{(x+1)}}{\cancel{(3x+2)(3x-2)}}$

$=9x+6.$

32. $\left(2x+1+\dfrac{1}{2x}\right)\div\dfrac{(8x^3-1)^2}{2x^2+2x}\cdot\left(4x-2+\dfrac{3}{x+1}\right)$

$$=\frac{\cancel{4x^2+2x+1}}{\cancel{2}\cancel{x}}\cdot\frac{\cancel{2}\cancel{x}\cancel{(x+1)}}{(2x-1)^2\cancel{(4x^2+2x+1)^2}}\cdot\frac{\cancel{4x^2+2x+1}}{\cancel{x+1}}$$

$$=\frac{1}{(2x-1)^2}.$$

33. $\dfrac{27a^3+8b^3}{ax}\cdot\left(\dfrac{3a}{2b}\div\dfrac{2b}{3a}\right)\div\left[\left(3a-2b+\dfrac{4b^2}{3a}\right)\cdot\dfrac{9ab}{x}\right]$

$$=\frac{(3a+2b)\cancel{(9a^2-6ab+4b^2)}}{\cancel{a}\cancel{x}}\cdot\frac{\cancel{9}\overset{a}{\cancel{a^2}}}{4b^2}\cdot\frac{3\cancel{a}\cancel{x}}{\cancel{(9a^2-6ab+4b^2)}\cancel{9}\cancel{a}b}$$

$$=\frac{3a(3a+2b)}{4b^3}.$$

Page 173

1. $\dfrac{3+\frac{1}{7}}{1-\frac{1}{7}}=\dfrac{\overset{11}{\cancel{22}}}{\cancel{7}}\cdot\dfrac{\cancel{7}}{\underset{3}{\cancel{6}}}=\dfrac{11}{3}.$ **3.** $\dfrac{9-\frac{4}{25}}{3-\frac{2}{5}}=\dfrac{\overset{17}{\cancel{221}}}{\underset{5}{\cancel{25}}}\cdot\dfrac{\cancel{5}}{\cancel{13}}=\dfrac{17}{5}.$

2. $\dfrac{\frac{6}{7}-2}{1-\frac{3}{7}}=\dfrac{\overset{-2}{\cancel{-8}}}{\cancel{7}}\cdot\dfrac{\cancel{7}}{\cancel{4}}=-2.$ **4.** $\dfrac{(\frac{3}{2})^2-16}{\frac{3}{2}+4}=\left(\dfrac{9}{4}-16\right)\div\dfrac{11}{2}=\dfrac{\overset{-5}{\cancel{-55}}}{\underset{2}{\cancel{4}}}\cdot\dfrac{\cancel{2}}{\cancel{11}}=-\dfrac{5}{2}.$

5. $\dfrac{3^3-(\frac{1}{2})^3}{3^2+3\cdot\frac{1}{2}+(\frac{1}{2})^2}=\left(27-\dfrac{1}{8}\right)\div\left(9+\dfrac{3}{2}+\dfrac{1}{4}\right)=\dfrac{\overset{5}{\cancel{215}}}{\underset{2}{\cancel{8}}}\cdot\dfrac{\cancel{4}}{\cancel{43}}=\dfrac{5}{2}.$

6. $\dfrac{(\frac{3}{5})^2+\frac{3}{5}-2}{\frac{3}{5}-1}=\left(\dfrac{9}{25}+\dfrac{3}{5}-2\right)\div\left(\dfrac{3}{5}-1\right)=\dfrac{\overset{13}{\cancel{-26}}}{\underset{5}{\cancel{25}}}\cdot\dfrac{\cancel{5}}{-\cancel{2}}=\dfrac{13}{5}.$

7. $\dfrac{(\frac{2}{3})^3-(\frac{3}{2})^3}{(\frac{2}{3})^2+(\frac{2}{3})(\frac{3}{2})+(\frac{3}{2})^2}=(\frac{8}{27}-\frac{27}{8})\div(\frac{4}{9}+1+\frac{9}{4})$

$$=\frac{\overset{-5}{\cancel{-665}}}{\underset{3\cdot 2}{\cancel{27\cdot 8}}}\cdot\frac{\overset{4}{\cancel{36}}}{\cancel{133}}=\frac{-5}{6}.$$

8. $\dfrac{(\frac{7}{2})^3-(\frac{1}{3})^3}{(\frac{7}{2})^2+\frac{7}{2}\cdot\frac{1}{3}+(\frac{1}{3})^2}=\frac{7}{2}-\frac{1}{3}=\frac{21}{6}-\frac{2}{6}=\frac{19}{6}.$

9. $\dfrac{\frac{2}{3}}{5}+\dfrac{2}{\frac{3}{5}}=\dfrac{2}{15}+\dfrac{10}{3}=\dfrac{52}{15}.$

10. $\dfrac{c-\frac{1}{c}}{\frac{c-1}{c^2}}=\dfrac{\cancel{c^2-1}}{\cancel{c}}\cdot\dfrac{\cancel{c^2}}{\cancel{c-1}}=c(c+1).$ 12. $\dfrac{x+\frac{x^2}{y}}{1+\frac{x}{y}}=\dfrac{\cancel{xy+x^2}}{\cancel{y}}\cdot\dfrac{\cancel{y}}{\cancel{x+y}}=x.$

11. $\dfrac{2-\frac{c^2}{2}}{\frac{2-c}{4}}=\dfrac{\cancel{4-c^2}}{\cancel{2}}\cdot\dfrac{\cancel{4}}{\cancel{2-c}}=2(2+c).$ 13. $\dfrac{\frac{b^2}{5}-5}{1+\frac{b}{5}}=\dfrac{\cancel{b^2-25}}{\cancel{5}}\cdot\dfrac{\cancel{5}}{\cancel{b+5}}=b-5.$

14. $\dfrac{\frac{c^2}{d^2}-\frac{d^2}{c^2}}{\frac{1}{c^2}+\frac{1}{d^2}}=\dfrac{\cancel{c^4-d^4}}{\cancel{c^2d^2}}\cdot\dfrac{\cancel{c^2d^2}}{\cancel{c^2+d^2}}=c^2-d^2.$

15. $\dfrac{\frac{a^2}{b}-\frac{b^2}{a}}{a+b+\frac{b^2}{a}}=\dfrac{\cancel{a^3-b^3}}{\cancel{a}b}\cdot\dfrac{\cancel{a}}{\cancel{a^2+ab+b^2}}=\dfrac{a-b}{b}.$

16. $\dfrac{\frac{x}{y}}{z}-\dfrac{x}{\frac{y}{z}}=\dfrac{x}{yz}-\dfrac{xz}{y}=\dfrac{x}{yz}-\dfrac{xz^2}{yz}=\dfrac{x-xz^2}{yz}.$

17. $\dfrac{\frac{x}{y}+5+\frac{6y}{x}}{3+\frac{2x}{y}-\frac{9y}{x}}=\dfrac{x^2+5xy+6y^2}{xy}\cdot\dfrac{xy}{2x^2+3xy-9y^2}$

$=\dfrac{(x+2y)\cancel{(x+3y)}}{\cancel{xy}}\cdot\dfrac{\cancel{xy}}{\cancel{(x+3y)}(2x-3y)}$

$=\dfrac{x+2y}{2x-3y}.$

18. $\dfrac{\frac{8}{a}-a^2}{1+\frac{2}{a}+\frac{a}{2}}=\dfrac{\overset{2-a}{\cancel{8-a^3}}}{\cancel{a}}\cdot\dfrac{2\cancel{a}}{\cancel{2a+4+a^2}}=4-2a.$

19. $\dfrac{1-\frac{y}{x}}{\frac{x^2+y^2}{x}-2y}=\dfrac{\cancel{x-y}}{\cancel{x}}\cdot\dfrac{\cancel{x}}{\underset{x-y}{\cancel{x^2+y^2-2xy}}}=\dfrac{1}{x-y}.$

20. $\dfrac{\frac{(x+3y)^2}{6xy}-2}{\left(\frac{1}{y}-\frac{3}{x}\right)^2}=\dfrac{\cancel{x^2-6xy+9y^2}}{6\cancel{xy}}\cdot\dfrac{\overset{xy}{\cancel{x^2y^2}}}{\cancel{(x-3y)^2}}=\dfrac{xy}{6}.$

21. $$\frac{\left(\dfrac{3a-b}{2a^3}\right)^2}{\dfrac{(b-3a)^2}{4a^2}} = \frac{\cancel{(3a-b)^2}}{\underset{a^4}{\cancel{4}\cancel{a^6}}} \cdot \frac{\cancel{4}\cancel{a^2}}{\cancel{(b-3a)^2}} = \frac{1}{a^4}.$$

22. $$\frac{\dfrac{x-2y}{2y}+\dfrac{x+2y}{x}}{\dfrac{x-2y}{x}-\dfrac{x+2y}{2y}} = \frac{\cancel{x^2+4y^2}}{\cancel{2}\cancel{x}\cancel{y}} \cdot \frac{\cancel{2}\cancel{x}\cancel{y}}{\underset{-1}{-\cancel{(x^2+4y^2)}}} = -1.$$

23. $$\frac{\dfrac{1}{a}-\dfrac{b}{a+b}}{\dfrac{1}{a}+\dfrac{1}{b}} = \frac{a+b-ab}{\cancel{a}(a+b)} \cdot \frac{\cancel{a}b}{a+b} = \frac{b(a+b-ab)}{(a+b)^2}.$$

24. $$\frac{\dfrac{1}{x^2}-\dfrac{1}{xy}+\dfrac{1}{y^2}}{\dfrac{1}{x^3}+\dfrac{1}{y^3}} = \frac{\cancel{y^2-xy+x^2}}{\cancel{x^2}\cancel{y^2}} \cdot \frac{\overset{xy}{\cancel{x^3}\cancel{y^3}}}{\underset{x+y}{\cancel{x^3+y^3}}} = \frac{xy}{x+y}.$$

25. $$\frac{\dfrac{1}{5b^2}+\dfrac{1}{2ab}+\dfrac{1}{5a^2}}{\dfrac{1}{2a}+\dfrac{1}{b}} = \frac{\overset{a+2b}{\cancel{2a^2+5ab+2b^2}}}{\underset{5ab}{\cancel{10}\cancel{a^2}\cancel{b^2}}} \cdot \frac{\cancel{2}\cancel{a}\cancel{b}}{\cancel{2a+b}} = \frac{a+2b}{5ab}.$$

26. $$\frac{\dfrac{x}{x-1}-\dfrac{2x}{x-2}}{\dfrac{2x}{x-2}-\dfrac{3x}{x-3}} = \frac{-\cancel{x^2}}{(x-1)\cancel{(x-2)}} \cdot \frac{\cancel{(x-2)}(x-3)}{-\cancel{x^2}} = \frac{x-3}{x-1}.$$

27. $$\frac{\dfrac{2(b+d)-3(2a-c)}{2ab-bc+2ad-cd}}{\dfrac{2}{2a-c}-\dfrac{3}{b+d}} = \frac{\cancel{2b+2d-6a+3c}}{\cancel{(2a-c)(b+d)}} \cdot \frac{\cancel{(2a-c)(b+d)}}{\cancel{2b+2d-6a+3c}} = 1.$$

28. $$\frac{\left(\dfrac{5x+4y}{4x}\right)^2-\dfrac{5y}{x}}{\dfrac{(5x+4y)^2}{4x}-20y} = \frac{\cancel{(5x-4y)^2}}{\underset{4x}{\cancel{16}\cancel{x^2}}} \cdot \frac{\cancel{4}\cancel{x}}{\cancel{(5x-4y)^2}} = \frac{1}{4x}.$$

29. $$\frac{2+\dfrac{12-6a}{2a^2-a-6}}{2a+\dfrac{21}{2a+3}-7} = \frac{\cancel{4}\cancel{a}\cancel{(a-2)}}{\cancel{(2a+3)}(a-2)} \cdot \frac{\cancel{2a+3}}{\cancel{4}\cancel{a}\cancel{(a-2)}} = \frac{1}{a-2}.$$

Page 177

1. $$\frac{x}{2}+\frac{x}{3}=10.$$

Multiplying both members by 6, the L.C.M. of the denominators, and canceling,

$$3x+2x=60.$$

Whence $x=12.$

2. $$\frac{x}{4}+\frac{x}{8}=3.$$

Multiplying both members by 8, the L.C.M. of the denominators, and canceling,

$$2x+x=24.$$

Whence $x=8.$

3. $$\frac{x}{3}-\frac{x}{8}=5.$$

Multiplying both members by 24, the L.C.M. of the denominators, and canceling,

$$8x-3x=120.$$

Whence $x=24.$

4. $$x+\frac{x}{5}=6.$$

Multiplying both members by 5, the L.C.M. of the denominators, and canceling,

$$5x+x=30.$$

Whence $x=5.$

5. $$\frac{x}{5}+\frac{x}{3}-\frac{x}{15}=14.$$

L.C.M. $=15.$

$$3x+5x-x=210.$$

Whence $x=30.$

6. $$\frac{x+5}{4}-\frac{2x+4}{9}=1.$$

L.C.M. $=36.$

$$9(x+5)-4(2x+4)=36.$$

Whence $x=7.$

7. $$\frac{x-2}{2}+\frac{3x+2}{2}=6.$$

L.C.M. $=2.$

$$x-2+3x+2=12.$$

Whence $x=3.$

8. $$\frac{x+3}{4}+\frac{4x-5}{5}=5.$$

L.C.M. $=20.$

$$5(x+3)+4(4x-5)=100.$$

Whence $x=5.$

9. $$\frac{x+1}{2}+\frac{x-2}{3}-\frac{x+3}{4}=2.$$

L.C.M. $=12.$

$$6(x+1)+4(x-2)-3(x+3)=24.$$

Whence $x=5.$

10. $$\tfrac{2}{3}x+\tfrac{5}{4}x=\tfrac{23}{24}.$$

L.C.M. $=24.$

$$16x+30x=23.$$

Whence $x=\tfrac{1}{2}.$

11. $$\tfrac{4}{3}x+\tfrac{2}{5}x=26.$$

L.C.M. $=15.$

$$20x+6x=390.$$

Whence $x=15.$

12. $$\tfrac{3}{2}x+\tfrac{4}{3}x=\tfrac{17}{9}.$$

L.C.M. $=18.$

$$27x+24x=34.$$

Whence $x=\tfrac{2}{3}.$

13. $$\frac{1}{x}+\frac{2}{x}=3.$$

L.C.M. $=x.$

$$1+2=3x.$$

Whence $x=1.$

14. $$\frac{3}{x}-\frac{2}{x}=6.$$

L.C.M. $=x.$

$$3-2=6x.$$

Whence $x=\tfrac{1}{6}.$

15. $$\frac{5}{x}+\frac{4}{x}=9.$$

$$\text{L.C.M.}=x.$$

$$5+4=9x.$$

Whence $x=1.$

16. $$\frac{2}{x}+\frac{16}{15}=\frac{19}{3x}.$$

$$\text{L.C.M.}=15x.$$

$$30+16x=95.$$

Whence $x=\frac{65}{16}.$

17. $$\frac{2x+3}{5}-\frac{1}{3}(x-3)=2.$$

$$\text{L.C.M.}=15.$$

$$3(2x+3)-5(x-3)=30.$$

Whence $x=6.$

18. $$\frac{5-x}{2}+\frac{2x+3}{3}=4.$$

$$\text{L.C.M.}=6.$$

$$3(5-x)+2(2x+3)=24.$$

Whence $x=3.$

19. $$\frac{2x+13}{3}-\frac{6-x}{4}=1.$$

$$\text{L.C.M.}=12.$$

$$4(2x+13)-3(6-x)=12.$$

Whence $x=-2.$

20. $$\frac{3}{4}(x-1)+\frac{5x-7}{4}=\frac{3}{2}.$$

$$\text{L.C.M.}=4.$$

$$3(x-1)+(5x-7)=6.$$

Whence $x=2.$

21. $$\frac{5x+12}{6}-\frac{4}{11}(2x+7)+\frac{1}{3}=0.$$

$$\text{L.C.M.}=66.$$

$$11(5x+12)-24(2x+7)+22=0.$$

Whence $x=2.$

22. $$\frac{x-7}{6}+\frac{1}{2}\left(2-\frac{x}{2}\right)=\frac{3x-22}{6}.$$

Removing parenthesis,

$$\frac{x-7}{6}+1-\frac{x}{4}=\frac{3x-22}{6}.$$

$$\text{L.C.M.}=12.$$

$$2(x-7)+12-3x=2(3x-22).$$

Whence $x=6.$

23. $$\frac{x}{a}+x=a+1.$$

$$\text{L.C.M.}=a.$$

$$x+ax=(a+1)a.$$

$$(a+1)x=(a+1)a.$$

$$x=a.$$

24. $$\frac{x}{a}-\frac{x}{3a}=6.$$

$$\text{L.C.M.}=3a.$$

$$3x-x=18a.$$

Whence $x=9a.$

25. $$\frac{x}{a}-\frac{2x}{5a}=\frac{3}{5b}.$$

$$\text{L.C.M.}=5ab.$$

$$5bx-2bx=3a.$$

Whence $x=\frac{a}{b}.$

26. $$6-\frac{x}{c}=\frac{1}{3}(c-3x)+\frac{17c}{3}.$$

Removing parenthesis, $$6-\frac{x}{c}=\frac{c}{3}-x+\frac{17c}{3}.$$

Combining, $$6-\frac{x}{c}=6c-x.$$

$$\text{L.C.M.}=c.$$

$$6c-x=6c^2-cx.$$

Transposing, $$cx-x=6c^2-6c.$$

Factoring, $$x(c-1)=6c(c-1).$$

Whence $$x=6c.$$

27. $$n\left(\frac{2}{3}-\frac{n}{5}\right)-\frac{x}{3}=\frac{n}{5}(5n-3x).$$

Removing parenthesis, $$\frac{2n}{3}-\frac{n^2}{5}-\frac{x}{3}=n^2-\frac{3nx}{5}.$$

$$\text{L.C.M.}=15.$$

$$10n-3n^2-5x=15n^2-9nx.$$

Transposing and grouping, $$(9n-5)x=18n^2-10n.$$

Dividing by $9n-5$, $$x=2n.$$

28. $$2x+2a+\frac{3x+4}{a}=\frac{2(2a^2-1)}{a}-1.$$

$$\text{L.C.M.}=a.$$

$$2ax+2a^2+3x+4=4a^2-2-a.$$

$$(2a+3)x=2a^2-a-6.$$

$$x=a-2.$$

29. $$\frac{x+a}{a}-\frac{x+b}{b}=a^2(b-a).$$

$$\text{L.C.M.}=ab.$$

$$b(x+a)-a(x+b)=a^3b(b-a).$$

Regrouping, $$(b-a)x=a^3b(b-a).$$

$$x=a^3b.$$

30. $$\frac{2a-3x}{6a}+\frac{5a-2x}{5a}+\frac{41}{30}=0.$$

$$\text{L.C.M.}=30a.$$

$$5(2a-3x)+6(5a-2x)+41a=0.$$

Whence $$x=3a.$$

31. $$\frac{6x}{a}-\frac{1}{2}\left(\frac{2a}{3}-12x\right)+\frac{19a}{3}=-6.$$

Removing parenthesis, $$\frac{6x}{a}-\frac{a}{3}+6x+\frac{19a}{3}=-6.$$

$$\text{L.C.M.}=3a.$$

$$18x-a^2+18ax+19a^2=-18a.$$

Grouping, $$18(1+a)x=-18(a+a^2).$$

Dividing, $$x=-a.$$

32. $$(x+5)(x-6)=x(x-\tfrac{5}{2}).$$

Removing parentheses, $$x^2-x-30=x^2-\tfrac{5}{2}x.$$

Transposing and combining, $$\tfrac{3}{2}x=30.$$

Whence $$x=20.$$

33. $$(x+7)(x+18)=x(x-\tfrac{1}{5}).$$

Removing parentheses, $$x^2+25x+126=x^2-\tfrac{1}{5}x.$$

Transposing and combining, $$\tfrac{126}{5}x=-126.$$

$$126x=(-126)(5).$$

$$x=-5.$$

34. $$(4x+3)(6+x)=4x(x-\tfrac{1}{6})+45\tfrac{2}{3}.$$

Removing parentheses, $$4x^2+27x+18=4x^2-\tfrac{2}{3}x+\tfrac{137}{3}.$$

Transposing and combining, $$\frac{83x}{3}=\frac{83}{3}.$$

Whence $$x=1.$$

Page 179

1. Let $n=$ the number.

Then $$\frac{n}{3}+\frac{n}{12}=15.$$

Whence $$n=36.$$

2. Let $n=$ the number.

Then $$\frac{n}{4}-\frac{n}{9}=10.$$

Whence $$n=72.$$

3. Let $n=$ the greater number.

Then $38-n=$ the less number,

and $$\frac{n}{10}=\frac{38-n}{9}.$$

Whence $$n=20;\ 38-n=18.$$

4. Let $x =$ the length of the rectangle in meters.
Then $\frac{2}{3}x =$ the width in meters,
and $2x + \frac{4}{3}x = 200.$
Whence $x = 60;\ \frac{2}{3}x = 40.$

5. Let $x =$ the number.
Then $$\frac{4+x}{7} = \frac{x}{5}.$$
Whence $x = 10.$

6. Let $x =$ the first integer.
Then $$\frac{x}{2} = \frac{x+1+x+2}{5}.$$
Whence $x = 6.$

7. Let $x =$ the first integer.
Then $$\frac{x}{12} = \frac{x+2+x+4}{26}.$$
Whence $x = 36.$

8. Let $x =$ the number.
Then $$\frac{5+x}{7+x} = \frac{5}{6}.$$
Whence $x = 5.$

9. Let $x =$ the less part.
Then $24 - x =$ the greater part,
and $$\frac{(24-x)-x}{4} = 4.$$
Whence $x = 4;\ 24 - x = 20.$

10. Let $x =$ the number.
Then $\frac{1}{4}(3x - 4) = \frac{1}{7}(5x - 4).$
Whence $x = 12.$

11. Let $x =$ the greater part.
Then $108 - x =$ the less part,
and $$\frac{108-x}{x} = \frac{2}{7}.$$
Whence $x = 84;\ 108 - x = 24.$

12. Let $x =$ the greater number.
Then $24 - x =$ the less number,
and $$\frac{x-(24-x)}{24} = \frac{x}{60}.$$
Whence $x = 15;\ 24 - x = 9.$

13. Let $b =$ B's age in years.

Then $\frac{5}{2}b =$ A's age in years,

and $\frac{5}{2}b + 10 = 2(b + 10)$.

Whence $b = 20$; $\frac{5}{2}b = 50$.

14. Let $x =$ the husband's age in years at marriage.

Then $\frac{4}{5}x =$ the wife's age in years at marriage,

and $\frac{4}{5}x + 20 = \frac{8}{9}(x + 20)$.

Whence $x = 25$; $\frac{4}{5}x = 20$.

15. Let $b =$ B's age in years now.

Then $b + 12 =$ A's age in years now,

and $b - 8 = \frac{5}{8}(b + 12 - 8)$.

Whence $b = 28$; $b + 12 = 40$.

16. Let the number of moons each planet has be represented by the initial of the planet.

Then $U + J + S + M + N = 26$.

By the conditions of the problem this may be written

$$U + (U + 5) + (2U + 2) + (U - 2) + \tfrac{1}{2}(U - 2) = 26.$$

Whence
$$U = 4.$$
$$J = U + 5 = 9.$$
$$S = 2U + 2 = 10.$$
$$M = U - 2 = 2.$$
$$N = \tfrac{1}{2}(U - 2) = 1.$$

17. The altitude of the trapezoid is 8, and one base is 12.

Let $x =$ the other base.

Then $$\frac{8(12 + x)}{2} = \frac{24 \cdot 12}{2}.$$

Whence $x = 24$.

18. Let $x =$ distance to target in feet.

Then $\frac{x}{1925} =$ time of flight in seconds,

and $\frac{x}{1100} =$ time for return of sound in seconds.

Therefore $$\frac{x}{1925} + \frac{x}{1100} = 3.$$

Whence $$x = 2100;\ \frac{x}{1925} = \frac{2100}{1925} = \frac{12}{11}.$$

19. Let $x =$ velocity in feet per second.

Then $\frac{7920}{x} =$ time of flight in seconds,

and $\frac{7920}{1100} =$ time for return of sound in seconds.

Therefore $$\frac{7920}{x} + \frac{7920}{1100} = \frac{47}{5}.$$

Whence $x = 3600$ number of feet per second.

Page 182

2. $$\frac{x-3}{3+x} = 4.$$

Multiplying both members by $3 + x$, the L.C.M. of the denominators, and canceling,

$$x - 3 = 4(3 + x).$$

Whence $x = -5$.

3. $$\frac{x-5}{x-2} = \frac{1}{2}.$$

Multiplying both members by $2(x-2)$, the L.C.M. of the denominators, and canceling,

$$2(x - 5) = x - 2.$$

Whence $x = 8$.

4. $$\frac{3x+4}{2x-2} = -2.$$

L.C.M. $= 2x - 2$.

$$3x + 4 = -2(2x - 2).$$

Whence $x = 0$.

5. $$\frac{4}{6-5x} = \frac{1}{x}.$$

L.C.M. $= x(6 - 5x)$.

$$4x = 6 - 5x.$$

Whence $x = \frac{2}{3}$.

7. $$\frac{x-3}{x} = \frac{2x}{x-2}.$$

L.C.M. $= x(x - 2)$.

$$(x - 3)(x - 2) = 2x^2.$$

Whence

$$(x - 1)(x + 6) = 0.$$

$$x = 1, -6.$$

8. $$\frac{x-1}{x+3} = x.$$

L.C.M. $= x + 3$.

$$x - 1 = x(x + 3).$$

Whence

$$(x + 1)^2 = 0.$$

$$x = -1, -1.$$

9. $$\frac{x}{5-x} = \frac{2x}{4x-5}.$$

L.C.M. $= (5 - x)(4x - 5)$.

$$x(4x - 5) = 2x(5 - x).$$

Whence

$$3x(2x - 5) = 0.$$

$$x = 0, \tfrac{5}{2}.$$

10. $$\frac{2}{x+1} = \frac{4}{4+x}.$$

L.C.M. $= (x + 1)(4 + x)$.

$$2(4 + x) = 4(x + 1).$$

Whence $x = 2$.

11. $$\frac{1}{2-x} = \frac{3}{3-x}.$$

L.C.M. $= (2 - x)(3 - x)$.

$$3 - x = 3(2 - x).$$

Whence $x = \frac{3}{2}$.

12. $$\frac{x-5}{4-5x} = -6x.$$

L.C.M. $= 4 - 5x$.

$$x - 5 = -6x(4 - 5x).$$

Whence

$$5(3x - 1)(2x - 1) = 0.$$

$$x = \tfrac{1}{2}, \tfrac{1}{3}.$$

13.
$$\frac{x+7}{3-x}+\frac{3x}{2}=x.$$
$$\text{L.C.M.}=2(3-x).$$
$$2(x+7)+3x(3-x)=2x(3-x).$$
Whence
$$(x-7)(x+2)=0.$$
$$x=7,\ -2.$$

14.
$$\frac{2}{x-6}=\frac{x-12}{x+6}+\frac{x}{x+6}\cdot$$
$$\text{L.C.M.}=(x-6)(x+6).$$
$$2(x+6)=(x-12)(x-6)+x(x-6).$$
Whence
$$(x-3)(x-10)=0.$$
$$x=3,\ 10.$$

15.
$$\frac{x+3}{x-4}=\frac{x+9}{x-5}.$$
$$\text{L.C.M.}=(x-4)(x-5).$$
$$(x+3)(x-5)=(x+9)(x-4).$$
Whence
$$x=3.$$

16.
$$\frac{4x-1}{x+2}=\frac{7-2x}{x+4}.$$
$$\text{L.C.M.}=(x+4)(x+2).$$
$$(4x-1)(x+4)=(7-2x)(x+2).$$
Whence
$$(x-1)(x+3)=0.$$
$$x=1,\ -3.$$

17.
$$\frac{4}{2x-3}+\frac{3x+2}{3}=x.$$
$$\text{L.C.M.}=3(2x-3).$$
$$12+(3x+2)(2x-3)=3x(2x-3).$$
Whence
$$x=-\tfrac{3}{2}.$$

18.
$$\frac{x+6}{3-x}=\frac{5+2x}{7x-5}.$$
$$\text{L.C.M.}=(3-x)(7x-5).$$
$$(x+6)(7x-5)=(5+2x)(3-x).$$
Whence
$$9(x-1)(x+5)=0.$$
$$x=1,\ -5.$$

19.
$$\frac{3x}{8} - \frac{4}{x-4} = \frac{3x-4}{8}.$$
$$\text{L.C.M.} = 8(x-4).$$
$$3x(x-4) - 32 = (3x-4)(x-4).$$
Whence
$$x = 12.$$

20.
$$\frac{2}{4x+1} - \frac{1}{5x-2} - \frac{1}{3x+4} = 0.$$
$$\text{L.C.M.} = (4x+1)(5x-2)(3x+4).$$
$$2(5x-2)(3x+4) - (4x+1)(3x+4) - (4x+1)(5x-2) = 0.$$
Whence
$$2(x-3)^2 = 0.$$
$$x = 3, 3.$$

21.
$$\frac{47}{220} - \frac{7}{4(x+3)} = \frac{3}{5(x+3)}.$$
$$\text{L.C.M.} = 220(x+3).$$
$$(47)(x+3) - (7)(55) = (3)(44).$$
Whence
$$x = 8.$$

22.
$$\frac{4x}{x+3} - \frac{6}{2(x+3)} = \frac{x^2+11}{3(x+3)}.$$
Canceling,
$$\frac{4x}{x+3} - \frac{3}{x+3} = \frac{x^2+11}{3(x+3)}.$$
$$\text{L.C.M.} = 3(x+3).$$
$$12x - 9 = x^2 + 11.$$
Whence
$$(x-2)(x-10) = 0.$$
$$x = 2, 10.$$

23.
$$\frac{4-x}{x-5} + \frac{7}{5} = \frac{3}{5-x}.$$
Rewriting the last fraction,
$$\frac{4-x}{x-5} + \frac{7}{5} = \frac{-3}{x-5}.$$
$$\text{L.C.M.} = 5(x-5).$$
$$5(4-x) + 7(x-5) = -15.$$
Whence
$$x = 0.$$

24.
$$\frac{x}{x-3} + 3 = \frac{1}{3-x}.$$
Rewriting the last fraction,
$$\frac{x}{x-3} + 3 = \frac{-1}{x-3}.$$
$$\text{L.C.M.} = x - 3.$$
$$x + 3(x-3) = -1.$$
Whence
$$x = 2.$$

25.
$$\frac{3x}{x-2}=\frac{x-3}{2-x}-x.$$

Rewriting,
$$\frac{3x}{x-2}=\frac{3-x}{x-2}-x.$$
$$\text{L.C.M.}=x-2.$$
$$3x=3-x-x(x-2).$$

Whence
$$(x-1)(x+3)=0.$$
$$x=1,\ -3.$$

26. From Hint,
$$3x(x+3)-16=x(x-3)-2x.$$

Whence
$$x=1,\ -8.$$

27.
$$\frac{1}{3-x}+\frac{2}{x+3}=\frac{6x}{x^2-9}.$$

Rewriting,
$$\frac{1}{3-x}+\frac{2}{3+x}+\frac{6x}{9-x^2}=0.$$
$$\text{L.C.M.}=9-x^2.$$
$$(3+x)+2(3-x)+6x=0.$$

Whence
$$x=-\tfrac{9}{5}.$$

28.
$$\frac{2x}{x+1}+\frac{3}{x-1}+\frac{24}{1-x^2}=0.$$
$$\frac{2x}{x+1}+\frac{3}{x-1}-\frac{24}{x^2-1}=0.$$
$$2x(x-1)+3(x+1)-24=0.$$

Whence
$$x=3,\ -\tfrac{7}{2}.$$

29.
$$\frac{3}{x-4}=\frac{5x-12}{x^2-16}-\frac{4}{16-x^2}.$$
$$\frac{3}{x-4}=\frac{5x-12+4}{x^2-16}=\frac{5x-8}{x^2-16}.$$
$$3(x+4)=5x-8.$$

Whence
$$x=10.$$

30.
$$\frac{x+2}{x-2}=\frac{10-2x^2}{4-x^2}-\frac{7}{x^2-4}.$$
$$\frac{x+2}{x-2}=\frac{-10+2x^2-7}{x^2-4}=\frac{2x^2-17}{x^2-4}.$$
$$(x+2)^2=2x^2-17.$$

Whence
$$x=7,\ -3.$$

31.
$$\frac{1+x}{3-x}+\frac{x-2}{2-x}=\frac{x-1}{x^2-5x+6}.$$
$$\frac{1+x}{3-x}+\frac{x-2}{2-x}=\frac{x-1}{6-5x+x^2}.$$
$$(1+x)(2-x)+(x-2)(3-x)=x-1.$$
Whence
$$x=1,\ \tfrac{3}{2}.$$

32.
$$\frac{x-4}{2x-5}+\frac{2x-15}{2x+4}=\frac{8x^2-20x-31}{4x^2-2x-20}.$$
$$(x-4)(2x+4)+(2x-15)(2x-5)=8x^2-20x-31.$$
Whence
$$2(x+15)(x-3)=0.$$
$$x=3,\ -15.$$

Page 184

1. $.4x=6.$

Multiplying by 10,
$$4x=60.$$
$$x=15.$$

2. $.3x+.5=.8.$

Multiplying by 10,
$$3x+5=8.$$
$$x=1.$$

3. $.3x+4=.25.$

Multiplying by 100,
$$30x+400=25.$$
$$x=-12.5.$$

4. $.75-.7x=.26.$
$$75-70x=26.$$
$$x=.7.$$

5. $.92+.3x=5.12.$
$$92+30x=512.$$
$$x=14.$$

6. $3.75=2.15-.5x.$
$$375=215-50x.$$
$$x=-3.2.$$

7. $.06x-4.5=1.68.$
$$6x-450=168.$$
$$x=103.$$

8. $.03x+.16=.58.$
$$3x+16=58.$$
$$x=14.$$

9. $.04x=.1x+2.4.$
$$4x=10x+240.$$
$$x=-40.$$

10. $.8x-2.7=.55x.$
$$80x-270=55x.$$
$$x=10.8.$$

11. $.15x-.4x=8x-49.5.$
$$15x-40x=800x-4950.$$
$$4950=825x.$$
$$x=6.$$

12. $1.7x+3.14=-9.66-1.5x.$
$$170x+314=-966-150x.$$
$$320x=-1280.$$
$$x=-4.$$

13.

$$3x + 7 - 1.25x = 8.845 + .52x.$$
$$3000x + 7000 - 1250x = 8845 + 520x.$$
$$1230x = 1845.$$
$$x = 1.5.$$

14.

$$1.7x + .17 - .03x = 4.73 + 1.1x.$$
$$170x + 17 - 3x = 473 + 110x.$$
$$x = 8.$$

15.

$$.12(2x + .5) - .2(1.5x - 2) = .4.$$
$$.24x + .06 - .3x + .4 = .4.$$
$$24x + 6 - 30x + 40 = 40.$$
$$x = 1.$$

16.

$$6(3x - 1.1) - 8.4(.7x - 3) = 6x + .24.$$
$$18x - 6.6 - 5.88x + 25.2 = 6x + .24.$$
$$1800x - 660 - 588x + 2520 = 600x + 24.$$
$$612x = -1836.$$
$$x = -3.$$

17.

$$\frac{.7x}{4} - \frac{.19}{3} = \frac{.1x}{6}.$$

Clearing of decimals, $$\frac{70x}{4} - \frac{19}{3} = \frac{10x}{6}.$$

Canceling, $$\frac{35x}{2} - \frac{19}{3} = \frac{5x}{3}.$$

Whence $$105x - 38 = 10x.$$

$$x = \frac{2}{5} = .4.$$

18.

$$\frac{.3x - 6.2}{4} - \frac{6.75 - .4x}{5} = \frac{3.5}{2}.$$
$$\frac{30x - 620}{4} - \frac{675 - 40x}{5} = \frac{350}{2}.$$
$$\frac{15x - 310}{2} - 135 + 8x = 175.$$
$$15x - 310 - 2(135 - 8x) = 350.$$

Whence $$x = 30.$$

NOTE. Examples 17 and 18 make it plain that an equation should usually be cleared of decimals before it is cleared of fractions, since advantageous cancellations are likely to result.

19. $$\frac{.14x+3.2}{6}-\frac{x-.75}{5}=\frac{2.4x-1.5}{10}.$$

$$\frac{14x+320}{6}-\frac{100x-75}{5}=\frac{240x-150}{10}.$$

$$\frac{7x+160}{3}-(20x-15)=24x-15.$$

$$7x+160-3(20x-15)=3(24x-15).$$

Whence $$x=2.$$

20. From Hint, $$\frac{600x-269}{40}+\frac{150x}{24}=3.9.$$

$$\frac{600x-269}{40}+\frac{25x}{4}=3.9.$$

$$600x-269+250x=156.$$

$$x=\tfrac{1}{2}.$$

21. $$\frac{1.8x-2}{1.7}=\frac{5.7-.7x}{1.8}.$$

Multiplying the numerator and denominator of each fraction by 10,

$$\frac{18x-20}{17}=\frac{57-7x}{18}.$$

$$18(18x-20)=17(57-7x).$$

$$x=3.$$

22. $$\frac{3.2x}{.5}+\frac{4.5x}{12.5}=1.52+6x.$$

$$\frac{32x}{5}+\frac{45x}{125}=1.52+6x.$$

$$\frac{32x}{5}+\frac{9x}{25}=1.52+6x.$$

$$640x+36x=152+600x \text{ (cf. Ex. 18, Note).}$$

$$x=2.$$

23. $$\frac{.3(3-.4x)}{.16}-\frac{.5(.2x-6)}{.8}=5.$$

$$\frac{.9-.12x}{.16}-\frac{.1x-3}{.8}=5.$$

Multiplying the numerator and denominator of the first fraction by 100, of the second by 10,

$$\frac{90-12x}{16}-\frac{x-30}{8}=5.$$

$$90-12x-2(x-30)=80.$$

$$x=5.$$

24.

$$\frac{.3(5-x)}{6.25}=\frac{1.5-10x}{14}+.56x.$$

$$\frac{30(5-x)}{625}=\frac{15-100x}{140}+.56x.$$

$$\frac{6(5-x)}{125}=\frac{3-20x}{28}+.56x.$$

$$168(5-x)=125(3-20x)+3500(.56x).$$

Whence $465=-372x$, and $x=-1.25$.

Page 188

1. Let $P=$ the principal.

Then $.06P=$ the interest for one year,

and $P+.06P=371.$

$$106P=37{,}100.$$

$$P=350.$$

2. Here $P+3(.04P)=476.$

$$112P=47{,}600.$$

$$P=425.$$

3. If $x=$ the number of years,

then $(x)(325)(.06)=39.$

$$19.50x=39.$$

$$x=2.$$

4. If $x=$ the number of years,

then $(x)(480)(.065)=156.$

$$31.2x=156.$$

$$x=5.$$

6. Let $x=$ the rate of interest.

Then $\$725x=$ the interest for one year,

and $\$2900x=$ the interest for four years.

Therefore $2900x=145.$

$$x=\tfrac{5}{100}, \text{ or } 5\%.$$

7. As in Ex. 6, $(250)(6)(x)=317.50-250=67.50.$

$$1500x=67.50.$$

$$x=.045=4\tfrac{1}{2}\%.$$

8. As in Ex. 6, if $x =$ the number of years,

$$(300)(x)(.05) = 600 - 300 = 300.$$
$$15x = 300.$$
$$x = 20.$$

9. As in Ex. 6, if $x =$ the number of years,

$$(500)(x)(.06) = 1500 - 500 = 1000.$$
$$30x = 1000.$$
$$x = 33\tfrac{1}{3}.$$

11. Let $x =$ the number of dollars at 5%.
Then $1500 - x =$ the number of dollars at 4%,
and $.05x + .04(1500 - x) = 69.$
Whence $x = 900;\ 1500 - x = 600.$

12. Let $x =$ the number of dollars at 5%.
Then $x + 100 =$ the number of dollars at 6%,
and $.05x + .06(x + 100) = 50.$
Whence $x = 400;\ x + 100 = 500.$

13. Let $x =$ the number of dollars at 4%.
Then $3600 - x =$ the number of dollars at 5%,
and $.04x = .05(3600 - x).$
Whence $x = 2000;\ 3600 - x = 1600.$

14. Let $x =$ the number of dollars at 5%.
Then $240 - x =$ the number of dollars at 6%,
and $.05x + 10 = .06(240 - x).$
Whence $x = 40;\ 240 - x = 200.$

15. Let $x =$ the number of dollars at 6%.
Then $4300 - x =$ the number of dollars at 5%,
and $.06x + 1.60 = .05(4300 - x).$
Whence $x = 1940;\ 4300 - x = 2360.$

16. Let $x =$ the number of dollars at 5%.
Then $5360 - x =$ the number of dollars at 6%,
and $.05x - 63.40 = .06(5360 - x).$
Whence $x = 3500;\ 5360 - x = 1860.$

17. Let $x =$ the number of dollars at 4%.
Then $4560 - x =$ the number of dollars at 6%,
and $.04x + .06(4560 - x) = 202.40.$
Whence $x = 3560;\ 4560 - x = 1000.$

Page 190

1. $$2ax - 2a^2 = 6a^2 - 2ax.$$
$$4ax = 8a^2.$$
$$x = 2a.$$

2. $$5c^2 - 4cx = c(5x - 4c).$$
$$5c^2 - 4cx = 5cx - 4c^2.$$
$$9c^2 = 9cx.$$
$$c = x.$$

3. $$3a^2x - 4b^2 = b^2 - 2a^2x.$$
$$5a^2x = 5b^2.$$
$$x = \frac{b^2}{a^2}.$$

4. $$2(x + 3) - 4a = 6.$$
$$2x + 6 - 4a = 6.$$
$$2x = 4a.$$
$$x = 2a.$$

5. $$3(x - 1) - 9a = 6.$$
$$3x - 3 - 9a = 6.$$
$$3x = 9a + 9.$$
$$x = 3a + 3.$$

6. $$3(x - 2) - 4h = 2(x - 2).$$
$$3x - 6 - 4h = 2x - 4.$$
$$x = 4h + 2.$$

7. $$3(2x - a) = 2(x - a).$$
$$6x - 3a = 2x - 2a.$$
$$4x = a.$$
$$x = \frac{a}{4}.$$

8. $$4(x - b) = 2(x + b).$$
$$4x - 4b = 2x + 2b.$$
$$2x = 6b.$$
$$x = 3b.$$

9. $$4a(6x - 3b) = 3b(8 - 4a).$$
$$24ax - 12ab = 24b - 12ab.$$
$$24ax = 24b.$$
$$x = \frac{b}{a}.$$

10. $$ax + bx = a^2 + ab.$$
$$(a + b)x = (a + b)a.$$
$$x = a.$$

11. $$cx + b^2 = bx + bc.$$
$$cx - bx = bc - b^2.$$
$$(c - b)x = (c - b)b.$$
$$x = b.$$

12. $$ax + b^2 = a^2 - bx.$$
$$ax + bx = a^2 - b^2.$$
$$(a + b)x = (a - b)(a + b).$$
$$x = a - b.$$

13. $$6ab + 15cx - 10bc - 9ax = 0.$$
$$15cx - 9ax = 10bc - 6ab.$$
$$(15c - 9a)x = (5c - 3a)2b.$$
$$x = \frac{2b}{3}.$$

14. $$12 - 15a + 16x = 20ax.$$
$$16x - 20ax = 15a - 12.$$
$$(16 - 20a)x = 15a - 12.$$
$$x = \frac{15a - 12}{16 - 20a} = -\frac{12 - 15a}{16 - 20a} = -\frac{3}{4}.$$

15.
$$ax + bx + cx = ak + bk + ck.$$
$$(a + b + c)x = (a + b + c)k.$$
$$x = k.$$

16.
$$2ax + 2a + cx + c + x + 1 = 0.$$
$$(2a + c + 1)x = -(2a + c + 1).$$
$$x = -1.$$

17.
$$6ab + kx + 4a^2 = 2ax + 3bk + 2ak.$$
$$kx - 2ax = (3b + 2a)k - (3b + 2a)2a.$$
$$(k - 2a)x = (k - 2a)(3b + 2a).$$
$$x = 3b + 2a.$$

18.
$$5ax - 5a^2 - 10ab = 3ac + 6bc - 3cx.$$
$$5ax + 3cx = 3c(a + 2b) + 5a(a + 2b).$$
$$(5a + 3c)x = (5a + 3c)(a + 2b).$$
$$x = a + 2b.$$

19.
$$\frac{x}{2b} = a.$$
$$x = 2ab.$$

20.
$$\frac{3ab}{x} = b.$$
$$3ab = bx.$$
$$x = 3a.$$

21.
$$\frac{c}{x} + \frac{3c}{2x} = \frac{5}{4}.$$
$$4c + 6c = 5x.$$
$$10c = 5x.$$
$$x = 2c.$$

22.
$$\frac{8a}{3x} + \frac{8a}{x} = \frac{3}{2} + \frac{5a}{3x}.$$
$$16a + 48a = 9x + 10a.$$
$$54a = 9x.$$
$$x = 6a.$$

23.
$$\frac{x}{a} + \frac{x}{b} = a + b.$$
$$ax + bx = ab(a + b).$$
$$x = ab.$$

24.
$$\frac{d^2}{x} - d = \frac{a^2}{x} + a.$$
$$d^2 - dx = a^2 + ax.$$
$$d^2 - a^2 = (a + d)x.$$
$$x = d - a.$$

25.
$$\frac{hx}{2k} - 4k^2 = \frac{2kx}{h} - h^2.$$
$$h^2x - 8k^3h = 4k^2x - 2kh^3.$$
$$2kh^3 - 8k^3h = (4k^2 - h^2)x.$$
$$x = -2kh.$$

26.
$$\frac{2x}{a} + \frac{a - 4x}{3} - 3a = -4.$$
$$6x + a^2 - 4ax - 9a^2 = -12a.$$
$$6x - 4ax = 8a^2 - 12a.$$
$$(6 - 4a)x = (4a - 6)2a.$$
$$x = -2a.$$

27.
$$\frac{x}{a}+\frac{x}{c}+ac=bc+ab+\frac{x}{b}.$$
$$(bc+ab-ac)\,x=abc\,(bc-ac+ab).$$
$$x=abc.$$

28.
$$\frac{3\,a-4\,x}{5\,a}+\frac{3\,a-2\,x}{4\,a}=\frac{1}{20}.$$
$$4(3\,a-4\,x)+5(3\,a-2\,x)=a.$$
$$26\,a=26\,x.$$
$$x=a.$$

29.
$$\frac{bx}{2}-\frac{3\,a}{5}\left(x-\frac{2\,ab}{3}\right)=ab\left(\frac{b}{2}-\frac{a}{5}\right).$$
$$\frac{bx}{2}-\frac{3\,ax}{5}+\frac{6\,a^2\,b}{15}=\frac{ab^2}{2}-\frac{a^2b}{5}.$$
$$15\,bx-18\,ax+12\,a^2b=15\,ab^2-6\,a^2b.$$
$$(15\,b-18\,a)\,x=15\,ab^2-18\,a^2b.$$
$$x=ab.$$

30.
$$\frac{x-a}{x-b}=\frac{b}{a}.$$
$$ax-a^2=bx-b^2.$$
$$(a-b)\,x=a^2-b^2.$$
$$x=a+b.$$

31.
$$\frac{b}{b(b-x)}+\frac{3}{d(b-x)}+\frac{d+3}{2\,db}=0.$$
$$2\,db+6\,b+(d+3)(b-x)=0.$$
$$2\,db+6\,b+bd+3\,b=(d+3)\,x.$$
$$x=3\,b.$$

32.
$$\frac{c}{k(x+c)}+\frac{k}{c(x-k)}=\frac{c^2-kc+2\,k^2}{2\,kc(x-k)}.$$
$$2\,c^2(x-k)+2\,k^2(x+c)=(x+c)(c^2-kc+2\,k^2).$$
$$2\,c^2x-2\,c^2k+2\,k^2x+2\,k^2c=c^2x-kcx+2\,k^2x+c^3-kc^2+2\,k^2c.$$
$$(c^2+kc)\,x=c^3+c^2k=(c^2+kc)\,c.$$
$$x=c.$$

33.
$$\frac{1}{ab}+1-\frac{ab}{x}=\frac{1}{x}.$$
$$x+abx-a^2b^2=ab.$$
$$(1+ab)\,x=(1+ab)\,ab.$$
$$x=ab.$$

Page 191

1. $$K = 2\pi RH.$$
$$\frac{K}{2\pi H} = R.$$

2. $$A = \frac{ab}{2}.$$
$$\frac{2A}{b} = a.$$

3. $$C = 2\pi R.$$
$$\frac{C}{2\pi} = R.$$

4. $$d = rt.$$
(1) $$\frac{d}{r} = t.$$
(2) $$\frac{d}{t} = r.$$

5. $$\frac{a}{A} = \frac{D}{360}.$$
(1) $$a = \frac{AD}{360}.$$
(2) Whence $$A = \frac{360\,a}{D}.$$

6. $$\frac{D}{360} = \frac{l}{C}.$$
$$CD = 360\,l.$$
$$C = \frac{360\,l}{D}.$$

7. $$C = \frac{E}{R + r}.$$
$$CR + Cr = E.$$
$$Cr = E - CR.$$
$$r = \frac{E - CR}{C}.$$

8. $$C = \frac{E}{R + nr}.$$
$$CR + Cnr = E.$$
$$Cnr = E - CR.$$
Whence (1) $$n = \frac{E - CR}{Cr},$$
and (2) $$r = \frac{E - CR}{Cn}.$$

9. $$C = \frac{ne}{R + nr}.$$
$$CR + C\,nr = ne.$$
$$Cnr = ne - CR.$$
Whence (1) $$r = \frac{ne - CR}{Cn}.$$
and $$Cnr - ne = -CR.$$
$$(e - Cr)\,n = CR.$$
(2) $$n = \frac{CR}{e - Cr}.$$

10. $$C = \tfrac{5}{9}(F - 32).$$
$$\tfrac{9}{5}C = F - 32.$$
$$\tfrac{9}{5}C + 32 = F.$$

11. $$\frac{W_1}{W_2} = \frac{L_2}{L_1}.$$
$$W_1L_1 = W_2L_2.$$
$$\frac{W_1L_1}{L_2} = W_2.$$

12. $$A = P(1 + rt).$$
$$A = P + Prt.$$
$$A - P = Prt.$$
Whence $$\frac{A - P}{Pr} = t,$$
and $$\frac{A - P}{Pt} = r.$$

13. $$\frac{V_1}{V_2} = \frac{P_2}{P_1}.$$
$$\frac{V_1P_1}{V_2} = P_2.$$

14. $$s = \frac{n(a+l)}{2}.$$

$$2s = an + ln.$$

Whence (1) $$n = \frac{2s}{a+l}.$$

and $$2s - an = ln.$$

(2) $$l = \frac{2s - an}{n}.$$

15. $$s = \frac{rl - a}{r - 1}.$$

$$rs - s = rl - a.$$

Whence (1) $$a = rl - rs + s.$$

(2) $$l = \frac{rs - s + a}{r}.$$

(3) $$r = \frac{s - a}{s - l}.$$

16. $$\frac{D}{180} = \frac{a}{\pi}.$$

$$\frac{\pi D}{180} = a.$$

17. $$V_1 = V_0(1 + .00365\,t_1).$$

$$V_1 - V_0 = (V_0)(.00365\,t_1).$$

$$\frac{V_1 - V_0}{.00365\,V_0} = t_1.$$

18. $$A = \frac{(b_1 + b_2)a}{2}.$$

$$2A = ab_1 + ab_2.$$

$$2A - ab_1 = ab_2.$$

$$b_2 = \frac{2A - ab_1}{a}.$$

Page 193

1. $$l_1 W_1 = l_2 W_2 \text{ in feet and pounds.}$$

Letting subscript 1 pertain to A,

$$(4)(96) = (6)(W_2).$$

Whence $$W_2 = 64 \text{ pounds.}$$

2. $$l_1 W_1 = l_2 W_2 \text{ in inches and pounds.}$$

Letting subscript 1 pertain to B,

$$(80)(120) = (l_2)(100).$$

Whence $$l_2 = 96 \text{ inches.}$$

3. Letting x and $9 - x$ be the distances from the fulcrum in feet,

$$125x = 100(9 - x).$$

Whence $x = 4$ feet, A's distance,

and $9 - x = 5$ feet, B's distance.

4. Letting x and $210 - x$ be the weights in pounds and reducing the distances to inches, $$45x = 60(210 - x).$$

Whence $x = 120$ pounds, A's weight.

and $210 - x = 90$ pounds, B's weight.

5. Let $x =$ B's distance in feet from the fulcrum,

and $y =$ B's weight in pounds.

Then $$xy = (24 - x)(\tfrac{2}{3}y).$$

Or dividing by y, $$x = 16 - \tfrac{2}{3}x.$$

Whence $$x = \tfrac{48}{5} \text{ feet} = 9.6 \text{ feet.}$$

Page 194

1. Let $x =$ the less part.
Then $240 - x =$ the greater part,
and $\dfrac{240 - x}{x} = 7.$
Whence $x = 30;\ 240 - x = 210.$

2. Let $x =$ the greater part.
Then $68 - x =$ the less part,
and $\frac{2}{3}x = \frac{3}{4}(68 - x).$
Whence $x = 36;\ 68 - x = 32.$

3. Let $x =$ the first part.
Then $\frac{4}{3} - x =$ the second part,
and $\frac{1}{3}x = \frac{1}{5}(\frac{4}{3} - x).$
Whence $x = \frac{1}{2};\ \frac{4}{3} - x = \frac{5}{6}.$

4. $x = 23;\ 98 - x = 75.$

5. Let $x =$ the less part.
Then $120 - x =$ the greater part,
and $\dfrac{120 - x}{x} = 5 + \dfrac{12}{x}.$
Whence $x = 18;\ 120 - x = 102.$

6. Let $x =$ the less number.
Then $1797 - x =$ the greater number,
and $\dfrac{1797 - x}{x} = 70 + \dfrac{22}{x}.$
Whence $x = 25;\ 1797 - x = 1772.$

7. Let $x =$ the smaller part.
Then $\frac{11}{6} - x =$ the greater part,
and $x(\frac{11}{6} - x) - \frac{1}{3} = x^2.$
Whence $(4x - 1)(3x - 2) = 0,$
and $x = \frac{1}{4};\ \frac{11}{6} - x = \frac{19}{12};$
or $x = \frac{2}{3};\ \frac{11}{6} - x = \frac{7}{6}.$

8. Let $x =$ the less number.
Then $16 - x =$ the greater number,
and $4(16 - x) - 50 = \frac{1}{2}(28 - x).$
Whence $x = 0;\ 16 - x = 16.$

9. Let $x =$ the boy's age in years now.
Then $x = \frac{3}{5}(x + 10).$
Whence $x = 15.$

10. Let $x =$ the man's age in years now.

Then $\frac{1}{6}(x - 12) = \frac{1}{8}(x + 8)$.

Whence $x = 72$.

11. Let $n =$ the number of nickels.

Then $60 - n =$ the number of quarters,

and $5n + 25(60 - n) = 1100$.

Whence $n = 20;\ 60 - n = 40$.

12. Let $d =$ the number of dimes.

Then $23 - d =$ the number of quarters,

and $10d + 25(23 - d) = 305$.

Whence $d = 18;\ 23 - d = 5$.

13. Let $x =$ the certain even number.

Then $\left(\frac{x}{2}\right)^2 + 14 = \frac{1}{4}(x + 2)(x + 4)$.

Whence $x = 8;\ x + 2 = 10;\ x + 4 = 12$.

14. Let $x =$ the width in feet.

Then $4x =$ the length in feet,

and $(4x - 4)(x + \frac{3}{2}) = (4x)(x) + 11$.

Whence $x = 8\frac{1}{2};\ 4x = 34$.

15. Let $x =$ the length,

and $\frac{2}{3}x =$ the breadth.

Then $(2x)(\frac{2}{3}x - 14) = (x)(\frac{2}{3}x) - (\frac{1}{3}x)^2$.

Whence $x = 36$, root 0 rejected;

$\frac{2}{3}x = 24$.

16. Let $x =$ the side in feet.

Then $4x =$ the perimeter in feet,

and $\frac{x^2}{9} =$ the area in square yards.

Therefore $(15)\left(\frac{x^2}{9}\right) = (20)(4x)$.

Whence $x = 48$, root 0 rejected.

17. Let $x =$ the width in yards.

Then $2x =$ the length in yards,

and $(50)(6x) = \frac{1}{2}(15)(2x^2)$.

Whence $x = 20$, root 0 rejected;

$2x = 40$.

18. Let $x =$ the width of the picture in inches.
Then $\frac{5}{2}x =$ the length of the picture in inches.
Therefore $(x)(\frac{5}{2}x) =$ the area of the unframed picture in square inches,
and $(x + 4)(\frac{5}{2}x + 4) =$ the area of the framed picture in square inches.
But $(x + 4)(\frac{5}{2}x + 4) - (x)(\frac{5}{2}x) = 128.$
Whence $x = 8$; $\frac{5}{2}x = 20.$

19. Let $x =$ the number of dozen bought.
Then $16x =$ the expenditure in cents.
3 for 5 = 20 cents per dozen, etc.
Hence $(\frac{1}{5}x)(20) + (\frac{4}{5}x)(18) =$ the total receipts in cents.
But $(\frac{1}{5}x)(20) + (\frac{4}{5}x)(18) - 16x = 48.$
Whence $x = 20.$

21. As in Ex. 20, $\frac{1}{6} + \frac{1}{9} = \frac{1}{x}.$
Whence $x = \frac{18}{5}.$

22. As in Ex. 20,
$$\frac{1}{6} + \frac{1}{8} + \frac{1}{8} + \frac{1}{12} = \frac{1}{x}.$$
Whence $x = 2.$

23. As in Ex. 20, $\frac{1}{3} + \frac{2}{9} = \frac{1}{x}.$
Whence $x = \frac{9}{5}.$

24. Let $x =$ the number of days by B alone.
Then $\frac{1}{8} + \frac{1}{x} = \frac{5}{24}.$
Whence $x = 12.$

25. Let $x =$ the number of days by C alone.
Then $\frac{4}{21} + \frac{5}{21} + \frac{1}{x} = \frac{4}{7}.$
Whence $x = 7.$

26. From Hint, the part done by A in 6 days + the part done by B in 3 days = 1.
Let $x =$ the number of days by B alone.
Then $\frac{6}{8} + \frac{3}{x} = 1.$
Whence $x = 12.$

27. As in Ex. 26, let $x =$ the number of days A and B worked together.

Then $$\frac{1}{4} + \frac{x}{4} + \frac{x}{5} = 1.$$

Whence $$x = \tfrac{15}{9} = 1\tfrac{2}{3}.$$

29. A travels 48 miles; B travels 72 miles.

Let $x =$ B's rate in miles per hour,

and $x - 4 =$ A's rate in miles per hour.

Then $\dfrac{72}{x} =$ B's time in hours,

and $\dfrac{48}{x-4} =$ A's time in hours.

But $$\frac{72}{x} = \frac{48}{x-4}.$$

Whence $$x = 12;\ x - 4 = 8.$$

30. Let $x =$ the rate going in miles per hour,

and $x + 5 =$ the rate returning in miles per hour,

Then $\dfrac{360}{x} =$ the number of hours going,

and $\dfrac{360}{x+5} =$ the number of hours returning.

But $$\frac{360}{x} - 1 = \frac{360}{x+5}.$$

Whence $$x = 40;\ x + 5 = 45.$$

31. Let $x =$ the rate going in miles per hour.

Then $x + 5 =$ the rate returning in miles per hour,

and, as in Ex. 30, $$\frac{120}{x} - 4 = \frac{120}{x+5}.$$

Whence $$x = 10;\ x + 5 = 15.$$

32. Let $x =$ the rate of the automobile in miles per hour.

Then, noting that the automobile ran for $11\frac{1}{3} - 3\frac{1}{3} = 8$ hours,
$$(15)(11\tfrac{1}{3}) = (8)(x).$$

Whence $$x = 21\tfrac{1}{4}.$$

34. Let $x =$ the number of hours after A starts.

Then $x - \frac{5}{2} =$ B's time in hours.

But $$\frac{7x}{2} + \frac{19}{2}\left(x - \frac{5}{2}\right) = 25.$$

Whence $x = 3\frac{3}{4}$.

35. Let $x =$ A's rate in miles per hour.

Then $x + 2 =$ B's rate in miles per hour,

and since B *traveled* $\frac{5}{2}$ hours less than A, while each covered 60 miles,

$$\frac{60}{x} = \frac{60}{x+2} + \frac{5}{2}.$$

Whence $x = 6$, root -8 rejected.

36. From Hint, $5(4 - x) = 3(4 + x)$.

Whence $x = 1$.

37. Let $x =$ the distance upstream in miles.

Then $$\frac{x}{\frac{13}{2}} + \frac{x}{\frac{5}{2}} = 6.$$

Whence $x = 10\frac{5}{6}$.

38. Let $x =$ the number of hours returning.

Then $15 - x =$ the number of hours going,

and $(15 - x)(\frac{16}{3}) = (x)(\frac{8}{3})$.

Whence $x = 10$.

39. Let $x =$ the rate of the aëroplane in miles per hour.

Then $$\frac{24}{x - 15} + \frac{24}{x + 15} = \frac{2}{3}.$$

Whence $(x - 75)(x + 3) = 0$,

and $x = 75$, root -3 rejected.

40. Let $x =$ the number of sheep bought.

Then $\frac{48}{x} =$ the price paid for each,

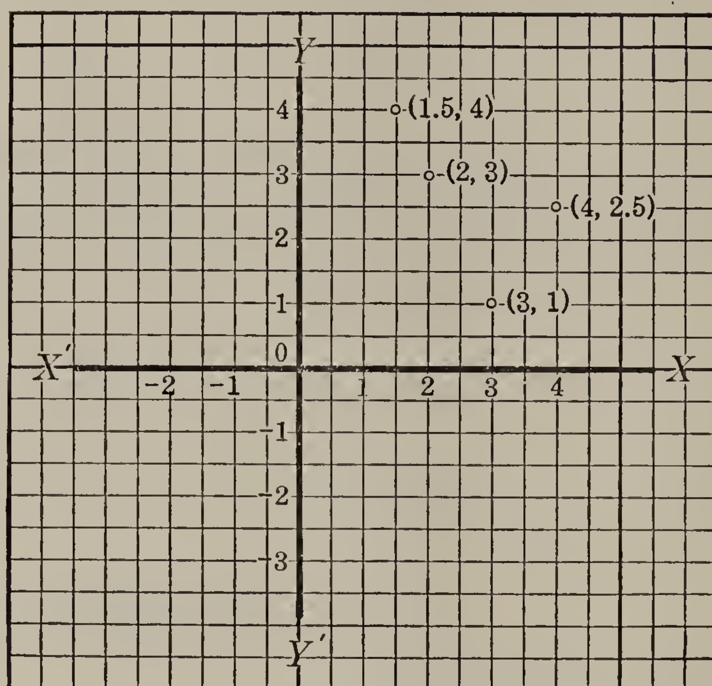
and $\frac{50}{x-2} =$ the price for which each was sold.

Then $$\frac{48}{x} = \frac{50}{x-2} - 1.$$

Whence $x = 12$, root -8 rejected.

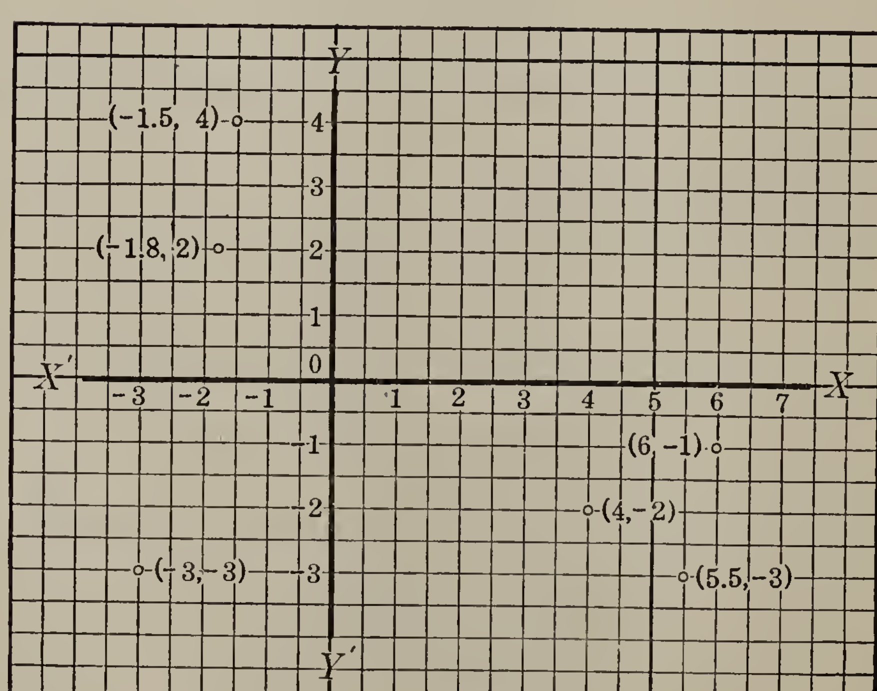
Page 204

1.



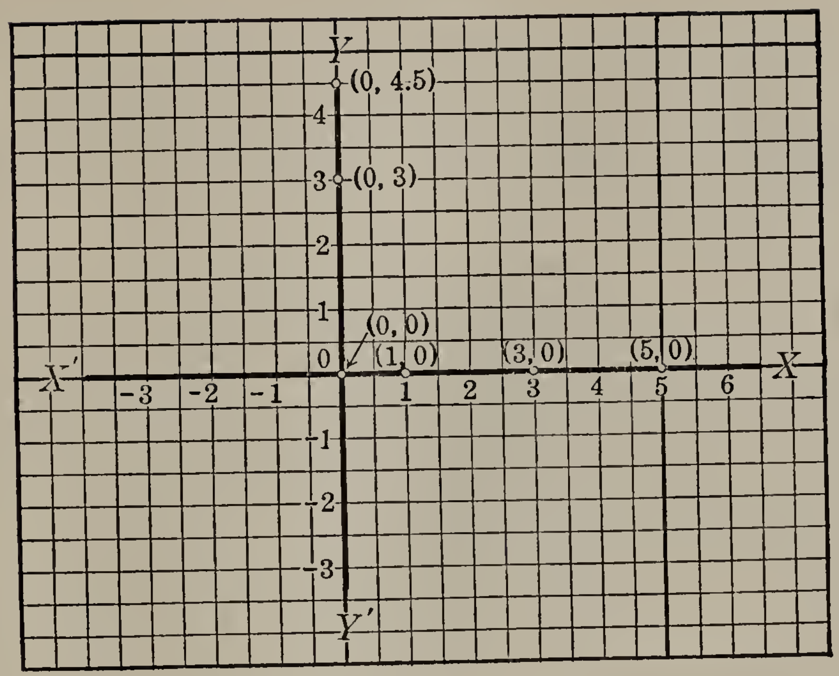
Y
X'
X
Y'
0
4
3
2
1
-1
-2
-3
-2
-1
1
2
3
4
(1.5, 4)
(2, 3)
(4, 2.5)
(3, 1)

2.

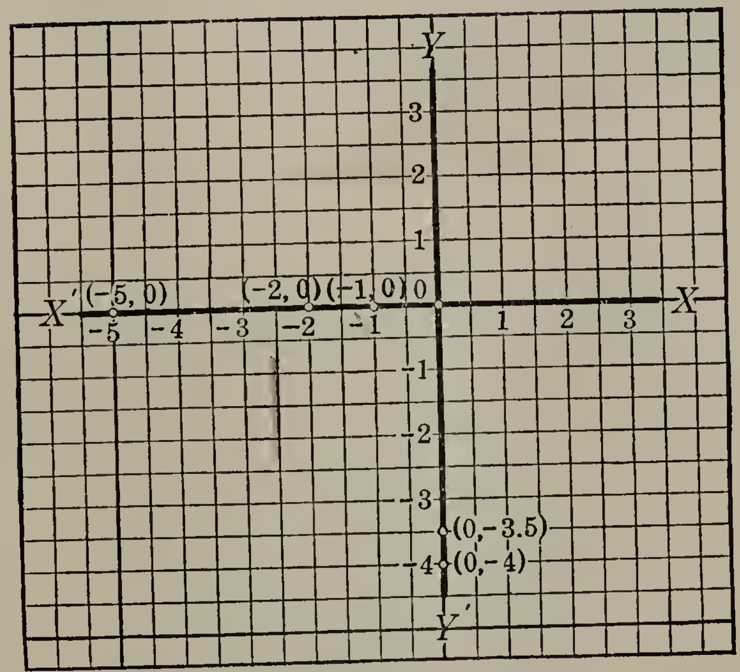


Y
X'
X
Y'
0
4
3
2
1
-1
-2
-3
-3
-2
-1
1
2
3
4
5
6
7
(-1.5, 4)
(-1.8, 2)
(6, -1)
(4, -2)
(-3, -3)
(5.5, -3)

3.



4.



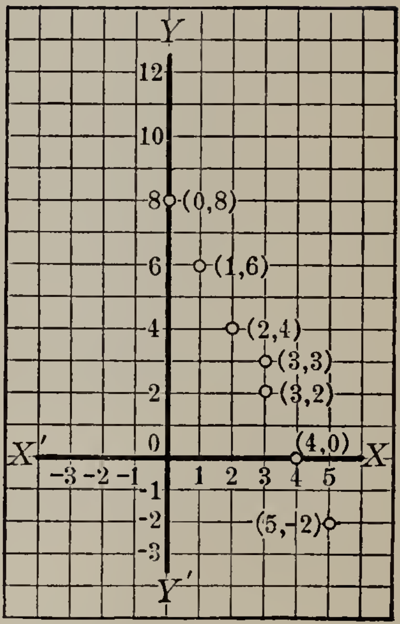
5. If the x-distance of a point is zero, it is located on the y-axis; if both coördinates are zero, it is located at the origin.

Page 205

1. For $2x + y = 8$.

If $x =$	4	2	0	5	3	1
then $y =$	0	4	8	-2	2	6

The six points are in a straight line.



Points whose x- and y-distances satisfy $2x + y = 8$ seem to lie on the straight line drawn through the points located above.

The graph of $2x + y = 8$ seems to be the straight line drawn through the points whose coördinates satisfy the equation.

The values $x = 3$, $y = 3$ do not satisfy $2x + y = 8$.

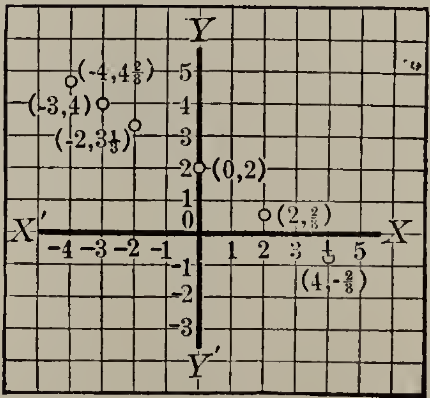
The point (3, 3) is not on the graph of $2x + y = 8$.

If the x- and y-distances of a point satisfy $2x + y = 8$, the point appears to be located on the graph of $2x + y = 8$.

If the x- and y-distances of a point do not satisfy $2x + y = 8$, the point is not on the graph of the equation.

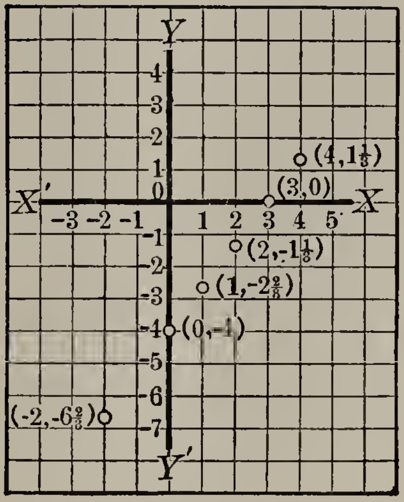
2. For $2x + 3y = 6$.

If $x =$	4	2	0	-2	-3	-4
then $y =$	$-\frac{2}{3}$	$\frac{2}{3}$	2	$3\frac{1}{3}$	4	$4\frac{2}{3}$



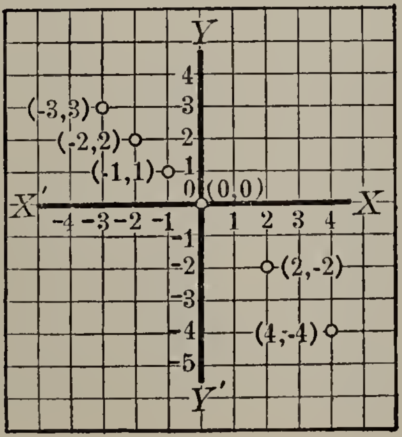
3. For $4x - 3y = 12$.

If $x =$	4	3	2	1	0	-2
then $y =$	$1\frac{1}{3}$	0	$-1\frac{1}{3}$	$-2\frac{2}{3}$	-4	$-6\frac{2}{3}$



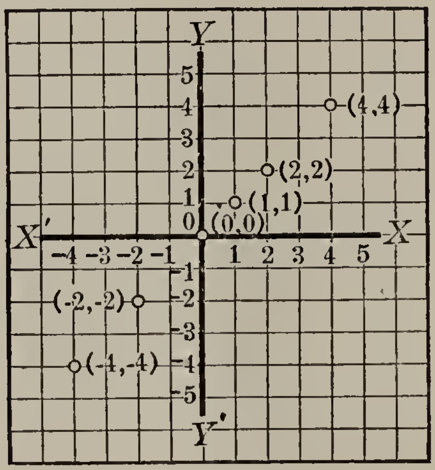
4. For $x + y = 0$.

If $x =$	4	2	0	-1	-2	-3
then $y =$	-4	-2	0	1	2	3



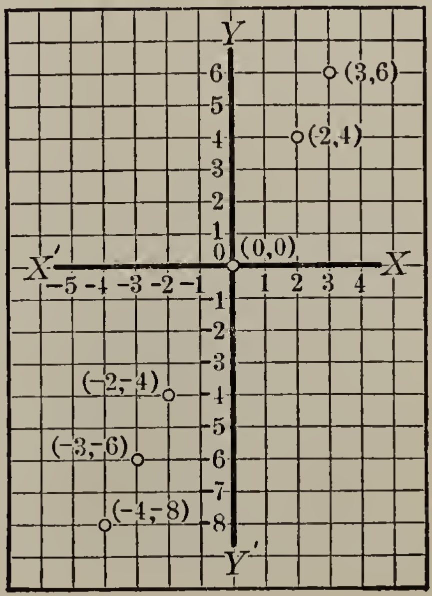
5. For $x - y = 0$.

If $x =$	4	2	1	0	-2	-4
then $y =$	4	2	1	0	-2	-4



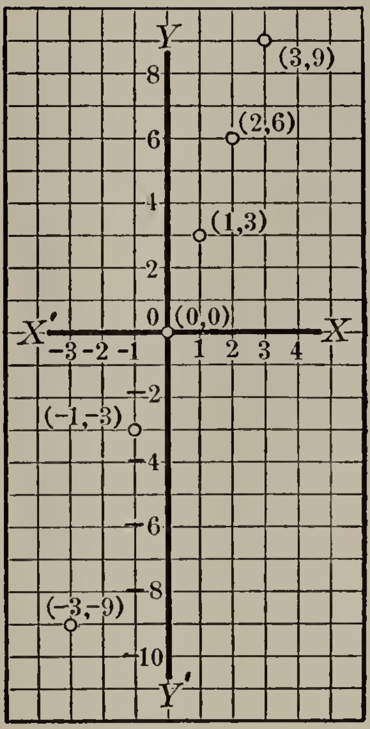
6. For $2x = y$.

If $x =$	3	2	0	-2	-3	-4
then $y =$	6	4	0	-4	-6	-8



7. For $y = 3x$.

If $x =$	3	2	1	0	-1	-3
then $y =$	9	6	3	0	-3	-9

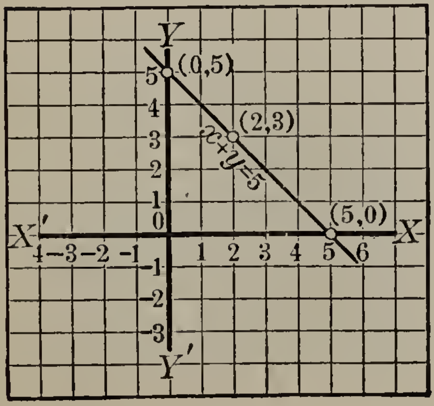


Yes, a line can be drawn through the points located in each of Exercises 2–7 inclusive.

Page 207

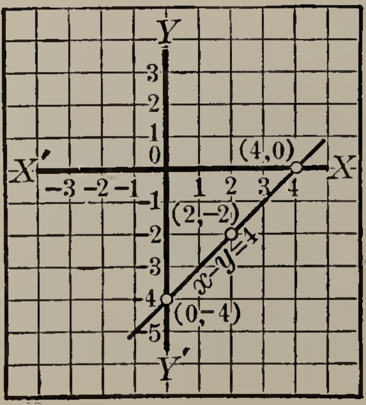
1. For $x + y = 5$.

If $x =$	5	0	2
then $y =$	0	5	3



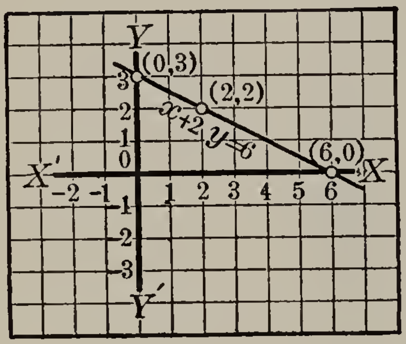
2. For $x - y = 4$.

If $x =$	0	4	2
then $y =$	-4	0	-2



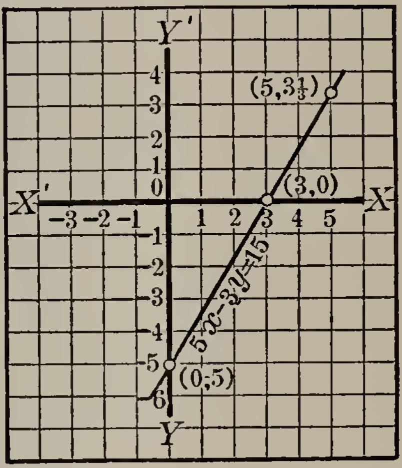
3. For $x + 2y = 6$.

If $x =$	0	6	2
then $y =$	3	0	2



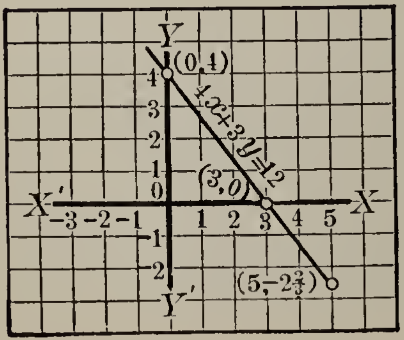
5. For $5x - 3y = 15$.

If $x =$	0	3	5
then $y =$	-5	0	$3\frac{1}{3}$



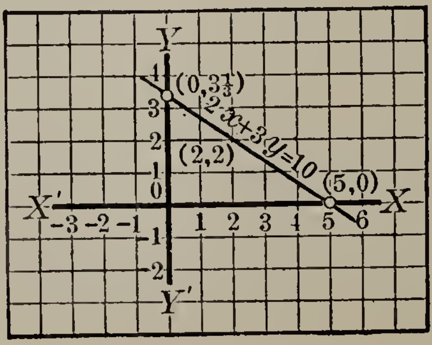
4. For $4x + 3y = 12$.

If $x =$	0	3	5
then $y =$	4	0	$-2\frac{2}{3}$



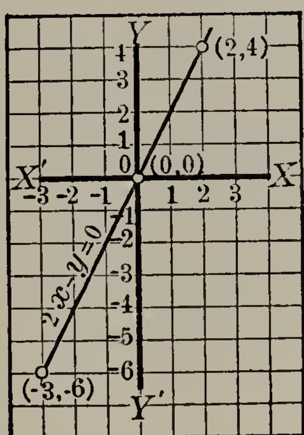
6. For $2x + 3y = 10$.

If $x =$	0	5	2
then $y =$	$3\frac{1}{3}$	0	2



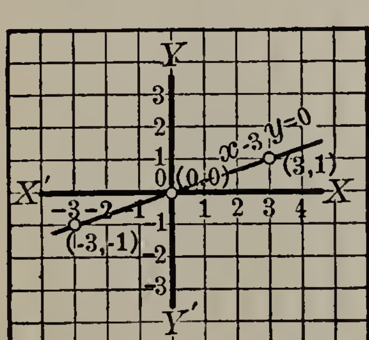
7. For $2x - y = 0$.

If $x =$	0	2	-3
then $y =$	0	4	-6



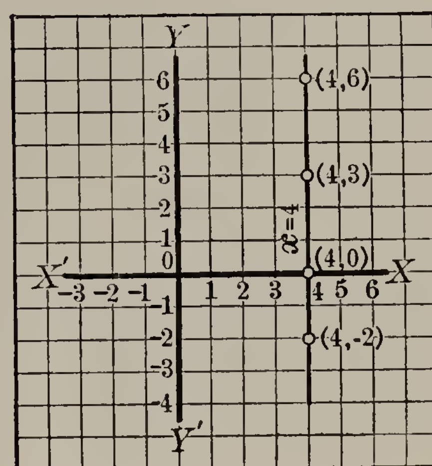
8. For $x - 3y = 0$.

If $x =$	0	3	-3
then $y =$	0	1	-1



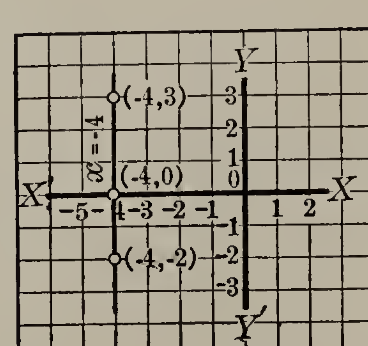
9. For $x = 4$, or $x + 0y = 4$.

If $x =$	4	4	4	4
then $y =$	3	6	0	-2



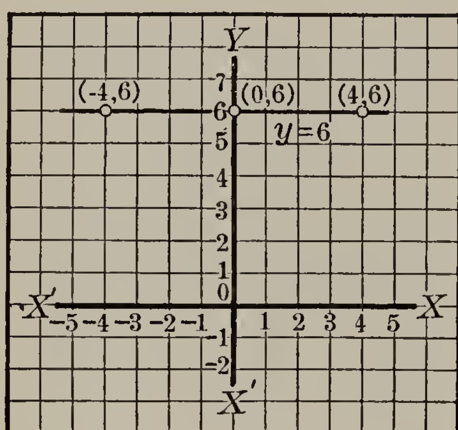
10. For $x = -4$, or $x + 0y = -4$.

If $x =$	-4	-4	-4
then $y =$	0	3	-2



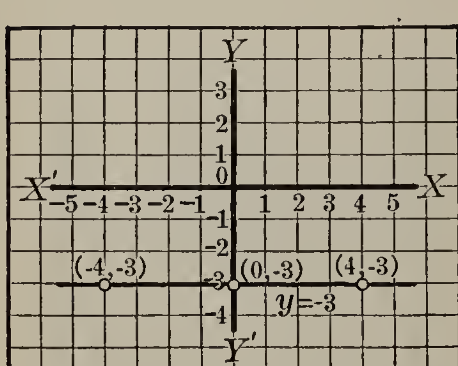
11. For $y = 6$, or $0x + y = 6$.

If $x =$	0	4	-4
then $y =$	6	6	6



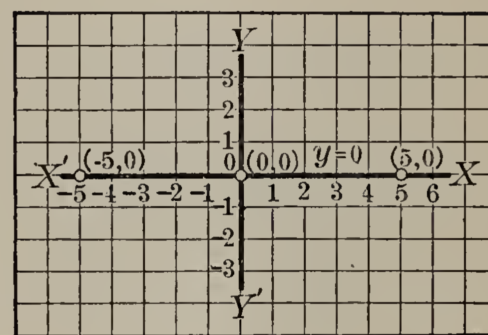
12. For $y = -3$, or $0x + y = -3$.

If $x =$	0	4	-4
then $y =$	-3	-3	-3



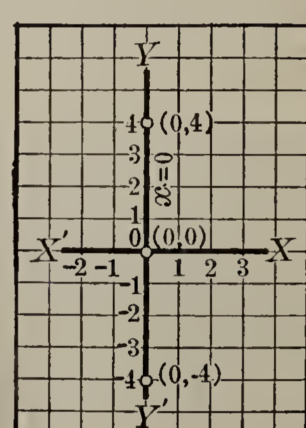
13. For $y = 0$, or $0x + y = 0$.

If $x =$	0	5	-5
then $y =$	0	0	0



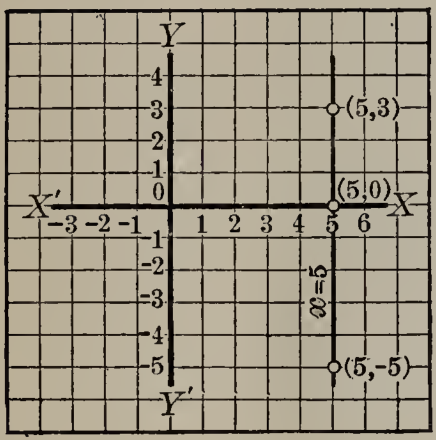
14. For $x = 0$, or $x + 0y = 0$.

If $x =$	0	0	0
then $y =$	0	4	-4



15. For $x = 5$, or $x + 0y = 5$.

If $x =$	5	5	5
then $y =$	0	3	-5



16. Yes.

17. Yes.

18. No, its values do not satisfy the equation; no, its values do not satisfy the equation; yes, since its values do satisfy the equation.

19. Yes, point (6, 5) lies on $5x - 3y = 15$.

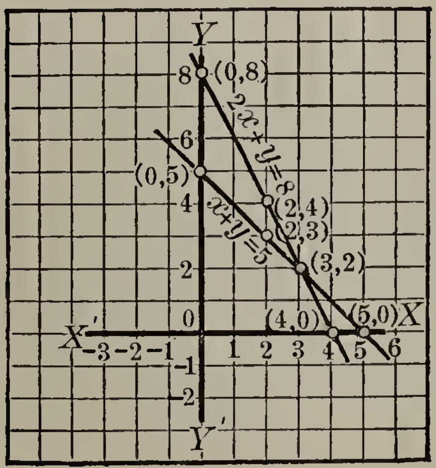
Page 209

1. For $x + y = 5$.

If $x =$	0	5	2
then $y =$	5	0	3

For $2x + y = 8$.

If $x =$	0	4	2
then $y =$	8	0	4



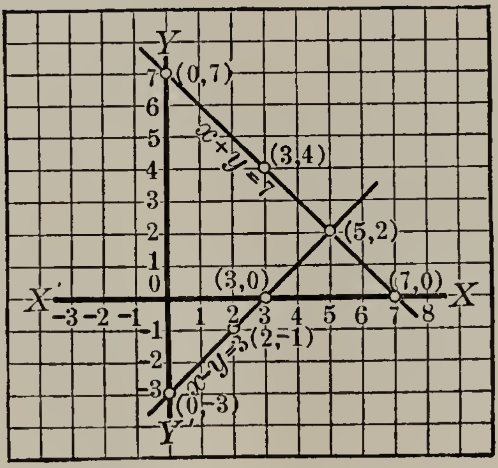
Ans. (3, 2).

2. For $x + y = 7$.

If $x =$	0	7	3
then $y =$	7	0	4

For $x - y = 3$.

If $x =$	0	3	2
then $y =$	-3	0	-1



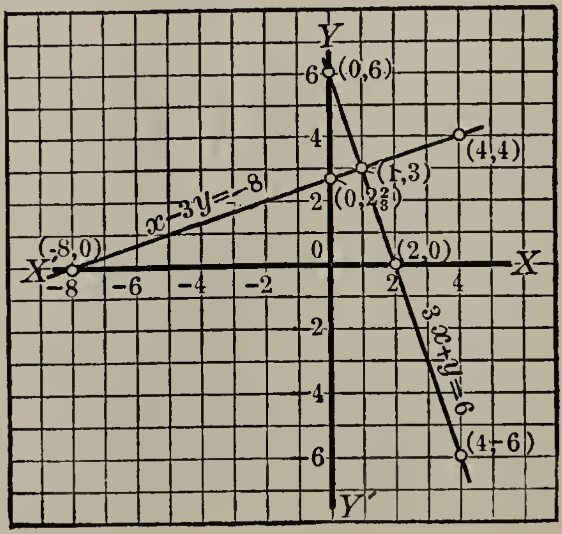
Ans. (5, 2).

3. For $3x + y = 6$.

If $x =$	0	2	4
then $y =$	6	0	-6

For $x - 3y = -8$.

If $x =$	0	-8	4
then $y =$	$2\frac{2}{3}$	0	4



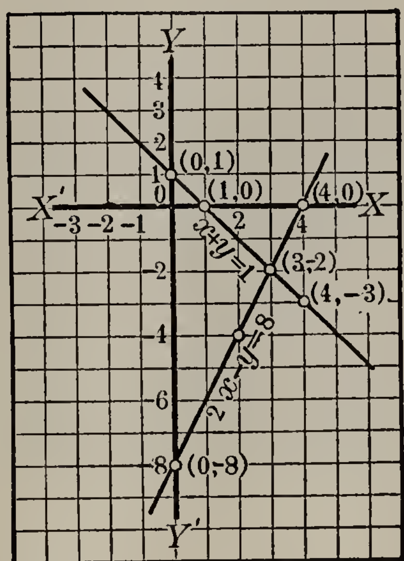
Ans. (1, 3).

4. For $x + y = 1$.

If $x =$	0	1	4
then $y =$	1	0	-3

For $2x - y = 8$.

If $x =$	0	4	2
then $y =$	-8	0	-4



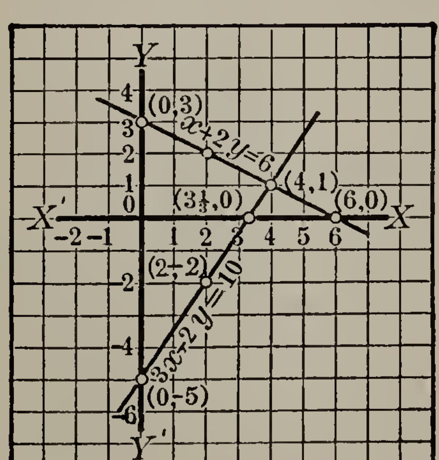
Ans. $(3, -2)$.

5. For $3x - 2y = 10$.

If $x =$	0	$3\frac{1}{3}$	2
then $y =$	-5	0	-2

For $x + 2y = 6$.

If $x =$	0	6	2
then $y =$	3	0	2



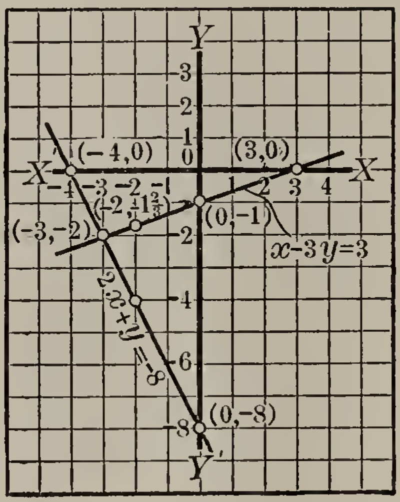
Ans. $(4, 1)$.

6. For $x - 3y = 3$.

If $x =$	0	3	-2
then $y =$	-1	0	$-1\frac{2}{3}$

For $2x + y = -8$.

If $x =$	0	-4	-2
then $y =$	-8	0	-4



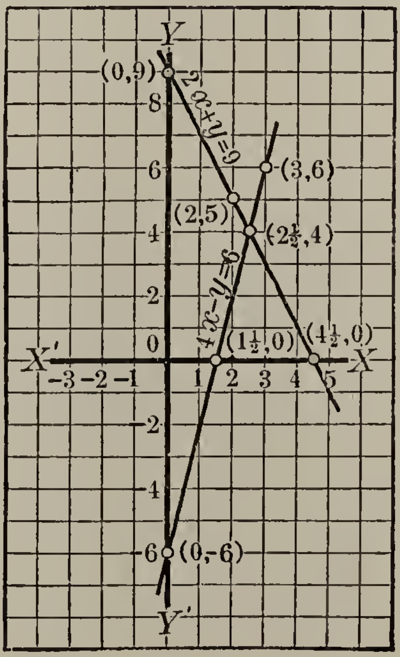
Ans. $(-3, -2)$.

7. For $4x - y = 6$.

If $x =$	0	$1\frac{1}{2}$	3
then $y =$	-6	0	6

For $2x + y = 9$.

If $x =$	0	$4\frac{1}{2}$	2
then $y =$	9	0	5



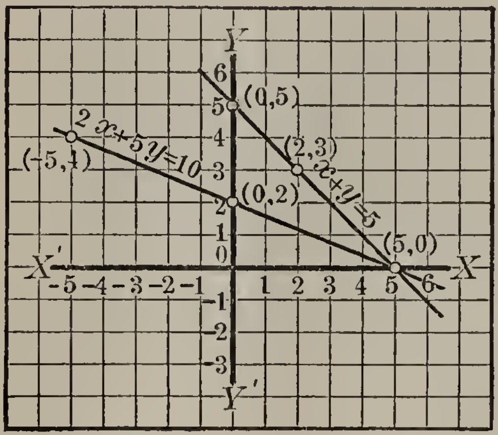
Ans. $(2\frac{1}{2}, 4)$.

8. For $2x + 5y = 10$.

If $x =$	0	5	-5
then $y =$	2	0	4

For $x + y = 5$.

If $x =$	0	5	2
then $y =$	5	0	3



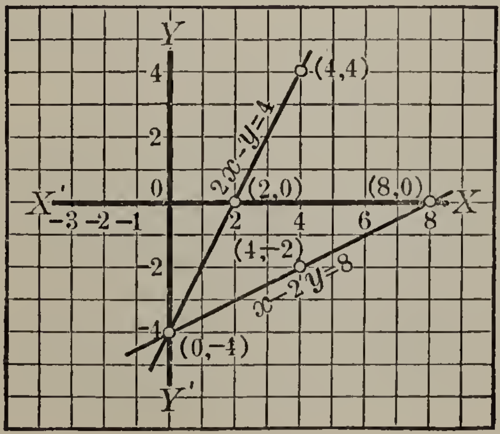
Ans. (5, 0).

9. For $x - 2y = 8$.

If $x =$	0	8	4
then $y =$	-4	0	-2

For $2x - y = 4$.

If $x =$	0	2	4
then $y =$	-4	0	4



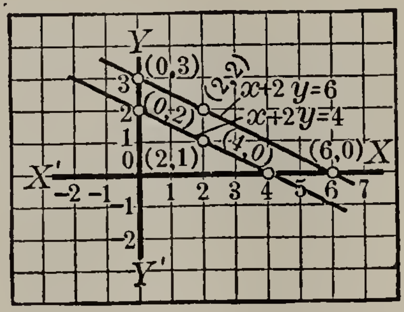
Ans. (0, -4).

10. For $x + 2y = 4$.

If $x =$	0	4	2
then $y =$	2	0	1

For $x + 2y = 6$.

If $x =$	0	6	2
then $y =$	3	0	2



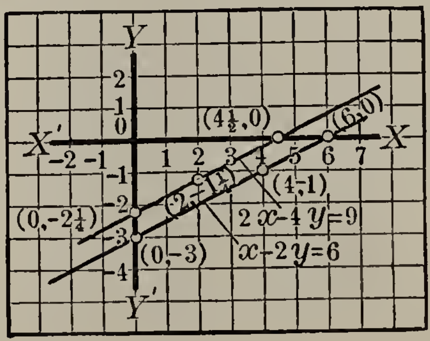
Ans. No set of roots.

11. For $x - 2y = 6$.

If $x =$	0	6	4
then $y =$	-3	0	-1

For $2x - 4y = 9$.

If $x =$	0	$4\frac{1}{2}$	2
then $y =$	$-2\frac{1}{4}$	0	$-1\frac{1}{4}$

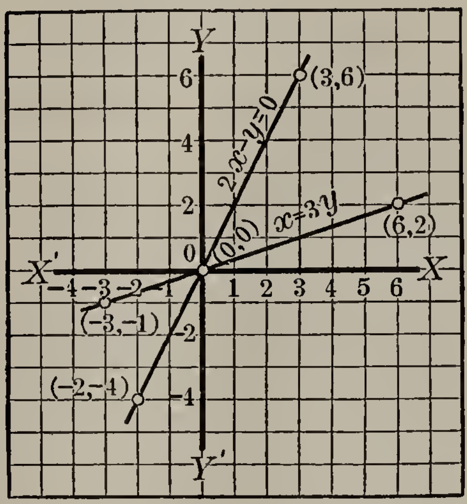


Ans. No set of roots.

12. For $2x - y = 0$. For $x = 3y$.

If $x =$	0	3	-2
then $y =$	0	6	-4

If $x =$	0	6	-3
then $y =$	0	2	-1



Ans. (0, 0).

Page 211

1. For 1885 the curve indicates a population of 57,000,000; for 1905 the curve indicates a population of 84,000,000.

2. The lower rate of increase is due largely to the casualties of the Civil War.

3. For 1915 the graph indicates a population of 99,000,000; for 1920 the graph indicates a population of 107,000,000.

4. Decreased immigration and deaths due to the great war will probably cause a lower population figure for 1920 than that indicated by the curve.

Page 212

1. The wind velocity at midnight, August 15, was 23 miles per hour; the wind velocity at midnight, August 16, was 76 miles per hour.

2. The wind velocity was 40 miles per hour at noon, August 16; at 2, 4, 6.30, 10, 12 P.M., August 17; at 8.30, 9.30, A.M., August 18. The wind velocity was 80 miles per hour at 10 and 11.30 P.M., August 16; at 12.30 and 4.30 A.M., August 17.

3. The barometric pressure at midnight, August 15, was 29.75 inches; at midnight, August 16, 28.7 inches; at 2 A.M., August 17, 28.65 inches.

4. The lowest reading of the barometer was 28.65 inches. The highest wind velocity was 91 miles per hour. These readings were recorded shortly after 2 A.M., August 17.

Page 214 (First set)

1. The graph shows a population of 67,000,000 for 1892; of 87,000,000 for 1907; of 95,000,000 for 1912.

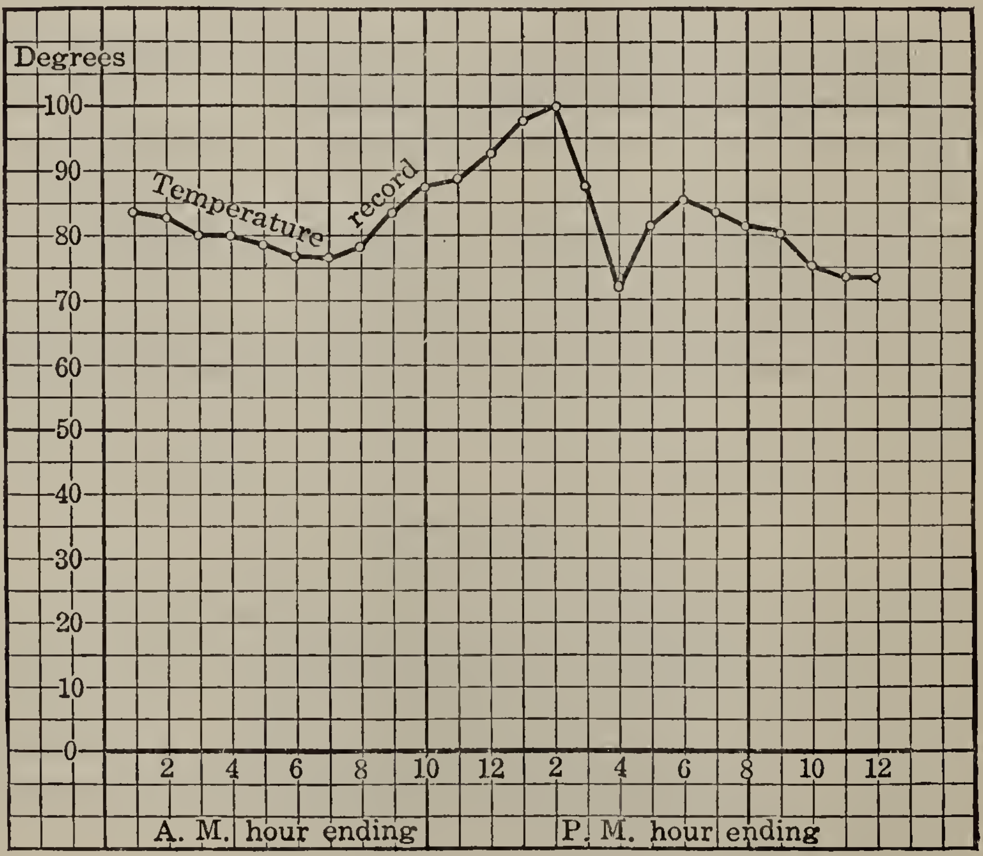
2. Since 1901 the rate of increase in the production of wheat has practically kept pace with the rate of increase of the population.

3. Save for the last five-year interval the graph reveals a tendency toward a decrease in the number of bushels of wheat exported.

4. The graph does not indicate that under normal conditions the exportation of wheat will increase to any considerable extent. The large wheat crop of 1915 and the increased demand due to the war show an abnormal tendency for the last five-year interval.

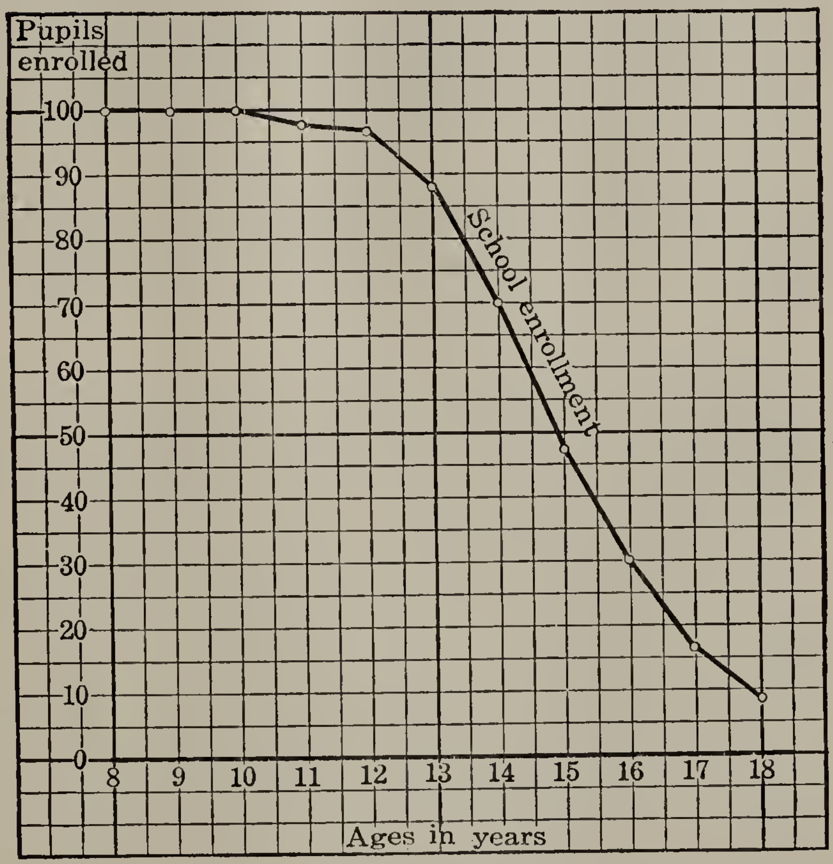
Page 214 (Second set)

1.



From the fall in temperature it appears that the storm must have occurred shortly after 2 P.M. The fall in temperature after 2 P.M. was due to atmospheric changes which took place in the storm area. The storm seems to have ceased at 4 P.M., thus allowing the sun's rays to reach the earth in the vicinity of the instrument.

2.



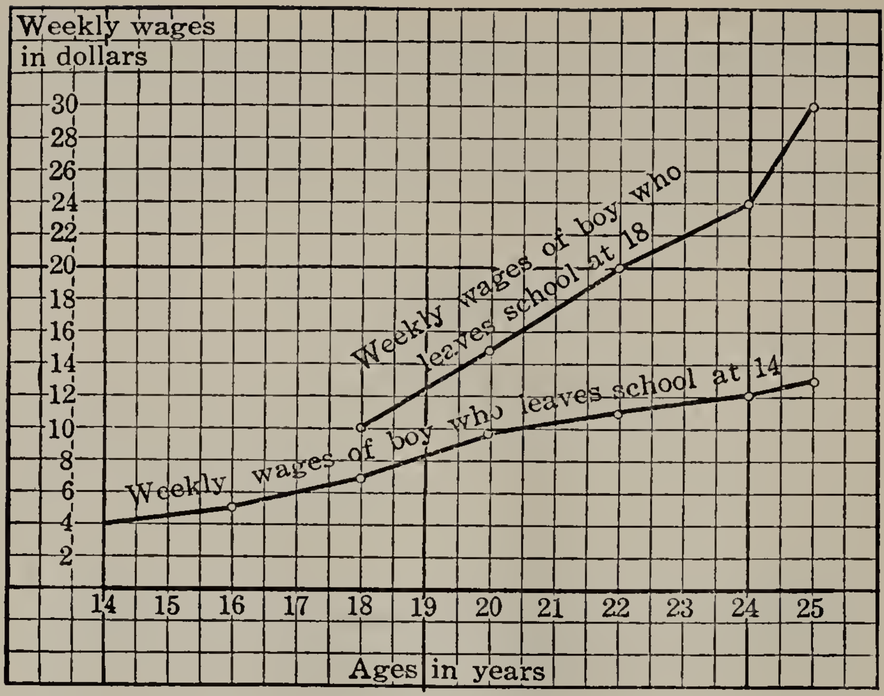
The tendency to leave school first becomes pronounced between the ages 12 and 13.

Yes, the tendency to leave school appears greater on the part of 14- and 15-year-olds.

The more decided drop in the curve between 14 and 15 shows the greater loss in school enrollment.

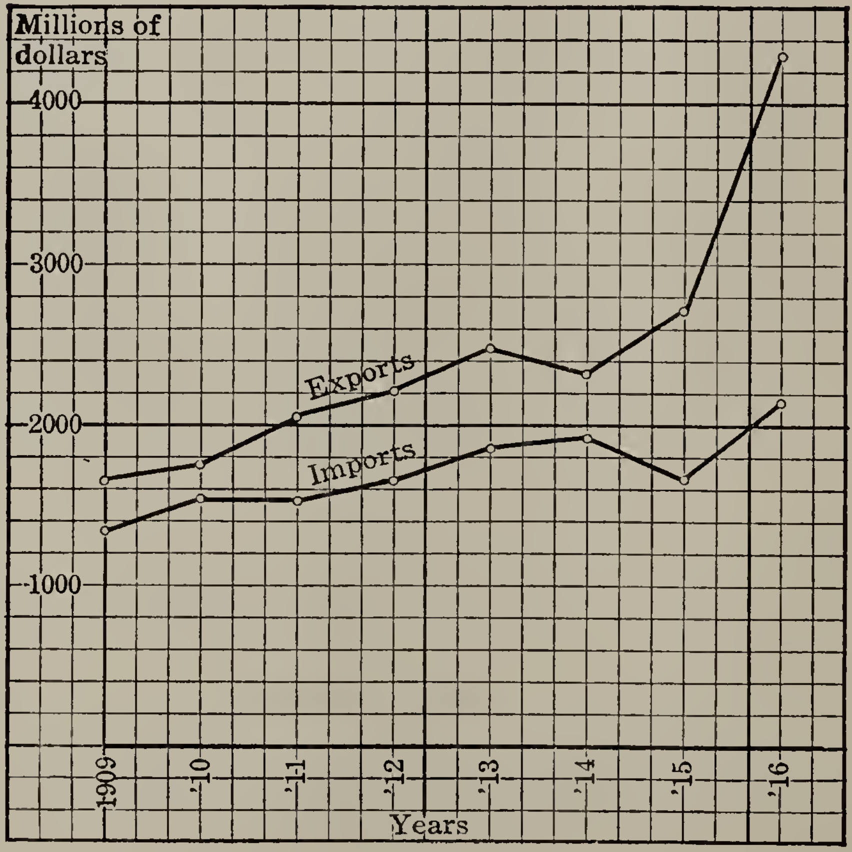
This loss is due in part to the fact that many more pupils leave school at the close of the grammar-school period than earlier.

3.



The graph shows that the boy who stays in school till he is 18 years of age receives a greater increase in weekly wages after that time than does the boy who leaves school and begins work at 14 years.

4.

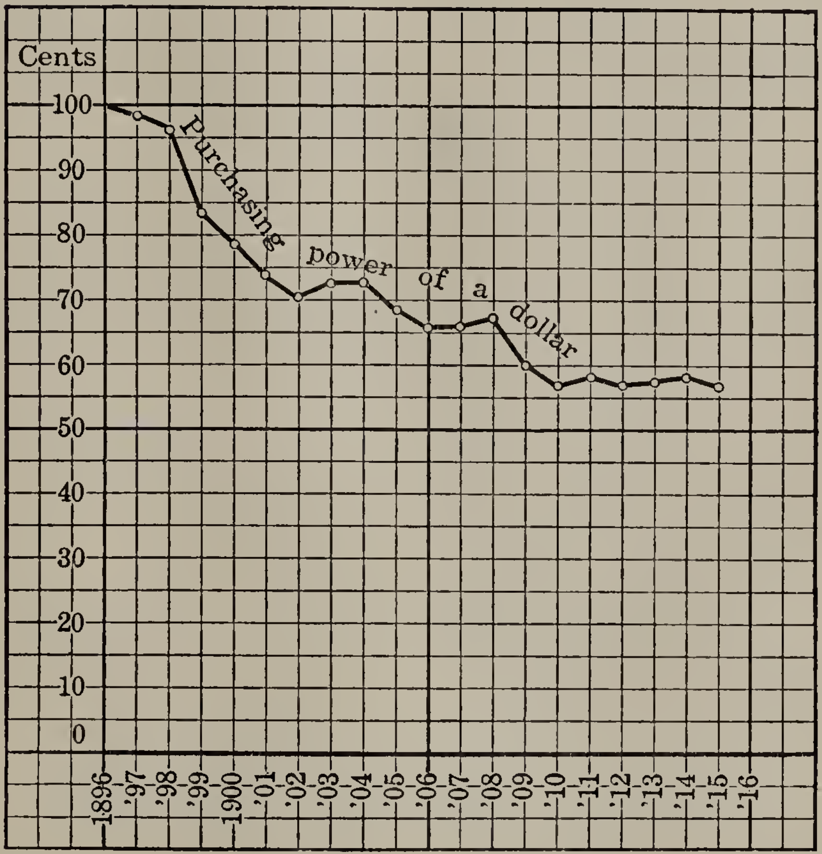


The decreased imports of 1915 were due to the fact that the United States was unable to get certain articles from certain nations at war.

The decreased exports of 1914 were due to the fact that certain nations at war were unable to get their normal amounts of merchandise from the United States.

The increased exports of 1915 are due in a large measure to the increase in the manufacture and shipping of war supplies.

5.



The purchasing power of a dollar has been almost constantly decreasing since 1896.

More merchandise could be bought for a certain sum in 1900 than in 1910. Approximately one third more could have been bought for a given sum at the earlier date.

Let $\quad x =$ the 1896 equivalent of \$1500 in 1910.

Then $$\frac{1500}{x} = \frac{100}{57.8}.$$

Whence $\quad x = \$867.$

CR

Page 221

1. $x + y = 3,$ (1)
$x - y = 1.$ (2)
(1) + (2), $2x = 4.$
$x = 2.$
Substituting in (1),
$2 + y = 3.$
$y = 1.$

2. $x + y = 7,$ (1)
$x - y = 1.$ (2)
(1) + (2), $2x = 8.$
$x = 4.$
Substituting in (1), $y = 3.$

3. $2x + y = 9,$ (1)
$x - y = 6.$ (2)
(1) + (2), $3x = 15.$
$x = 5.$
Substituting in (1), $y = -1.$

4. $x + y = 5,$ (1)
$x - 3y = 1.$ (2)
(1) − (2), $4y = 4.$ (3)
$y = 1.$ (4)
Substituting in (1), $x = 4.$

5. $x + y = -3,$ (1)
$x - y = 3.$ (2)
[(1) + (2)] ÷ 2, $x = 0,$
and from (1), $y = -3.$

6. $x + 2y = 7,$ (1)
$5x - 2y = 11.$ (2)
[(1) + (2)] ÷ 6, $x = 3,$
and from (1), $y = 2.$

7. $2y + x = 4,$ (1)
$3y - x = 21.$ (2)
[(1) + (2)] ÷ 5, $y = 5,$
and from (1), $x = -6.$

8. $7r - s = 2,$ (1)
$6r - s = 3.$ (2)
(1) − (2), $r = -1,$
and from (1), $s = -9.$

9. $3x - y = 3,$ (1)
$5x + 2y = 16.$ (2)
[2(1) + (2)] ÷ 11, $x = 2,$
and from (1), $y = 3.$

10. $4x + y = 2,$ (1)
$x - 2y = 5.$ (2)
[2(1) + (2)] ÷ 9, $x = 1,$
and from (1), $y = -2.$

11. $x + 2y = 1,$ (1)
$3x + 10y = 4.$ (2)
[(2) − 3(1)] ÷ 4, $y = \frac{1}{4},$
and from (1), $x = \frac{1}{2}.$

12. $5m - 3n = 0,$ (1)
$15m + 12n = 75.$ (2)
[4(1) + (2)] ÷ 35, $m = \frac{15}{7},$
and from (1), $n = \frac{75}{21} = \frac{25}{7}.$

13. $x - 6y = 7,$ (1)
$12y - x = -1.$ (2)
[(1) + (2)] ÷ 6, $y = 1,$
and from (1), $x = 13.$

14. $2x + 25y = 70,$ (1)
$3x = 10y + 10.$ (2)
Transposing (2),
$3x - 10y = 10.$ (3)
[3(1) − 2(3)] ÷ 95, $y = 2,$
and from (1), $x = 10.$

15. $4p + q = 5,$ (1)
$p - 4q = 14.$ (2)
[4(1) + (2)] ÷ 17, $p = 2,$
and from (1), $q = -3.$

16. $12x + 5y = 6,$ (1)
$3x - 3y = 10.$ (2)
[(1) − 4(2)] ÷ 17, $y = -2,$
and from (2), $x = \frac{4}{3}.$

17. $3x - 4y = 14,\qquad (1)$

$3y - 4x = -14.\qquad (2)$

$[3(1) + 4(2)] \div (-7),\ x = 2,$

and from (2), $y = -2.$

18. $4x + 3y = 5,\qquad (1)$

$9y - 8x = 0.\qquad (2)$

$[2(1) + (2)] \div 15,\quad y = \frac{2}{3},$

and from (1), $x = \frac{3}{4}.$

19. $6u + 8v = 26,\qquad (1)$

$5u - 3v = 70.\qquad (2)$

$[5(1) - 6(2)] \div 58,\ v = -5,$

and from (1), $u = 11.$

20. $11x + 7y = 111,\qquad (1)$

$3y - 4x = 4.\qquad (2)$

$[4(1) + 11(2)] \div 61,\ y = 8,$

and from (1), $x = 5.$

21. $5y + 3z = 37,\qquad (1)$

$9z + 15y = 111.\qquad (2)$

$3(1) = (2).$

Hence the equations are indeterminate; any set of roots of (1), as

$y = 2,$

$z = 9,$

also satisfies (2).

Page 223

1. $x - 2y = 8,\qquad (1)$

$3x + 2y = 8.\qquad (2)$

From (1), $x = 2y + 8.\qquad (3)$

(2) then becomes $6y + 24 + 2y = 8.$

Whence $y = -2,$

and from (3), $x = 4.$

2. $x - 2y = -1,\qquad (1)$

$4x - y = 10.\qquad (2)$

From (1), $x = 2y - 1.\qquad (3)$

(2) then becomes $8y - 4 - y = 10.$

Whence $y = 2,$

and from (3), $x = 3.$

3. $14m - 2n = 1,\qquad (1)$

$n - 6m = 0.\qquad (2)$

From (2), $n = 6m.\qquad (3)$

(1) then becomes $14m - 12m = 1.$

Whence $m = \frac{1}{2},$

and from (3), $n = 3.$

4. $6x + 10y = 42,\qquad (1)$

$2y = 3x.\qquad (2)$

From (2), $y = \frac{3}{2}x.\qquad (3)$

(1) then becomes $6x + 15x = 42.$

Whence $x = 2,$

and from (3), $y = 3.$

5. $5x + 3y = -1,$ (1)

$2x - 6y = 8.$ (2)

From (2), $x = 3y + 4.$ (3)

(1) then becomes $15y + 20 + 3y = -1.$

Whence $y = -\frac{7}{6},$

and from (3), $x = \frac{1}{2}.$

6. $x + y = 7,$ (1)

$2x + 3y = 17.$ (2)

From (1), $y = 7 - x.$ (3)

(2) then becomes $2x + 21 - 3x = 17.$

Whence $x = 4,$

and from (3), $y = 3.$

7. $3x = 6y - 3,$ (1)

$y + 4x = 5.$ (2)

From (1), $x = 2y - 1.$ (3)

(2) then becomes $y + 8y - 4 = 5.$

Whence $y = 1,$

and from (3), $x = 1.$

8. $5x + 10y = 25,$ (1)

$5x - 3y = 9.$ (2)

From (1), $5x = 25 - 10y.$ (3)

(2) then becomes $25 - 10y - 3y = 9.$

Whence $y = \frac{16}{13},$

and from (3), $x = \frac{33}{13}.$

9. $20y - 3z = 1,$ (1)

$z - 6y = 0.$ (2)

From (2), $z = 6y.$ (3)

(1) then becomes $20y - 18y = 1.$

Whence $y = \frac{1}{2},$

and from (3), $z = 3.$

10. $3x + 12 = 3 + y,$ (1)

$y = x + 1.$ (2)

(1) then becomes $3x + 12 = 3 + x + 1.$

Whence $x = -4,$

and from (2), $y = -3.$

11. $18 + 2k = h,$ (1)

$h + k = -9.$ (2)

(2) then becomes $18 + 2k + k = -9.$

Whence $k = -9,$

and from (2), $h = 0.$

12. $$2x = 4y + 6, \quad (1)$$
$$7x + 3y = 4. \quad (2)$$

From (1), $$x = 2y + 3. \quad (3)$$

Then (2) becomes $$14y + 21 + 3y = 4.$$

Whence $$y = -1,$$

and from (3), $$x = 1.$$

Page 224

1. $$4x + \frac{2y}{3} = \frac{26}{3}, \quad (1)$$
$$3y - \frac{7x}{2} = -4. \quad (2)$$

Clearing of fractions, $$12x + 2y = 26, \quad (3)$$
$$-7x + 6y = -8. \quad (4)$$

$[3(1) - (2)] \div 43$, $$x = 2,$$

and from (4), $$y = 1.$$

2. $$\frac{5x}{6} + \frac{y}{4} = 7, \quad (1)$$
$$\frac{2x}{3} - \frac{y}{8} = 3. \quad (2)$$

Clearing of fractions, $$20x + 6y = 168, \quad (3)$$
$$16x - 3y = 72.$$

$[(3) + 2(4)] \div 52$, $$x = 6,$$

and from (3), $$y = 8.$$

3. $$x - \frac{3y}{5} = \frac{18}{5}, \quad (1)$$
$$7x + \frac{8y}{3} = -16. \quad (2)$$

Clearing of fractions, $$5x - 3y = 18, \quad (3)$$
$$21x + 8y = -48. \quad (4)$$

$[8(3) + 3(4)] \div 103$, $$x = 0,$$

and from (3), $$y = -6.$$

4. $$.4x + .9y = 5.7, \quad (1)$$
$$x - \frac{y}{2} = \frac{1}{2}. \quad (2)$$

Clearing of fractions and decimals,
$$4x + 9y = 57, \quad (3)$$
$$2x - y = 1. \quad (4)$$

$[(3) - 2(4)] \div 11$, $$y = 5,$$

and from (4), $$x = 3.$$

5. $$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}, \quad (1)$$

$$2y - 3x = 0. \quad (2)$$

From (1), $$3y + 2x = 13. \quad (3)$$

$[2(2) + 3(3)] \div 13$, $$y = 3,$$

and from (2), $$x = 2.$$

6. $$\frac{x}{3} + \frac{y}{4} = -\frac{7}{8}, \quad (1)$$

$$\frac{y}{3} - \frac{x}{4} = \frac{11}{12}. \quad (2)$$

Clearing of fractions, $$8x + 6y = -21, \quad (3)$$

$$-3x + 4y = 11. \quad (4)$$

$[2(3) - 3(4)] \div 25$, $$x = -3,$$

and from (4), $$y = \tfrac{1}{2}.$$

7. $$.04x + .3y = 1, \quad (1)$$

$$.5x - .25y = 4.5. \quad (2)$$

Clearing, $$4x + 30y = 100, \quad (3)$$

$$50x - 25y = 450. \quad (4)$$

$(3) \div 2$, $$2x + 15y = 50. \quad (5)$$

$(4) \div 25$, $$2x - y = 18. \quad (6)$$

$[(5) - (6)] \div 16$, $$y = 2,$$

and from (6), $$x = 10.$$

8. $$7x - 4y = 14, \quad (1)$$

$$\frac{2x + 7}{2} - y = 4. \quad (2)$$

From (2), $$2x - 2y = 1. \quad (3)$$

$[(1) - 2(3)] \div 3$, $$x = 4,$$

and from (3), $$y = \tfrac{7}{2}.$$

9. $$\frac{x}{3} + \frac{y}{4} = \frac{1}{6}, \quad (1)$$

$$\frac{x - 2}{5} - \frac{y + 1}{2} = \frac{1}{2}. \quad (2)$$

Clearing, $$4x + 3y = 2, \quad (3)$$

$$2x - 5y = 14. \quad (4)$$

$[(3) - 2(4)] \div 13$, $$y = -2,$$

and from (4), $$x = 2.$$

10. $$\frac{2x+6}{5}+\frac{y+4}{5}=3, \quad (1)$$

$$\frac{7x+1}{3}-\frac{11y-4}{7}=4. \quad (2)$$

From (1), $$2x+y=5. \quad (3)$$

From (2), $$49x-33y=65. \quad (4)$$

[33(3) + (4)] ÷ 115, $$x=2,$$

and from (3), $$y=1.$$

11. $$\frac{x+y}{2}-8=\frac{x-y}{3}, \quad (1)$$

$$\frac{x-y}{4}+\frac{x+y}{3}=11. \quad (2)$$

From (1), $$x+5y=48. \quad (3)$$

From (2), $$7x+y=132. \quad (4)$$

[(3) – 5(4)] ÷ (– 34), $$x=18,$$

and from (4), $$y=6.$$

12. $$\frac{x}{6}-\frac{y}{4}=1, \quad (1)$$

$$\frac{4+5x}{11}-\frac{3-2y}{5}+4=0. \quad (2)$$

From (1), $$2x-3y=12. \quad (3)$$

From (2), $$25x+22y=-207. \quad (4)$$

[25(3) – 2(4)] ÷ (– 119), $$y=-6,$$

and from (3), $$x=-3.$$

13. $$\frac{1}{x}+\frac{1}{y}=\frac{5}{6}, \quad (1)$$

$$\frac{1}{x}-\frac{1}{y}=\frac{1}{6}. \quad (2)$$

Adding, $$\frac{2}{x}=1;\ x=2.$$

Substituting in (1), $$\frac{1}{y}=\frac{1}{3};\ y=3.$$

14. $$\frac{1}{x}+\frac{1}{y}=\frac{4}{15}, \quad (1)$$

$$\frac{3}{x}+\frac{2}{y}=\frac{19}{15}. \quad (2)$$

[2(1) – (2)], $$-\frac{1}{x}=-\frac{11}{15};\ x=\frac{15}{11},$$

and from (1), $$\frac{1}{y}=-\frac{7}{15};\ y=-\frac{15}{7}.$$

15. $$\frac{6}{x} + \frac{7}{y} + \frac{3}{2} = 0, \qquad (1)$$

$$\frac{7}{x} - \frac{6}{y} = \frac{16}{3}. \qquad (2)$$

$[6(1) + 7(2)] \div 85,$ $$\frac{1}{x} = \frac{1}{3};\ x = 3,$$

and from (1), $$\frac{7}{y} = -\frac{7}{2};\ y = -2.$$

16. $$\frac{5}{x} + 12 = 17, \qquad (1)$$

$$\frac{2}{x} - 3y = 0. \qquad (2)$$

From (1), $$\frac{1}{x} = 1;\ x = 1.$$

Then (2) becomes $$3y = 2;\ y = \tfrac{2}{3}.$$

17. $$\frac{x-y}{2} = \frac{25}{6} - \frac{x+y}{3}, \qquad (1)$$

$$\frac{x+y-9}{2} + \frac{x-y+6}{3} = 0. \qquad (2)$$

From (1), $5x - y = 25.$

From (2), $5x + y = 15.$

Adding, etc., $x = 4,$

and from (2), $y = -5.$

18. $$\frac{x-2}{5} - \frac{y-10}{4} = \frac{10-x}{3}, \qquad (1)$$

$$\frac{y+2}{3} - \frac{2x+y}{16} = \frac{x+13}{8}. \qquad (2)$$

From (1), $32x - 15y = 74.$ (3)

From (2), $-12x + 13y = 46.$ (4)

$[3(3) + 8(4)] \div 59,$ $y = 10,$

and from (3), $x = 7.$

19. $$\frac{x+2}{7} + 8 - 2x = \frac{x-y}{4}, \qquad (1)$$

$$2y - \frac{3x-2y}{3} = 3x + 4. \qquad (2)$$

From (1), $-59x + 7y = -232.$ (3)

From (2), $-12x + 8y = 12.$ (4)

$[8(3) - 7(4)] \div (-388),$ $x = 5,$

and from (4), $y = 9.$

20. From Hint, $x + 9y = 28.$ (4)

$8x - y = 5.$ (5)

From (5), $y = 8x - 5.$ (6)

(4) then becomes $x + 72x - 45 = 28.$

Whence $x = 1,$

and from (6), $y = 3.$

21. $\dfrac{4x + y}{6x + y} = \dfrac{2}{5},$ (1)

$x + \dfrac{10y}{3} = \dfrac{71}{3}.$ (2)

From (1), $8x + 3y = 0.$ (3)

From (2), $3x + 10y = 71.$ (4)

From (3), $y = -\frac{8}{3}x.$ (5)

(4) then becomes $3x - \frac{80}{3}x = 71.$

Whence $x = -3,$

and from (5), $y = 8.$

22. $\dfrac{4x}{3y + 2} = 4,$ (1)

$\dfrac{2x}{4x - 3y + 1} = \dfrac{1}{4}.$ (2)

From (1), $4x - 12y = 8,$

or $x = 3y + 2.$ (3)

From (2), $4x + 3y = 1,$

or $4(3y + 2) + 3y = 1.$

Whence $y = -\frac{7}{15},$

and from (3), $x = \frac{3}{5}.$

23. $\dfrac{3x - 4y + 2}{x - y} = 3,$ (1)

$\dfrac{x + y}{x} = \dfrac{5}{3}.$ (2)

From (1), $3x - 4y + 2 = 3x - 3y,$

or $y = 2.$

(2) then becomes $3(x + 2) = 5x,$

or $x = 3.$

24. $\dfrac{x - 3}{y + 1} = \dfrac{x - 6}{y - 2},$ (1)

$\dfrac{x + y - 5}{x - y + 1} = 3.$ (2)

From (1), $(x-3)(y-2)=(x-6)(y+1),$

or $-3x+3y=-12.$

Whence $x=y+4.$ (3)

From (2), $x+y-5=3(x-y+1),$

$-2x+4y=8,$

or using (3), $-2y-8+4y=8.$

Whence $y=8,$

and from (3), $x=12.$

25. $\dfrac{5x+3y-5}{3}=y+\dfrac{5}{3},$ (1)

$\dfrac{25x}{10x+2y}=1.$ (2)

From (1), $5x=10;\ x=2.$

(2) then becomes $50=20+2y,$

$y=15.$

26. $\dfrac{r+5}{r+1}=\dfrac{s+2}{s-2},$ (1)

$\dfrac{11r+4}{r+s+1}=4.$ (2)

From (1), $-4r+4s=12.$ (3)

From (2), $7r-4s=0.$ (4)

$[(3)+(4)]\div 3,$ $r=4,$

and from (4), $s=7.$

27. $\dfrac{2}{x+y+2}=\dfrac{1}{3x+5y-5},$ (1)

$\dfrac{x}{4}+\dfrac{y}{2}=1.$ (2)

From (1), $5x+9y=12.$ (3)

From (2), $x+2y=4.$ (4)

$5(4)-(3),$ $y=8,$

and from (4), $x=-12.$

28. $\dfrac{4}{x-1}=\dfrac{3}{1-y},$ (1)

$\dfrac{5}{2y-5}=\dfrac{7}{2x-39}.$ (2)

From (1), $3x+4y=7.$ (3)

From (2), $10x-14y=160,$ (4)

or $5x-7y=80.$ (5)

$[5(3)-3(5)]\div 41,$ $y=-5,$

and from (3), $x=9.$

Page 227

1. Let x and y represent the numbers.

Then $x + y = 36,$

and $x - y = 20.$

Whence $x = 28;\ y = 8.$

2. Let $x =$ the greater number,

and $y =$ the less.

Then $\frac{x}{y} = 12,$

and $x + y = 39.$

Whence $y = 3;\ x = 36.$

3. Let $x =$ the greater number,

and $y =$ the less.

Then $x - y = 28,$

and $\frac{x}{y} = 5.$

Whence $y = 7;\ x = 35.$

4. Let $\frac{x}{y}$ represent the fraction.

Then $\frac{x + 6}{y + 12} = \frac{2}{3},$

and $\frac{x}{y} = \frac{3}{4}.$

Whence $x = 18;\ y = 24.$

Therefore $\frac{x}{y} = \frac{18}{24}.$

5. Let $n =$ the number of nickels,

and $q =$ the number of quarters.

Then $n + q = 63,$

and $5n + 25q = 835.$

Whence $n = 37;\ q = 26.$

6. Let $\frac{x}{y} =$ the fraction.

Then $x + y = 11,$

and $\frac{x - \frac{2}{3}}{y + \frac{4}{3}} = \frac{1}{4}.$

Whence $x = 3;\ y = 8.$

Therefore $\frac{x}{y} = \frac{3}{8}.$

7. Let $\frac{x}{y}$ = the fraction.

Then $y - x = 11,$

and $\frac{x + \frac{1}{2}}{y - \frac{3}{4}} = \frac{6}{19}.$

Whence $x = 4;\ y = 15.$

$$\frac{x}{y} = \frac{4}{15}.$$

8. Let x = the greater weight in pounds,

and y = the less weight in pounds.

Then $8x = 12y,$

and $6(x + 8) = 12y.$

Whence $x = 24;\ y = 16.$

9. Let x = the first weight in pounds,

and y = the second weight in pounds.

Then $10x = 8y,$

and $10(x - 4) = 6y.$

Whence $x = 16;\ y = 20.$

10. Let a and b be the ages in years now.

Then $a + 20 = 2(b + 20),$

and $a - 10 = 8(b - 10).$

Whence $b = 15;\ a = 50.$

11. Let l and w be the dimensions in feet.

Then $l = 2w + 8,$

and $2(l + w) = 184.$

Whence $w = 28;\ l = 64.$

12. Let x = the number of dollars at 6%,

and y = the number of dollars at 5%.

Then $x + y = 1500,$

and $.06x + .05y = 81.$

Whence $x = 600;\ y = 900.$

13. Let x = the number of dollars at $4\frac{1}{2}\%$,

and y = the number of dollars at $3\frac{1}{2}\%$.

Then $x + y = 3000,$

and $.045x = 2(.035y) - 3.$

Whence $x = 1800;\ y = 1200.$

14. Let $x =$ the number of dollars at 4%,
and $y =$ the number of dollars at 6%.
Then $x + y = 5000,$
and $4(.04x) = 3(.06y) - 16.$
Whence $x = 2600;\ y = 2400.$

15. Let r and m be the values in cents.
Then $5r = 10m - 5,$
and $12m = 4r + 100.$
Whence $m = 24;\ r = 47.$

16. Let $x =$ the value in cents on Aug. 1, 1914,
and $y =$ the value in cents on Jan. 1, 1916.
Then $6y = 5x,$
and $10x - 10y = 40.$
Whence $y = 20;\ x = 24.$

18. Let $x =$ the tens' digit,
and $y =$ the units' digit.
Then $x + y = 10,$
and $10x + y + 54 = 10y + x.$
Whence $x = 2;\ y = 8.$ The number is 28.

19. As in Ex. 18, $x = \frac{1}{2}y.$
$$10x + y + 36 = 10y + x.$$
Whence $x = 4;\ y = 8.$ The number is 48.

20. As in Ex. 18, $\dfrac{10x + y}{x + y} = 4.$
$$2(10x + y) - 9 = 10y + x.$$
Whence $x = 3;\ y = 6.$ The number is 36.

21. As in Ex. 18, $\dfrac{10x + y + 4}{x + y} = 3.$
$$\frac{2(10x + y)}{x} = 29.$$
Whence $x = 2;\ y = 9.$ The number is 29.

22. As in Ex. 18, $\dfrac{10x + y}{x + y} = 7.$
$$\frac{10y + x}{x + y + 3} = 3.$$
Whence $x = 6;\ y = 3.$ The number is 63.

23. $\dfrac{1}{6};\ \dfrac{1}{7a};\ \dfrac{3}{2};\ \dfrac{4x}{3};\ \dfrac{2}{5};\ \dfrac{5}{17}.$

24. Let $x =$ the less number,
and $y =$ the greater number.

Then $\frac{1}{x}$ will be greater than $\frac{1}{y}$.

Therefore $$\frac{1}{x} + \frac{1}{y} = \frac{11}{28},$$

and $$\frac{1}{x} - \frac{1}{y} = \frac{3}{28}.$$

Whence $$x = 4;\ y = 7.$$

25. Let $x =$ the greater number,
and $y =$ the less.

Then $\frac{1}{y}$ will be greater than $\frac{1}{x}$.

Therefore $$\frac{1}{y} - \frac{1}{x} = \frac{1}{24},$$

and $$\frac{x}{y} = \frac{16}{15}.$$

Whence $$x = \tfrac{8}{5};\ y = \tfrac{3}{2}.$$

26. Let $x =$ the original weight in the first pan in grams,
and $y =$ the original weight in the second pan in grams.

Then $$x - 5 = \tfrac{1}{2}(y + 5),$$
and $$x + 15 = y - 15.$$
Whence $$x = 45;\ y = 75.$$

27. Let $a =$ the number of dollars A had at first,
and $b =$ the number of dollars B had at first.
Then $$2(a - 30) = b + 30,$$
and $$\tfrac{1}{5}(a - 30 + 150) = b + 30 - 150.$$
Whence $$a = 130;\ b = 170.$$

28. Let $f =$ the circumference of the fore wheel in feet,
and $r =$ the circumference of the rear wheel in feet.

Then $$f = r - \tfrac{3}{2},$$
and $$\frac{286}{f} = \frac{325}{r}.$$

Whence $$f = 11;\ r = 12\tfrac{1}{2}.$$

29. Let $a =$ the number of days required by A alone,

and $b =$ the number of days required by B alone.

Then $$\frac{1}{a} + \frac{1}{b} = \frac{5}{36},$$

and $$\frac{3}{a} + \frac{3}{b} + \frac{7}{a} = 1.$$

Whence $a = 12$; $b = 18$.

30. From Hint, $$\frac{10}{x + y} = 2,$$

and $$\frac{10}{x - y} = 4.$$

Whence $x = \frac{15}{4}$; $y = \frac{5}{4}$.

31. Let $x =$ the rate of boat in still water in miles per hour,

and $y =$ the rate of current in miles per hour.

Then $$\frac{45}{x + y} = 3,$$

and $$\frac{15}{x - y} = 3.$$

Whence $x = 10$; $y = 5$.

32. Let $x =$ the rate of current in miles per hour,

and $y =$ the distance from B to A in miles.

Then $$\frac{y}{\frac{15}{2} + x} = 12,$$

and $$\frac{\frac{y}{2}}{\frac{15}{2} - x} = 9.$$

Whence $x = 1\frac{1}{2}$; $y = 108$.

33. Let $x =$ the rate of current in miles per hour,

and $y =$ the distance from A to C in miles.

Then $$\frac{y}{12 + x} = 7,$$

and $$\frac{y - 36}{12 - x} = 5.$$

Whence $x = 1$; $y = 91$.

Page 231

1. $$x + y = 3a, \quad (1)$$
$$x - y = a. \quad (2)$$
$[(1) + (2)] \div 2$, $$x = 2a.$$
From (1), $$y = a.$$

2. $$5x + 4y = 17a, \quad (1)$$
$$8x + y = 11a. \quad (2)$$
$[4(2) - (1)] \div 27$, $$x = a.$$
From (2), $$y = 3a.$$

3. $$3x - y = 10b, \quad (1)$$
$$4x + 9y = 3b. \quad (2)$$
From (1), $$y = 3x - 10b. \quad (3)$$
In (2), $$4x + 27x - 90b = 3b.$$
Whence $$x = 3b,$$
and from (3), $$y = -b.$$

4. $$5x - 4y = 10a - 4, \quad (1)$$
$$x - 2ay = 0. \quad (2)$$
From (2), $$x = 2ay. \quad (3)$$
In (1), $$10ay - 4y = 10a - 4.$$
Whence $$y = 1,$$
and in (3), $$x = 2a.$$

5. $$3p + 4q - a = p - q + 22a, \quad (1)$$
$$p + a - q = 3p - 2q + 8a. \quad (2)$$
From (1), $$2p + 5q = 23a. \quad (3)$$
From (2), $$-2p + q = 7a. \quad (4)$$
$[(3) + (4)] \div 6$, $$q = 5a,$$
and from (3), $$p = -a.$$

6. $$3x + 4y = 6c, \quad (1)$$
$$\frac{x}{c} - \frac{3y}{4c} = 2. \quad (2)$$
From (2), $$4x - 3y = 8c. \quad (3)$$
$[3(1) + 4(3)] \div 25$, $$x = 2c,$$
and from (1), $$y = 0.$$

7. $$\frac{2x}{5} + \frac{2y}{3} = \frac{6a}{5}, \quad (1)$$
$$\frac{y}{2} + \frac{x}{2} = a. \quad (2)$$
From (1), $$6x + 10y = 18a. \quad (3)$$

From (2), $x + y = 2a.$

Whence $y = 2a - x.$ (4)

Then in (3), $6x + 20a - 10x = 18a.$

Whence $x = \frac{a}{2},$

and in (4), $y = \frac{3a}{2}.$

8. $7.5m + 3n = 6b,$ (1)

$.25m + .5n = 0.$ (2)

From (1), $75m + 30n = 60b,$

or $5m + 2n = 4b.$ (3)

From (2), $m = -2n.$ (4)

Then in (3), $-10n + 2n = 4b.$

Whence $n = -\frac{1}{2}b,$

and in (4), $m = b.$

9. $3x - 3y - 4a = x - b,$ (1)

$x + y + a + b = 2(a + b).$ (2)

From (1), $2x - 3y = 4a - b.$ (3)

From (2), $x + y = a + b.$ (4)

[(3) + 3(4)] ÷ 5, $x = \frac{7a + 2b}{5},$

and from (4), $y = \frac{3b - 2a}{5}.$

10. $\frac{x}{2a} - \frac{2y}{a} = -10,$ (1)

$\frac{3x}{a} - \frac{7y}{4a} = \frac{3}{2}.$ (2)

From (1), $x - 4y = -20a.$

Whence $x = 4y - 20a.$ (3)

From (2), $12x - 7y = 6a,$

and using (3), $48y - 240a - 7y = 6a.$

Whence $y = 6a,$

and from (3), $x = 4a.$

11. $x + y = a,$

$x - y = b.$

Adding, $x = \frac{a + b}{2}.$

Subtracting, $y = \frac{a - b}{2}.$

CR

12.
$$ax + 3y = 3 + 6a, \qquad (1)$$
$$a^2x + y = 5a. \qquad (2)$$

$a(1) - (2)$, $$3ay - y = -2a + 6a^2,$$

or $$(3a - 1)y = (3a - 1)2a.$$

Whence $$y = 2a,$$

and from (2), $$x = \frac{3}{a}.$$

13.
$$cx + y = 3, \qquad (1)$$
$$c(y - 3) = x. \qquad (2)$$

From (1), $$y = 3 - cx. \qquad (3)$$

In (2), $$c(-cx) = x.$$

Whence $$x = 0,$$

and from (3), $$y = 3.$$

14.
$$4x - 3y = 12(a - b), \qquad (1)$$
$$3x - 2y = 9a - 8b. \qquad (2)$$

$3(2) - 2(1)$, $$x = 3a,$$

and from (2), $$y = 4b.$$

15.
$$4x + 2y = a, \qquad (1)$$
$$2x + 4y = b. \qquad (2)$$

$[2(2) - (1)] \div 6$, $$y = \frac{2b - a}{6},$$

and from (2), $$x = \frac{2a - b}{6}.$$

16.
$$2x - ay = b, \qquad (1)$$
$$8x - by = 4a. \qquad (2)$$

$4(1) - (2)$, $$(b - 4a)y = 4(b - a).$$
$$y = \frac{4(b - a)}{b - 4a},$$

and from (1), $$x = \frac{b^2 - 4a^2}{2(b - 4a)}.$$

17.
$$\frac{1}{x} - \frac{1}{y} = a, \qquad (1)$$
$$\frac{3}{x} - \frac{2}{y} = b. \qquad (2)$$

$3(1) - (2)$, $$-\frac{1}{y} = 3a - b. \qquad (3)$$
$$y = \frac{1}{b - 3a}.$$

Substituting from (3) in (1), $\frac{1}{x} = b - 2a.$

$$x = \frac{1}{b - 2a}.$$

18. $$\frac{a}{x} + \frac{b}{y} = c, \qquad (1)$$

$$\frac{a}{x} - \frac{b}{y} = d. \qquad (2)$$

Adding, $\frac{2a}{x} = c + d.$

$$x = \frac{2a}{c + d}.$$

$[(1) - (2)] \div 2$, $\frac{b}{y} = \frac{c - d}{2}.$

$$y = \frac{2b}{c - d}.$$

19. $$ax + by = a + b,$$

$$ax - by = a - b.$$

Adding, etc., $x = \frac{2a}{2a} = 1.$

Subtracting, etc., $y = \frac{2b}{2b} = 1.$

20. $$ax - by = c. \qquad (1)$$

$$3ax - 2by = 4c. \qquad (2)$$

$[(2) - 3(1)] \div b$, $y = \frac{c}{b},$

and from (1), $x = \frac{2c}{a}.$

21. $$ax + by = c, \qquad (1)$$

$$hx + ky = m. \qquad (2)$$

$h(1) - a(2)$, $(bh - ak)y = ch - am.$

$$y = \frac{ch - am}{bh - ak}.$$

$k(1) - b(2)$, $(ak - bh)x = ck - bm.$

$$x = \frac{ck - bm}{ak - bh}.$$

Page 233

1. Solving, $x = \dfrac{20}{a-2}$.

2. Let $x =$ the decrease of the base in feet.

Then $\dfrac{(a+h)(b-x)}{2} = \dfrac{ab}{2}$.

Whence $x = \dfrac{bh}{a+h}$.

3. Let $x =$ the greater number,

and $y =$ the less.

Then $x + y = s$,

and $x - y = d$.

Whence $x = \dfrac{s+d}{2}$; $y = \dfrac{s-d}{2}$.

4. Let $x =$ the first number,

and $y =$ the second.

Then $x = ay$,

and $x - y = b$.

Whence $x = \dfrac{ab}{a-1}$; $y = \dfrac{b}{a-1}$.

5. Let $x =$ the first number,

and $y =$ the second.

Then $x + y = b$,

and $\dfrac{x}{y} = a$.

Whence $x = \dfrac{ab}{a+1}$; $y = \dfrac{b}{a+1}$.

6. Solving, $n = a + 2b$; $d = a + b$.

Then $\dfrac{n}{d} = \dfrac{a+2b}{a+b}$.

7. Let $n =$ the numerator,

and $d =$ the denominator.

Then $\dfrac{n+1}{d} = x$, and $\dfrac{n}{d-2} = y$.

Whence, solving for n and d, $n = \dfrac{y - 2xy}{x-y}$; $d = \dfrac{1-2y}{x-y}$.

8. Let $n =$ the numerator,
and $d =$ the denominator.

Then $\frac{n}{d} = b,$

and $\frac{n+2}{d} = c.$

Whence $n = \frac{2b}{c-b};\ d = \frac{2}{c-b}.$

9. Let $x =$ the distance, in feet, of first boy,
and $y =$ the distance, in feet, of second boy.

Then $x + y = l,$

and $ax = by.$

Whence $x = \frac{bl}{a+b};\ y = \frac{al}{a+b}.$

10. Let $x =$ the number originally at 80 cents,
and $y =$ the number originally at \$1.10.

Then $80x + 110y = 100h,$

and $110x + 80y = 100k.$

Whence $x = \frac{110k - 80h}{57};$

$y = \frac{110h - 80k}{57}.$

11. Let $x =$ the cost in cents of first book,
and $y =$ the cost in cents of second book.

Then $x + y = 100c,$

and $x - y = d.$

Whence $x = \frac{100c + d}{2};\ y = \frac{100c - d}{2}.$

12. Let $a =$ the number of dollars A has,
and $b =$ the number of dollars B has.

Then $a + b = k,$

and $a - h = b + h.$

Whence $a = \frac{k + 2h}{2};\ b = \frac{k - 2h}{2}.$

13. Let $a =$ the number of dollars A has,
and $b =$ the number of dollars B has.

Then $a - h = b + h,$

and $2(b - k) = a + k.$

Whence $a = 4h + 3k;\ b = 2h + 3k.$

14. Let $a =$ the number of dollars A has,
and $b =$ the number of dollars B has.
Then $a - 10 = b + 10 - h,$
and $3(b - k) = a + k.$
Whence $a = \dfrac{60 + 4k - 3h}{2};$
$b = \dfrac{20 + 4k - h}{2}.$

15. Let $a =$ the number of dollars A has,
and $b =$ the number of dollars B has.
Then $a + b = 40,$
and $a - h + k = b + h - k.$
Whence $a = 20 + h - k;\ b = 20 - h + k.$

16. Let $a =$ the number of dollars A has,
and $b =$ the number of dollars B has.
Then $a - r = \frac{1}{2}(b + r),$
and $b + r - 8 = \frac{3}{4}(a - r + 8).$
Whence $a = \dfrac{5r + 56}{5};\ b = \dfrac{112 - 5r}{5}.$

17. Let $x =$ the number of dollars at $a\%$,
and $b =$ the number of dollars at $b\%$.
Then $x + y = 1000,$
and $\dfrac{ax}{100} + \dfrac{by}{100} = c.$
Whence $x = \dfrac{1000b - 100c}{b - a};$
$y = \dfrac{1000a - 100c}{a - b}.$

18. Let $p =$ the number of dollars at 5%,
and $q =$ the number of dollars at 4%.
Then $p + q = x,$
and $.05p + .04q = y.$
Solving for p and q, $p = 100y - 4x,$
$q = 5x - 100y.$

19. Solving, $a = \dfrac{4c}{3};\ b = 4c.$

20. Let $a =$ the number of days required by A alone,

and $b =$ the number of days required by B alone.

Then $ah = b$,

and $\frac{1}{a} + \frac{1}{b} = \frac{1}{4}$.

Whence $a = \frac{4}{h}(h + 1)$; $b = 4(h + 1)$.

21. Let $a =$ the number of days required by A alone,

and $b =$ the number of days required by B alone.

Then $\frac{1}{a} + \frac{1}{b} = \frac{1}{d}$,

and $\frac{6}{a} = \frac{2}{3}$.

Whence $a = 9$; $b = \frac{9d}{9 - d}$.

22. Let $a =$ the number of days required by A alone,

and $b =$ the number of days required by B alone.

Then $\frac{1}{a} + \frac{1}{b} = \frac{1}{5}$,

and $\frac{k}{a} = \frac{2}{5}$.

Whence $a = \frac{5k}{2}$; $b = \frac{5k}{k - 2}$.

23. Let $a =$ the number of days required by A alone,

and $b =$ the number of days required by B alone.

Then $\frac{1}{a} + \frac{1}{b} = \frac{1}{n}$,

and $2a = b$.

Whence $a = \frac{3}{2}n$; $b = 3n$.

24. Let $a =$ the number of days required by A alone,

and $b =$ the number of days required by B alone.

Then $$\frac{1}{a} + \frac{1}{b} = \frac{1}{p}.$$

and $$qa = b.$$

Whence $$a = \frac{p}{q}(q+1);\ b = p(q+1).$$

Page 237

(In the exercises solved below the given equations may be combined in other orders than the one shown.)

1.
$$x + y + z = 9, \qquad (1)$$
$$x - y - z = -3, \qquad (2)$$
$$x + y - z = 5. \qquad (3)$$
$[(1) + (2)] \div 2,$ $\quad x = 3. \qquad (4)$

$[(1) - (3)] \div 2,$ $\quad z = 2. \qquad (5)$

Substituting from (4) and (5) in (1), $\quad y = 4.$

2.
$$x + y + z = 3, \qquad (1)$$
$$2x + y - z = 6, \qquad (2)$$
$$x - y - 2z = 0. \qquad (3)$$
$(1) + (2),$ $\quad 3x + 2y = 9. \qquad (4)$

$2(1) + (3),$ $\quad 3x + y = 6. \qquad (5)$

$(4) - (5),$ $\quad y = 3.$

Then from (5), $\quad x = 1,$

and from (1), $\quad z = -1.$

3.
$$x + y + z = 0, \qquad (1)$$
$$3x + 2y + 3z = -1, \qquad (2)$$
$$x - y - 2z = -8. \qquad (3)$$
$(1) + (3),$ $\quad 2x - z = -8. \qquad (4)$

$(2) - 2(1),$ $\quad x + z = -1. \qquad (5)$

$[(4) + (5)] \div 3,$ $\quad x = -3.$

Then from (5), $\quad z = 2,$

and from (1), $\quad y = 1.$

4.
$$3A + B - C = -8, \qquad (1)$$
$$A - 4B + 2C = 9, \qquad (2)$$
$$2A + 3B + 3C = 13. \qquad (3)$$
$(1) - 3(2),$ $\quad 13B - 7C = -35. \qquad (4)$

$(3) - 2(2),$ $\quad 11B - C = -5. \qquad (5)$

$[(4) - 7(5)] \div -64,$ $\quad B = 0.$

Then from (5), $\quad C = 5,$

and from (2), $\quad A = -1.$

5.
$$3x - 2y + 4z = 9, \qquad (1)$$
$$2x + 3y - 2z = -3, \qquad (2)$$
$$5x + 2y + 3z = 6. \qquad (3)$$
$(1) + 2(2),$ $\quad 7x + 4y = 3. \qquad (4)$

$3(2) + 2(3),$ $\quad 16x + 13y = 3. \qquad (5)$

$[13(4) - 4(5)] \div 27,$ $\quad x = 1.$

Then from (4), $\quad y = -1,$

and from (1), $\quad z = 1.$

6.
$$6h + 5k + 4m = 9, \qquad (1)$$
$$4h + 6k + 5m = 5, \qquad (2)$$
$$5h + 4k + 6m = 16. \qquad (3)$$
$3(2) - 2(1),$ $\quad 8k + 7m = -3. \qquad (4)$

$5(2) - 4(3),$ $\quad 14k + m = -39. \qquad (5)$

$[7(5) - (4)] \div 90,$ $\quad k = -3.$

Then from (5), $\quad m = 3,$

and from (2), $\quad h = 2.$

7.
$$x + 3y + 2z = 17, \qquad (1)$$
$$y - 4z = -5, \qquad (2)$$
$$x + 2y = -8. \qquad (3)$$
$(1) - (3),$ $\quad y + 2z = 25. \qquad (4)$

$[(4) - (2)] \div 6,$ $\quad z = 5.$

Then from (4), $\quad y = 15.$

and from (3), $\quad x = -38.$

8. $3x - y - 2z = -2,$ (1)

$6x + z = 4,$ (2)

$3y - 4z = -11.$ (3)

(2) − 2(1), $2y + 5z = 8.$ (4)

[3(4) − 2(3)] ÷ 23, $z = 2.$

Then from (4), $y = -1,$

and from (1), $x = \frac{1}{3}.$

9. $x + y = 1,$ (1)

$y + z = 3,$ (2)

$z + x = 8.$ (3)

(1) − (2), $x - z = -2.$ (4)

[(3) + (4)] ÷ 2, $x = 3.$

Then from (1), $y = -2,$

and from (3), $z = 5.$

10. $2p_1 - 3p_2 = 4,$ (1)

$3p_1 + p_3 = 5,$ (2)

$p_2 - 2p_3 = 2.$ (3)

2(2) + (3), $6p_1 + p_2 = 12.$ (4)

[(1) + 3(4)] ÷ 20, $p_1 = 2.$

Then from (2), $p_3 = -1,$

and from (3), $p_2 = 0.$

11. $5x - 7y + 4z = 3,$ (1)

$6x + 3y - 5z = -3,$ (2)

$4x + 6y + 3z = 25.$ (3)

5(1) + 4(2), $49x - 23y = 3.$ (4)

4(3) − 3(1), $x + 45y = 91.$ (5)

49(5), $49x + 2205y = 4459.$ (6)

[(6) − (4)] ÷ 2228, $y = 2.$

Then from (4), $x = 1,$

and from (3), $z = 3.$

12. $3A_1 + 2A_2 + 4A_3 = 9,$ (1)

$A_1 - A_2 - 2A_3 = 3,$ (2)

$A_1 - 3A_2 + 2A_3 = 2.$ (3)

(1) − 3(2), $5A_2 + 10A_3 = 0,$

or $A_2 + 2A_3 = 0.$ (4)

(2) − (3), $2A_2 - 4A_3 = 1.$ (5)

[(5) + 2(4)] ÷ 4, $A_2 = \frac{1}{4}.$

Then from (4), $A_3 = -\frac{1}{8},$

and from (2), $A_1 = 3.$

13. $\frac{1}{x} + \frac{1}{y} - \frac{1}{z} = 1,$ (1)

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{2}{3},$ (2)

$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 0.$ (3)

[(2) − (1)] ÷ 2, $\frac{1}{z} = -\frac{1}{6}, z = -6.$

[(1) + (3)] ÷ 2, $\frac{1}{x} = \frac{1}{2}, x = 2.$

[(2) − (3)] ÷ 2, $\frac{1}{y} = \frac{1}{3}, y = 3.$

14. $\frac{2}{p} + \frac{10}{q} - \frac{3}{r} = -3,$ (1)

$\frac{4}{p} + \frac{5}{q} + \frac{6}{r} = 15,$ (2)

$\frac{1}{p} + \frac{5}{q} - \frac{1}{r} = -\frac{1}{2}.$ (3)

2(1) − (2), $\frac{15}{q} - \frac{12}{r} = -21.$ (4)

(1) − 2(3), $-\frac{1}{r} = -2, r = \frac{1}{2}.$

Then in (4), $\frac{1}{q} = \frac{1}{5}, q = 5,$

and in (3), $\frac{1}{p} = \frac{1}{2}, p = 2.$

15. $\frac{2}{x} + \frac{3}{z} = \frac{34}{3}.$ (1)

$\frac{3}{x} + \frac{4}{y} = 24,$ (2)

$\frac{4}{z} + \frac{5}{y} = 28.$ (3)

3(1) − 2(2), $\frac{9}{z} - \frac{8}{y} = -14.$ (4)

[9(3) − 4(4)] ÷ 77, $\frac{1}{y} = 4, y = \frac{1}{4}.$

Then from (2), $\frac{1}{x} = \frac{8}{3}, x = \frac{3}{8},$

and from (1), $\frac{1}{z} = 2, z = \frac{1}{2}.$

Page 238

1. Let x, y, and z represent the three numbers respectively.

Then $x + y = 54.$

$y + z = 65.$

$x + z = 59.$

Whence $x = 24, y = 30, z = 35.$

2. Let x, y, and z represent the three numbers respectively.

Then $x + y + z = 70.$

$x + y = 45.$

$\frac{x}{y} = 5.$

Whence $x = 37\frac{1}{2}, y = 7\frac{1}{2}, z = 25.$

3. Let x, y, and z represent the three sides in feet.

Then $x + y + z = 60.$

$x = y.$

$z = x + 6.$

Whence $x = 18, y = 18, z = 24.$

4. Let x, y, and z represent the three sides in feet.

Then $x + y = 53.$

$x - y = 19.$

$x + y + z = 62.$

Whence $x = 36, y = 17, z = 9.$

5. Let x, y, and z represent the three sides in feet.

Then $x + y = 41.$

$y + z = 46.$

$x + z = 57.$

Whence $x = 26, y = 15, z = 31.$

6. Let x and y equal the first two numbers, and let z equal the third.

Then $x + y + z = 24.$

$\frac{x}{y} = 3.$

$\frac{x + y}{z} = 5.$

Whence $x = 15, y = 5, z = 4.$

7. Let x, y, and z represent the number of degrees in the three angles respectively.

Then $x = y.$

$x + y = z.$

$x + y + z = 180.$

Whence $x = 45, y = 45, z = 90.$

8. We have at once

$A - 12 = B.$

$B - 21 = C.$

$A + B + C = 180.$

Whence $A = 75, B = 63, C = 42.$

9. Let x, y, and z represent the number of degrees in the three angles respectively.

Then $x + y = z + 30.$

$z = 15(x - y).$

$x + y + z = 180.$

Whence $x = 55, y = 50, z = 75.$

10. Let a, b, and c represent the number of days required by A, B, and C respectively.

Then $\frac{1}{a} + \frac{1}{b} = \frac{1}{6}.$

$\frac{1}{a} + \frac{1}{c} = \frac{1}{8}.$

$\frac{1}{b} + \frac{1}{c} = \frac{1}{12}.$

Whence $a = \frac{48}{5}, b = 16, c = 48.$

11. Let a, b, and c represent the number of hours required by the pipes respectively.

Then $\frac{1}{a} + \frac{1}{b} = \frac{1}{6}.$

$\frac{1}{a} + \frac{1}{c} = \frac{1}{8}.$

$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{4}.$

Whence $a = 24, b = 8, c = 12.$

12. Let $\quad x =$ the hundreds' digit,

$y =$ the tens' digit,

and $\quad z =$ the units' digit.

Then

$$x + y + z = 19.$$
$$z - y = 3.$$
$$495 + 100x + 10y + z = 100z + 10y + x.$$

Whence $\quad x = 4,\ y = 6,\ z = 9.$

13. As in Ex. 12,

$$100x + 10y + z - 54 = 100x + 10z + y.$$
$$100x + 10y + z + 360 = 100y + 10x + z.$$
$$x + y + z = 11.$$

Whence $\quad x = 3,\ y = 7,\ z = 1.$

Page 242

1.
$$\begin{array}{r|l}
 & a^2 + 6a + 9 \;\big|\; \underline{a + 3} \\
 & a^2 \\ \hline
2a + 3 & 6a + 9 \\
 & \underline{6a + 9}
\end{array}$$

2.
$$\begin{array}{r|l}
 & x^2 + 10ax + 25a^2 \;\big|\; \underline{x + 5a} \\
 & x^2 \\ \hline
2x + 5a & 10ax + 25a^2 \\
 & \underline{10ax + 25a^2}
\end{array}$$

3.
$$\begin{array}{r|l}
 & x^4 - 8x^2 + 16 \;\big|\; \underline{x^2 - 4} \\
 & x^4 \\ \hline
2x^2 - 4 & -8x^2 + 16 \\
 & \underline{-8x^2 + 16}
\end{array}$$

4.
$$\begin{array}{r|l}
 & a^4 + 4a^2x^2 + 4x^4 \;\big|\; \underline{a^2 + 2x^2} \\
 & a^4 \\ \hline
2a^2 + 2x^2 & 4a^2x^2 + 4x^4 \\
 & \underline{4a^2x^2 + 4x^4}
\end{array}$$

5.
$$\begin{array}{r|l}
 & a^4 + 2a^3 + 3a^2 + 2a + 1 \;\big|\; \underline{a^2 + a + 1} \\
 & a^4 \\ \hline
2a^2 + a & 2a^3 + 3a^2 \\
 & \underline{2a^3 + a^2} \\
2a^2 + 2a + 1 & 2a^2 + 2a + 1 \\
 & \underline{2a^2 + 2a + 1}
\end{array}$$

6.
$$\begin{array}{r|l}
 & x^4 - 8x^3 + 24x^2 - 32x + 16 \;\big|\; \underline{x^2 - 4x + 4} \\
 & x^4 \\ \hline
2x^2 - 4x & -8x^3 + 24x^2 \\
 & \underline{-8x^3 + 16x^2} \\
2x^2 - 8x + 4 & 8x^2 - 32x + 16 \\
 & \underline{8x^2 - 32x + 16}
\end{array}$$

7.
$$\begin{array}{l|l|l}
 & x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 & x^2 + 2xy + y^2 \\
 & x^4 & \\
2x^2 + 2xy & 4x^3y + 6x^2y^2 & \\
 & 4x^3y + 4x^2y^2 & \\
2x^2 + 4xy + y^2 & 2x^2y^2 + 4xy^3 + y^4 & \\
 & 2x^2y^2 + 4xy^3 + y^4 &
\end{array}$$

8.
$$\begin{array}{l|l|l}
 & c^4 - 10c^3 + 21c^2 + 20c + 4 & c^2 - 5c - 2 \\
 & c^4 & \\
2c^2 - 5c & -10c^3 + 21c^2 & \\
 & -10c^3 + 25c^2 & \\
2c^2 - 10c - 2 & -4c^2 + 20c + 4 & \\
 & -4c^2 + 20c + 4 &
\end{array}$$

9.
$$\begin{array}{l|l|l}
 & n^6 - 6n^4 + 10n^3 + 9n^2 - 30n + 25 & n^3 - 3n + 5 \\
 & n^6 & \\
2n^3 - 3n & -6n^4 + 10n^3 + 9n^2 & \\
 & -6n^4 \qquad\qquad + 9n^2 & \\
2n^3 - 6n + 5 & 10n^3 \qquad - 30n + 25 & \\
 & 10n^3 \qquad - 30n + 25 &
\end{array}$$

10.
$$\begin{array}{l|l|l}
 & c^4 - 6c^3 + 13c^2 - 12c + 4 & c^2 - 3c + 2 \\
 & c^4 & \\
2c^2 - 3c & -6c^3 + 13c^2 & \\
 & -6c^3 + 9c^2 & \\
2c^2 - 6c + 2 & 4c^2 - 12c + 4 & \\
 & 4c^2 - 12c + 4 &
\end{array}$$

11.
$$\begin{array}{l|l|l}
 & 4a^6 + 12a^5 + 5a^4 - 22a^3 - 23a^2 + 8a + 16 & 2a^3 + 3a^2 - a - 4 \\
 & 4a^6 & \\
4a^3 + 3a^2 & +12a^5 + 5a^4 & \\
 & +12a^5 + 9a^4 & \\
4a^3 + 6a^2 - a & -4a^4 - 22a^3 - 23a^2 & \\
 & -4a^4 - 6a^3 + a^2 & \\
4a^3 + 6a^2 - 2a - 4 & -16a^3 - 24a^2 + 8a + 16 & \\
 & -16a^3 - 24a^2 + 8a + 16 &
\end{array}$$

12.
$$\begin{array}{l|l|l}
 & c^4 - 4c^3a + 6c^2a^2 - 4ca^3 + a^4 & c^2 - 2ca + a^2 \\
 & c^4 & \\
2c^2 - 2ca & -4c^3a + 6c^2a^2 & \\
 & -4c^3a + 4c^2a^2 & \\
2c^2 - 4ca + a^2 & 2c^2a^2 - 4ca^3 + a^4 & \\
 & 2c^2a^2 - 4ca^3 + a^4 &
\end{array}$$

13.

$$
\begin{array}{l|l}
 & 25y^4 + 30xy^3 - 11x^2y^2 - 12x^3y + 4x^4 \;\underline{|\,5y^2 + 3xy - 2x^2} \\
 & 25y^4 \\ \hline
10y^2 + 3xy & 30xy^3 - 11x^2y^2 \\
 & 30xy^3 + 9x^2y^2 \\ \hline
10y^2 + 6xy - 2x^2 & -20x^2y^2 - 12x^3y + 4x^4 \\
 & -20x^2y^2 - 12x^3y + 4x^4 \\ \hline
\end{array}
$$

14.

$$
\begin{array}{l|l}
 & 9a^6 - 36a^4x - 24a^3x^2 + 36a^2x^2 + 48ax^3 + 16x^4 \;\underline{|\,3a^3 - 6ax - 4x^2} \\
 & 9a^6 \\ \hline
6a^3 - 6ax & -36a^4x - 24a^3x^2 \\
 & -36a^4x \qquad + 36a^2x^2 \\ \hline
6a^3 - 12ax - 4x^2 & -24a^3x^2 \qquad + 48ax^3 + 16x^4 \\
 & -24a^3x^2 \qquad + 48ax^3 + 16x^4 \\ \hline
\end{array}
$$

15.

$$
\begin{array}{l|l}
 & 9c^4 - 12abc^3 - 2a^2b^2c^2 + 4a^3b^3c + a^4b^4 \;\underline{|\,3c^2 - 2abc - a^2b^2} \\
 & 9c^4 \\ \hline
6c^2 - 2abc & -12abc^3 - 2a^2b^2c^2 \\
 & -12abc^3 + 4a^2b^2c^2 \\ \hline
6c^2 - 4abc - a^2b^2 & -6a^2b^2c^2 + 4a^3b^3c + a^4b^4 \\
 & -6a^2b^2c^2 + 4a^3b^3c + a^4b^4 \\ \hline
\end{array}
$$

16.

$$
\begin{array}{l|l}
 & c^6 - 4xc^3 + 2a^2xc^3 + a^4x^2 - 4a^2x^2 + 4x^2 \;\underline{|\,c^3 - 2x + a^2x} \\
 & c^6 \\ \hline
2c^3 - 2x & -4xc^3 + 2a^2xc^3 \\
 & -4xc^3 \qquad + 4x^2 \\ \hline
2c^3 - 4x + a^2x & +2a^2xc^3 + a^4x^2 - 4a^2x^2 \\
 & +2a^2xc^3 + a^4x^2 - 4a^2x^2 \\ \hline
\end{array}
$$

17.

$$
\begin{array}{l|l}
 & 4 - \frac{4c^2}{5} + \frac{c^4}{25} \;\underline{|\,2 - \frac{c^2}{5}} \\
 & 4 \\ \hline
4 - \frac{c^2}{5} & -\frac{4c^2}{5} + \frac{c^4}{25} \\
 & -\frac{4c^2}{5} + \frac{c^4}{25} \\ \hline
\end{array}
$$

18.

$$
\begin{array}{l|l}
 & \frac{9}{a^2} - 3 + \frac{a^2}{4} \;\underline{|\,\frac{3}{a} - \frac{a}{2}} \\
 & \frac{9}{a^2} \\ \hline
\frac{6}{a} - \frac{a}{2} & -3 + \frac{a^2}{4} \\
 & -3 + \frac{a^2}{4} \\ \hline
\end{array}
$$

19.

$$
\begin{array}{l|l}
 & x^4 - 4x^3 + 5x^2 - 2x + \frac{1}{4} \;\underline{|\,x^2 - 2x + \frac{1}{2}} \\
 & x^4 \\ \hline
2x^2 - 2x & -4x^3 + 5x^2 \\
 & -4x^3 + 4x^2 \\ \hline
2x^2 - 4x + \frac{1}{2} & x^2 - 2x + \frac{1}{4} \\
 & x^2 - 2x + \frac{1}{4} \\ \hline
\end{array}
$$

20.
$$a^4+4a^3+4a^2-\frac{4a^2}{3}-\frac{8a}{3}+\frac{4}{9}\;\Big|\;a^2+2a-\frac{2}{3}$$
$$a^4$$
$$2a^2+2a\;\Big|\;4a^3+4a^2$$
$$4a^3+4a^2$$
$$2a^2+4a-\frac{2}{3}\;\Big|\;-\frac{4a^2}{3}-\frac{8a}{3}+\frac{4}{9}$$
$$-\frac{4a^2}{3}-\frac{8a}{3}+\frac{4}{9}$$

21.
$$x^4-x^3+\frac{x^2}{4}+\frac{2x^2}{3}-\frac{x}{3}+\frac{1}{9}\;\Big|\;x^2-\frac{x}{2}+\frac{1}{3}$$
$$x^4$$
$$2x^2-\frac{x}{2}\;\Big|\;-x^3+\frac{x^2}{4}$$
$$-x^3+\frac{x^2}{4}$$
$$2x^2-x+\frac{1}{3}\;\Big|\;\frac{2x^2}{3}-\frac{x}{3}+\frac{1}{9}$$
$$\frac{2x^2}{3}-\frac{x}{3}+\frac{1}{9}$$

22.
$$\frac{25m^4}{4}+\frac{10m^3}{3}-\frac{127m^2}{18}-2m+\frac{9}{4}\;\Big|\;\frac{5m^2}{2}+\frac{2m}{3}-\frac{3}{2}$$
$$\frac{25m^4}{4}$$
$$5m^2+\frac{2m}{3}\;\Big|\;\frac{10m^3}{3}$$
$$\frac{10m^3}{3}+\frac{4m^2}{9}$$
$$5m^2+\frac{4m}{3}-\frac{3}{2}\;\Big|\;-\frac{15m^2}{2}-2m+\frac{9}{4}$$
$$-\frac{15m^2}{2}-2m+\frac{9}{4}$$

Page 247

1.
$$44'89\,\underline{|67}$$
$$36$$
$$127\;\Big|\;8\,89$$
$$8\,89$$

2.
$$51'84\,\underline{|72}$$
$$49$$
$$142\;\Big|\;2\,84$$
$$2\,84$$

3.
$$53'29\,\underline{|73}$$
$$49$$
$$143\;\Big|\;4\,29$$
$$4\,29$$

4.
$$62'41\,\underline{|79}$$
$$49$$
$$149\;\Big|\;13\,41$$
$$13\,41$$

5.
92′16 | 96
81
186 | 11 16
11 16

6.
1′61′29 | 127
1
22 | 61
44
247 | 1729
1729

7.
2′46′49 | 157
1
25 | 1 46
1 25
307 | 21 49
21 49

8.
4′34′36 | 208.413
4
408 | 34 36
32 64
4164 | 1 7200
1 6656
41681 | 54400
41681
416823 | 1271900
1250469

9.
5′33′61 | 231
4
43 | 133
129
461 | 461
461

10.
16′56′80 | 407.038
16
807 | 56 80
56 49
81403 | 310000
244209
814068 | 6579100
6512544

11.
22′37′29 | 473
16
87 | 637
609
943 | 2829
2829

12.
32′83′29 | 573
25
107 | 7 83
7 49
1143 | 34 29
34 29

13.
63
63
189
378
3969
256
42′25 | 65
36
125 | 625
625

16
16
96
16
256

$$\text{Area} = \frac{63 \times 16}{2} = 504.$$

14.
48
48
384
192
2304
3025
53′29 | 73
49
143 | 429
429

55
55
275
275
3025

$$\text{Area} = \frac{55 \times 48}{2} = 1320.$$

15.

104
104
416
104
10816

153
153
459
765
153
23409
10816
3′42′25 | 185
1
28 | 2 42
2 24
365 | 18 25
18 25

$$\text{Area} = \frac{104 \times 153}{2} = 7956.$$

16.

645
645
3225
2580
3870
416025

812
812
1624
812
6496
659344
416025
1′07′53′69 | 1037
1
203 | 7 53
6 09
2067 | 1 44 69
1 44 69

$$\text{Area} = \frac{645 \times 812}{2}$$
$$= 261870.$$

17.

109
109
981
109
11881
8281
3600 | 60
36
00

91
91
91
819
8281

$$\text{Area} = \frac{60 \times 91}{2}$$
$$= 2730.$$

18.

2.57
2.57
1799
1285
514
6.6049
.1024
6.5025 | 2.55
4
45 | 250
225
505 | 2525
2525

.32
.32
64
96
.1024

$$\text{Area} = \frac{2.55 \times .32}{2}$$
$$= .408.$$

19.

2.05
2.05
1025
410
4.2025
3.4969
.7056 | .84
.64
164 | 656
656

1.87
1.87
1309
1496
187
3.4969

$$\text{Area} = \frac{1.87 \times .84}{2}$$
$$= .7854.$$

20.

6.4271 | 2.5351
4
45 | 242
225
503 | 1771
1509
5065 | 26200
25325
50701 | 87500
50701

21.

884.3 | 29.7371
4
49 | 484
441
587 | 4330
4109
5943 | 22100
17829
59467 | 427100
416269
594741 | 1083100
594741

22.

.0869 | .2947
4
49 | 469
441
584 | 2800
2336
5887 | 46400
41209

23.

.00′32′10′ | .0566
25
106 | 710
636
1126 | 7400
6756

24.

3.00 | 1.732
1
27 | 200
189
343 | 1100
1029
3462 | 7100
6924

25.

6. | 2.449
4
44 | 200
176
484 | 2400
1936
4889 | 46400
44001

26.

2.6 | 1.612
1
26 | 160
156
321 | 400
321
3222 | 7900
6444

27.

$2\frac{3}{7} = 2.42'85'71$ | 1.558
1
25 | 142
125
305 | 1785
1525
3108 | 26071
24864

28.

$\frac{3}{17} = .17'64'70$ | .420
16
82 | 164
164
840 | 7000

29.

$\sqrt{(195)^2 + (28)^2}$
$= \sqrt{38025 + 784}$
$= \sqrt{3'88'09}$ | 197
1
29 | 288
261
387 | 2709
2709

CR

30. $\sqrt{(409)^2 - (391)^2}$
$= \sqrt{167281 - 152881}$
$= \sqrt{1'44'00}$
$= 120$, side.
Area $= 391 \times 120 = 46920$.

31. $\sqrt{(533)^2 - (92)^2}$
$= \sqrt{284089 - 8464}$
$= \sqrt{2756'25}\,\underline{|525}$

$$\begin{array}{r|l} & 25 \\ \hline 102 & 256 \\ & 204 \\ \hline 1045 & 5225 \\ & 5225 \end{array}$$

Perimeter $= 2 \times 92 + 2 \times 525 = 1234$.

32. $2\,l + 2\,w = 46$.
$l - w = 7$.
$\therefore l = 15$ and $w = 8$.
$\sqrt{(15)^2 + 8^2} = \sqrt{225 + 64}$
$= \sqrt{289}$
$= 17$ yards.

33. $s^2 + s^2 = (74)^2 = 5476$.
$s = \sqrt{27'38}\,\underline{|52.32}$

$$\begin{array}{r|l} & 25 \\ \hline 102 & 2\,38 \\ & 2\,04 \\ \hline 1043 & 3400 \\ & 3129 \\ \hline 10462 & 27100 \\ & 20924 \end{array}$$

34. $\sqrt{(52)^2 + (52)^2}$
$= \sqrt{2704 + 2704}$
$= \sqrt{54'08}\,\underline{|73.53}$

$$\begin{array}{r|l} & 49 \\ \hline 143 & 508 \\ & 429 \\ \hline 1465 & 7900 \\ & 7325 \\ \hline 14703 & 57500 \\ & 44109 \end{array}$$

35. $(2\,x)^2 - x^2 = (10)^2$.
$3\,x^2 = 100$.
$x^2 = \frac{100}{3}$.
$x = \sqrt{33.3333}$.

$33.3333\,\underline{|5.77}$

$$\begin{array}{r|l} & 25 \\ \hline 107 & 833 \\ & 749 \\ \hline 1147 & 8433 \\ & 8029 \end{array}$$

Hypotenuse $= 5.77 \times 2 = 11.54$.

36. $(3\,x)^2 - x^2 = 16^2$.
$8\,x^2 = 256$.
$x = \sqrt{32}$.
$= 5.656$.
$3\,x = 16.96$.

37. $x^2 + (2.4\,x)^2 = (52)^2$.
$x^2 + 5.76\,x^2 = 2704$.
$6.76\,x^2 = 2704$.
$x^2 = 400$.
$x = 20$, width.

Length $= 20 \times 2.4 = 48$.

38. $x^2 + \left(\frac{3\,x}{4}\right)^2 = (100)^2$.
$\frac{25\,x^2}{16} = 10{,}000$.
$x^2 = 6400$.
$x = 80$, length.
$\frac{3\,x}{4} = 60$, width.

Area $= 80 \times 60 = 4800$.

39. $(2\,x)^2 - x^2 = (10)^2$.
$3\,x^2 = 100$.
$x = \sqrt{\frac{100}{3}}$
$= 5.77$.

40.
$$\overline{BD}^2 + (3)^2 = 6^2.$$
$$\overline{BD}^2 = 36 - 9$$
$$= 27.$$
$$BD = \sqrt{27}$$
$$= 5.196.$$
$$\text{Area} = \frac{6 \times 5.196}{2}$$
$$= 15.58.$$

41.
$$AC = BC$$
$$= 8.$$
$$\overline{BD}^2 + (4)^2 = (8)^2.$$
$$\overline{BD}^2 = 64 - 16$$
$$= 48.$$
$$BD = \sqrt{48}$$
$$= 6.928.$$
$$\text{Area} = \frac{6.928 \times 8}{2}$$
$$= 27.71.$$

42. Let $AB = x.$

Then
$$(14)^2 + \left(\frac{x}{2}\right)^2 = x^2.$$
$$784 + x^2 = 4x^2.$$
$$x = \sqrt{\tfrac{784}{3}}$$
$$= 16.165.$$
$$\text{Area} = \frac{16.165 \times 14}{2}$$
$$= 113.15.$$

43.
$$s = 45 \div 3$$
$$= 15.$$
$$(15)^2 - (\tfrac{15}{2})^2 = a^2.$$
$$\therefore a = \sqrt{\tfrac{675}{4}}$$
$$= 12.99.$$

44.
$$s^2 - \left(\frac{s}{2}\right)^2 = (25)^2.$$
$$3s^2 = 2500.$$
$$s = \sqrt{\tfrac{2500}{3}}$$
$$= 28.86.$$

Page 253

1. $a^{\frac{3}{2}}.$
2. $a^{\frac{1}{2}}x^2.$
3. $3 \cdot 2^{\frac{1}{2}}x^{\frac{5}{2}}.$
4. $3x^{\frac{1}{2}}.$
5. $20a^{\frac{1}{2}}x.$
6. $a^{\frac{2}{3}}.$
7. $a^{\frac{1}{3}}x^{\frac{4}{3}}.$
8. $2^{\frac{4}{3}}x^{\frac{2}{3}}.$
9. $6x^{\frac{4}{3}}.$
10. $12a^{\frac{1}{3}}x.$
11. $2a^{\frac{1}{2}}x^{\frac{3}{4}}.$
12. $8x^{\frac{1}{4}}.$
13. $12x^2 \cdot a^{\frac{1}{3}}x = 12x^3a^{\frac{1}{3}}.$
14. $cd^{\frac{3}{2}}e^{\frac{3}{2}}.$
15. $3a^{\frac{3}{2}}x^{\frac{2}{3}}.$
16. $x^{\frac{a}{n}} \cdot y^{\frac{b}{n}}.$
17. $x^{\frac{a}{n}} \cdot x^{\frac{3a}{n}} = x^{\frac{4a}{n}}.$
18. $(-32)^{\frac{4}{5}} \cdot (-64)^{\frac{1}{3}} = 16 \cdot (-4) = -64.$
19. $6 \cdot \frac{1}{3} = 2.$
20. $3 \cdot 3 = 9.$
21. $\frac{1}{2} \cdot 4 = 2.$

Page 255

1. $\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}.$
2. $\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}.$
3. $\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}.$
4. $\sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}.$
5. $\sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}.$
6. $\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}.$
7. $\sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3}.$
8. $\sqrt{63} = \sqrt{9 \cdot 7} = 3\sqrt{7}.$
9. $\sqrt{98} = \sqrt{49 \cdot 2} = 7\sqrt{2}.$
10. $2\sqrt{72} = 2\sqrt{36 \cdot 2} = 12\sqrt{2}.$
11. $3\sqrt{80} = 3\sqrt{16 \cdot 5} = 12\sqrt{5}.$
12. $5\sqrt{128} = 5\sqrt{64 \cdot 2} = 40\sqrt{2}.$
13. $\sqrt{147} = \sqrt{49 \cdot 3} = 7\sqrt{3}.$
14. $3\sqrt{162} = 3\sqrt{81 \cdot 2} = 27\sqrt{2}.$

15. $10\sqrt{175} = 10\sqrt{25 \cdot 7} = 50\sqrt{7}$.
16. $\sqrt{192} = \sqrt{64 \cdot 3} = 8\sqrt{3}$.
17. $\sqrt{243} = \sqrt{81 \cdot 3} = 9\sqrt{3}$.
18. $\sqrt{245} = \sqrt{49 \cdot 5} = 7\sqrt{5}$.
19. $\sqrt{343} = \sqrt{49 \cdot 7} = 7\sqrt{7}$.
20. $\sqrt{363} = \sqrt{121 \cdot 3} = 11\sqrt{3}$.
21. $2\sqrt{720} = 2\sqrt{144 \cdot 5} = 24\sqrt{5}$.
22. $\sqrt{1250} = \sqrt{625 \cdot 2} = 25\sqrt{2}$.
23. $\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = 2\sqrt[3]{2}$.
24. $\sqrt[3]{24} = \sqrt[3]{8 \cdot 3} = 2\sqrt[3]{3}$.
25. $2\sqrt[3]{40} = 2\sqrt[3]{8 \cdot 5} = 4\sqrt[3]{5}$.
26. $3\sqrt[3]{56} = 3\sqrt[3]{8 \cdot 7} = 6\sqrt[3]{7}$.
27. $4\sqrt[3]{72} = 4\sqrt[3]{8 \cdot 9} = 8\sqrt[3]{9}$.
28. $5\sqrt[3]{96} = 5\sqrt[3]{8 \cdot 12} = 10\sqrt[3]{12}$.
29. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = 3\sqrt[3]{2}$.
30. $\sqrt[3]{81} = \sqrt[3]{27 \cdot 3} = 3\sqrt[3]{3}$.
31. $\sqrt[3]{135} = \sqrt[3]{27 \cdot 5} = 3\sqrt[3]{5}$.
32. $3\sqrt[3]{189} = 3\sqrt[3]{27 \cdot 7} = 9\sqrt[3]{7}$.
33. $7\sqrt[3]{128} = 7\sqrt[3]{64 \cdot 2} = 28\sqrt[3]{2}$.
34. $\sqrt[3]{192} = \sqrt[3]{64 \cdot 3} = 4\sqrt[3]{3}$.
35. $\sqrt[3]{250} = \sqrt[3]{125 \cdot 2} = 5\sqrt[3]{2}$.
36. $8\sqrt[3]{375} = 8\sqrt[3]{125 \cdot 3} = 40\sqrt[3]{3}$.
37. $\sqrt[3]{448} = \sqrt[3]{64 \cdot 7} = 4\sqrt[3]{7}$.
38. $2\sqrt[3]{625} = 2\sqrt[3]{125 \cdot 5} = 10\sqrt[3]{5}$.
39. $\sqrt{a^3} = \sqrt{a^2 \cdot a} = a\sqrt{a}$.
40. $\sqrt{a^5} = \sqrt{a^4 \cdot a} = a^2\sqrt{a}$.
41. $\sqrt{4x^3} = \sqrt{4x^2 \cdot x} = 2x\sqrt{x}$.
42. $\sqrt{8x^2} = \sqrt{4x^2 \cdot 2} = 2x\sqrt{2}$.
43. $\sqrt{8x^3} = \sqrt{4x^2 \cdot 2x} = 2x\sqrt{2x}$.
44. $\sqrt{a^3x^2} = \sqrt{a^2x^2 \cdot a} = ax\sqrt{a}$.
45. $2\sqrt{x^3a} = 2\sqrt{x^2 \cdot xa} = 2x\sqrt{ax}$.
46. $3\sqrt{8a^3x} = 3\sqrt{4a^2 \cdot 2ax} = 6a\sqrt{2ax}$.
47. $5x\sqrt{4a^5x^3} = 5x\sqrt{4a^4x^2 \cdot ax} = 10a^2x^2\sqrt{ax}$.
48. $\sqrt[3]{a^4} = \sqrt[3]{a^3 \cdot a} = a\sqrt[3]{a}$.
49. $\sqrt[3]{a^5} = \sqrt[3]{a^3 \cdot a^2} = a\sqrt[3]{a^2}$.
50. $2a\sqrt[3]{x^7} = 2a\sqrt[3]{x^6 \cdot x} = 2ax^2\sqrt[3]{x}$.
51. $\sqrt[3]{8x^4} = \sqrt[3]{8x^3 \cdot x} = 2x\sqrt[3]{x}$.
52. $\sqrt[3]{27x^5} = \sqrt[3]{27x^3 \cdot x^2} = 3x\sqrt[3]{x^2}$.
53. $3x\sqrt[3]{4a^3x} = 3x\sqrt[3]{a^3 \cdot 4x} = 3ax\sqrt[3]{4x}$.
54. $5x\sqrt[3]{4a^2x^3} = 5x\sqrt[3]{x^3 \cdot 4a^2} = 5x^2\sqrt[3]{4a^2}$.
55. $\sqrt[3]{27a^3x} = 3a\sqrt[3]{x}$.
56. $\sqrt[3]{56a^4x} = \sqrt[3]{8a^3 \cdot 7ax} = 2a\sqrt[3]{7ax}$.
58. $\sqrt{4 - 4\sqrt{3}} = \sqrt{4(1 - \sqrt{3})} = 2\sqrt{1 - \sqrt{3}}$.
59. $\sqrt{18 + 9\sqrt{3}} = \sqrt{9(2 + \sqrt{3})} = 3\sqrt{2 + \sqrt{3}}$.
60. $\sqrt{100 - 25\sqrt{5}} = \sqrt{25(4 - \sqrt{5})} = 5\sqrt{4 - \sqrt{5}}$.
61. $\sqrt{9\sqrt{2} - 27} = \sqrt{9(\sqrt{2} - 3)} = 3\sqrt{\sqrt{2} - 3}$.
62. $\sqrt{R^2 - R^2\sqrt{3}} = \sqrt{R^2(1 - \sqrt{3})} = R\sqrt{1 - \sqrt{3}}$.
63. $\sqrt{R^2 + 4R^2\sqrt{5}} = \sqrt{R^2(1 + 4\sqrt{5})} = R\sqrt{1 + 4\sqrt{5}}$.
64. $\sqrt{5^2\sqrt{3} - 25^2} = \sqrt{25(\sqrt{3} - 25)} = 5\sqrt{\sqrt{3} - 25}$.
65. $\sqrt{a^2x^2 + a^3\sqrt{x}} = \sqrt{a^2(x^2 + a\sqrt{x})} = a\sqrt{x^2 + a\sqrt{x}}$.

Page 256

1. $\sqrt{\frac{3}{4}} = \sqrt{\frac{1}{4} \cdot 3} = \frac{1}{2}\sqrt{3}.$
2. $\sqrt{\frac{5}{9}} = \sqrt{\frac{1}{9} \cdot 5} = \frac{1}{3}\sqrt{5}.$
3. $\sqrt{\frac{3}{16}} = \sqrt{\frac{1}{16} \cdot 3} = \frac{1}{4}\sqrt{3}.$
4. $\sqrt{\frac{1}{2}} = \sqrt{\frac{2}{4}} = \sqrt{\frac{1}{4} \cdot 2} = \frac{1}{2}\sqrt{2}.$
5. $\sqrt{\frac{2}{3}} = \sqrt{\frac{6}{9}} = \sqrt{\frac{1}{9} \cdot 6} = \frac{1}{3}\sqrt{6}.$
6. $\sqrt{\frac{3}{2}} = \sqrt{\frac{6}{4}} = \sqrt{\frac{1}{4} \cdot 6} = \frac{1}{2}\sqrt{6}.$
7. $\sqrt{\frac{3}{5}} = \sqrt{\frac{15}{25}} = \sqrt{\frac{1}{25} \cdot 15} = \frac{1}{5}\sqrt{15}.$
8. $\sqrt{\frac{5}{3}} = \sqrt{\frac{15}{9}} = \sqrt{\frac{1}{9} \cdot 15} = \frac{1}{3}\sqrt{15}.$
9. $\sqrt{\frac{4}{7}} = \sqrt{\frac{28}{49}} = \sqrt{\frac{4}{49} \cdot 7} = \frac{2}{7}\sqrt{7}.$
10. $\sqrt{\frac{7}{5}} = \sqrt{\frac{35}{25}} = \sqrt{\frac{1}{25} \cdot 35} = \frac{1}{5}\sqrt{35}.$
11. $\sqrt{\frac{5}{8}} = \sqrt{\frac{10}{16}} = \sqrt{\frac{1}{16} \cdot 10} = \frac{1}{4}\sqrt{10}.$
12. $\sqrt{\frac{5}{2}} = \sqrt{\frac{10}{4}} = \sqrt{\frac{1}{4} \cdot 10} = \frac{1}{2}\sqrt{10}.$
13. $3\sqrt{\frac{3}{7}} = 3\sqrt{\frac{21}{49}} = 3\sqrt{\frac{1}{49} \cdot 21} = \frac{3}{7}\sqrt{21}.$
14. $2\sqrt{\frac{2}{5}} = 2\sqrt{\frac{10}{25}} = 2\sqrt{\frac{1}{25} \cdot 10} = \frac{2}{5}\sqrt{10}.$
15. $\sqrt{\frac{8}{5}} = \sqrt{\frac{40}{25}} = \sqrt{\frac{4}{25} \cdot 10} = \frac{2}{5}\sqrt{10}.$
16. $\sqrt{\frac{5}{12}} = \sqrt{\frac{15}{36}} = \sqrt{\frac{1}{36} \cdot 15} = \frac{1}{6}\sqrt{15}.$
17. $3\sqrt{\frac{7}{12}} = 3\sqrt{\frac{21}{36}} = 3\sqrt{\frac{1}{36} \cdot 21} = \frac{1}{2}\sqrt{21}.$
18. $2\sqrt{\frac{5}{18}} = 2\sqrt{\frac{10}{36}} = 2\sqrt{\frac{1}{36} \cdot 10} = \frac{1}{3}\sqrt{10}.$
19. $3a\sqrt{\frac{3}{11}} = 3a\sqrt{\frac{33}{121}} = 3a\sqrt{\frac{1}{121} \cdot 33} = \frac{3a}{11}\sqrt{33}.$
20. $3a^2\sqrt{\frac{3}{32}} = 3a^2\sqrt{\frac{6}{64}} = 3a^2\sqrt{\frac{1}{64} \cdot 6} = \frac{3a^2}{8}\sqrt{6}.$
21. $\sqrt{\frac{3}{4}} = \sqrt{\frac{1}{4} \cdot 3} = \frac{1}{2}\sqrt{3}.$
22. $\sqrt{1 - (\frac{1}{3})^2} = \sqrt{\frac{8}{9}} = \sqrt{\frac{4}{9} \cdot 2} = \frac{2}{3}\sqrt{2}.$
23. $\sqrt{9 - (\frac{3}{2})^2} = \sqrt{\frac{27}{4}} = \sqrt{\frac{9}{4} \cdot 3} = \frac{3}{2}\sqrt{3}.$
24. $\sqrt{25 - (\frac{5}{2})^2} = \sqrt{\frac{75}{4}} = \sqrt{\frac{25}{4} \cdot 3} = \frac{5}{2}\sqrt{3}.$
25. $\sqrt{2 + (\frac{3}{8})^2} = \sqrt{\frac{137}{64}} = \frac{1}{8}\sqrt{137}.$
26. $\sqrt{81 - (\frac{9}{2})^2} = \sqrt{\frac{243}{4}} = \sqrt{\frac{81}{4} \cdot 3} = \frac{9}{2}\sqrt{3}.$
27. $\sqrt{121 - (\frac{11}{2})^2} = \sqrt{\frac{363}{4}} = \sqrt{\frac{121}{4} \cdot 3} = \frac{11}{2}\sqrt{3}.$
28. $\sqrt{169 - (\frac{13}{2})^2} = \sqrt{\frac{507}{9}} = \sqrt{\frac{169}{4} \cdot 3} = \frac{13}{2}\sqrt{3}.$
29. $\sqrt{\frac{R}{3}} = \sqrt{\frac{3R}{9}} = \frac{1}{3}\sqrt{3R}.$
30. $\sqrt{\frac{R^2}{3}} = \sqrt{\frac{3R^2}{9}} = \frac{R}{3}\sqrt{3}.$
31. $\sqrt{\frac{3x^2}{4}} = \frac{x}{2}\sqrt{3}.$
32. $\sqrt{R^2 + R^2} = \sqrt{2R^2} = R\sqrt{2}.$

33. $\sqrt{R^2-\left(\frac{R}{2}\right)^2}=\sqrt{\frac{3R^2}{4}}=\frac{R}{2}\sqrt{3}.$

34. $\sqrt{R^2-\left(\frac{R}{4}\right)^2}=\sqrt{\frac{15R^2}{16}}=\frac{R}{4}\sqrt{15}.$

35. $\sqrt{\frac{3R^2}{4}-\frac{R^2}{4}\sqrt{2}}=\sqrt{\frac{R^2}{4}(3-\sqrt{2})}=\frac{R}{2}\sqrt{3-\sqrt{2}}.$

36. $\sqrt{\frac{R^2}{4}+\frac{R^2x}{2}}=\sqrt{\frac{R^2}{4}(1+2x)}=\frac{R}{2}\sqrt{1+2x}.$

37. $\sqrt{\frac{R^2}{4}+\frac{R^2}{2}\sqrt{2}}=\sqrt{\frac{R^2}{4}(1+2\sqrt{2})}=\frac{R}{2}\sqrt{1+2\sqrt{2}}.$

38. $\sqrt{R^2\sqrt{2}-\frac{R^2}{2}}=\sqrt{\frac{4R^2\sqrt{2}-2R^2}{4}}=\sqrt{\frac{R^2}{4}(4\sqrt{2}-2)}=\frac{R}{2}\sqrt{4\sqrt{2}-2}.$

39. $\sqrt[3]{R^3-\left(\frac{R}{2}\right)^3}=\sqrt[3]{\frac{7R^3}{8}}=\frac{R}{2}\sqrt[3]{7}.$

40. $\sqrt[3]{16+8\sqrt{2}}=\sqrt[3]{8(2+\sqrt{2})}=2\sqrt[3]{2+\sqrt{2}}.$

41. $\sqrt[3]{81+3\sqrt{243}}=\sqrt[3]{81+27\sqrt{3}}=\sqrt[3]{27(3+\sqrt{3})}=3\sqrt[3]{3+\sqrt{3}}.$

Page 257

2. $\sqrt{8^2-4^2}=\sqrt{48}=4\sqrt{3}.$

   $\text{Area}=\frac{4\cdot 4\sqrt{3}}{2}=8\sqrt{3}.$

3. $\sqrt{R^2-\left(\frac{R}{2}\right)^2}=\sqrt{\frac{3R^2}{4}}=\frac{R}{2}\sqrt{3}.$

   $\text{Area}=\frac{\left(\frac{R}{2}\sqrt{3}\cdot\frac{R}{2}\right)}{2}=\frac{R^2}{8}\sqrt{3}.$

4. $\sqrt{12^2+12^2}=\sqrt{288}=12\sqrt{2}.$

5. $s^2+s^2=(12)^2.$
   $\therefore s=6\sqrt{2}.$

6. $s^2+s^2=4R^2.$
   $\therefore s=R\sqrt{2}.$

7. $x=\sqrt{36+36}=6\sqrt{2}.$

8. $h^2=8^2+8^2=128.$
   $\therefore h=8\sqrt{2}.$
   $h^2=(13)^2+(13)^2=338.$
   $\therefore h=13\sqrt{2}.$

9. $x^2=50.$
   $\therefore x=5\sqrt{2}.$

10. $x^2+x^2=169.$
   $\therefore x=\frac{13}{2}\sqrt{2}.$

11. $s^2=(20)^2-(10)^2=300.$
   $\therefore s=10\sqrt{3}.$

12. $s^2=(12)^2-(6)^2=108.$
   $s=6\sqrt{3}.$

13. $s^2=(20)^2-(10)^2=300.$
   $s=10\sqrt{3},\ h=20.$

14. $s^2=(24)^2-(12)^2=432.$
   $\therefore s=12\sqrt{3},\ h=24.$

15. $a^2=(12)^2-(6)^2=108.$
   $a=6\sqrt{3}.$
   $\text{Area}=\frac{12\cdot 6\sqrt{3}}{2}=36\sqrt{3}.$

16. $a^2 = S^2 - \left(\frac{S}{2}\right)^2 = \frac{3S^2}{4}.$

$a = \frac{S}{2}\sqrt{3}.$

$\text{Area} = \dfrac{S \cdot \frac{S}{2}\sqrt{3}}{2} = \frac{S^2}{4}\sqrt{3}.$

17. $s^2 - \left(\frac{s}{2}\right)^2 = 100.$

$\frac{3s^2}{4} = 100.$

$\therefore s = \frac{20}{3}\sqrt{3}.$

$\text{Area} = \dfrac{\frac{20}{3}\sqrt{3}\cdot 10}{2} = \frac{100}{3}\sqrt{3}.$

18. $s^2 + s^2 = (20)^2 = 400.$

$\therefore s = 10\sqrt{2}.$

19. $\text{Area} = \frac{s \cdot s}{2} = 50.$

$\therefore s = 10.$

$h^2 = (10)^2 + (10)^2.$

$h = 10\sqrt{2}.$

20. $h^2 = \left(\frac{R}{2}\right)^2 + \left(\frac{3R}{2}\right)^2 = \frac{10R^2}{4}.$

$\therefore h = \frac{R}{2}\sqrt{10}.$

21. $h^2 = R^2 - \left(\frac{R}{5}\right)^2 = \frac{24R^2}{25}.$

$\therefore h = \frac{2R}{5}\sqrt{6}.$

22. $x^2 + x^2 = R^2.$

$\therefore x = \frac{R}{2}\sqrt{2}.$

23. $x = \frac{R}{2}.$

$\therefore y^2 = R^2 - \left(\frac{R}{2}\right)^2 = \frac{3R^2}{4}.$

$y = \frac{R}{2}\sqrt{3}.$

24. $h^2 = R^2 + \left[\frac{R}{2}(a-1)\right]^2.$

$= \frac{R^2}{4}(a^2 - 2a + 5).$

$\therefore h = \frac{R}{2}\sqrt{a^2 - 2a + 5}.$

Page 260

1. $\sqrt{27} + \sqrt{12} = 3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3}.$
2. $\sqrt{45} - \sqrt{20} = 3\sqrt{5} - 2\sqrt{5} = \sqrt{5}.$
3. $2\sqrt{200} - 3\sqrt{8} = 20\sqrt{2} - 6\sqrt{2} = 14\sqrt{2}.$
4. $\sqrt{28} + \sqrt{63} = 2\sqrt{7} + 3\sqrt{7} = 5\sqrt{7}.$
5. $7\sqrt{18} - \sqrt{98} = 21\sqrt{2} - 7\sqrt{2} = 14\sqrt{2}.$
6. $\sqrt{75} - \sqrt{27} + 2\sqrt{48} = 5\sqrt{3} - 3\sqrt{3} + 8\sqrt{3} = 10\sqrt{3}.$
7. $\sqrt{2} + \sqrt{\frac{1}{2}} = \sqrt{2} + \frac{1}{2}\sqrt{2} = \frac{3}{2}\sqrt{2}.$
8. $\sqrt{3} - \sqrt{\frac{1}{3}} = \sqrt{3} - \frac{1}{3}\sqrt{3} = \frac{2}{3}\sqrt{3}.$
9. $\frac{3}{2} + \sqrt{\frac{3}{4}} = \frac{3}{2} + \frac{1}{2}\sqrt{3}.$
10. $\frac{2}{3} + \sqrt{\frac{4}{3}} = \frac{2}{3} + \frac{2}{3}\sqrt{3}.$
11. $\frac{R}{2} + \sqrt{\frac{9R^2}{2}} = \frac{R}{2} + \frac{3R}{2}\sqrt{2}.$
12. $\frac{3R}{2} - \sqrt{\frac{9R^2}{2}} = \frac{3R}{2} - \frac{3R}{2}\sqrt{2}.$
13. $R - \sqrt{\frac{3R^2}{4}} = R - \frac{R}{2}\sqrt{3}.$
14. $5\sqrt{\frac{1}{2}} - \frac{3}{2}\sqrt{2} = \frac{5}{2}\sqrt{2} - \frac{3}{2}\sqrt{2} = \sqrt{2}.$
15. $8\sqrt{\frac{9}{2}} - \sqrt{72} = 12\sqrt{2} - 6\sqrt{2} = 6\sqrt{2}.$

16. $\sqrt{\frac{1}{3}}+2\sqrt{\frac{4}{3}}-3\sqrt{\frac{2}{3}}=\frac{1}{3}\sqrt{3}+\frac{4}{3}\sqrt{3}-\sqrt{6}=\frac{5}{3}\sqrt{3}-\sqrt{6}.$

17. $\sqrt{\frac{2}{5}}-\sqrt{\frac{9}{10}}+2\sqrt{10}=\frac{1}{5}\sqrt{10}-\frac{3}{10}\sqrt{10}+2\sqrt{10}=\frac{19}{10}\sqrt{10}.$

18. $\sqrt{\frac{3}{10}}-\sqrt{120}-2\sqrt{\frac{6}{5}}=\frac{1}{10}\sqrt{30}-2\sqrt{30}-\frac{2}{5}\sqrt{30}=-\frac{23}{10}\sqrt{30}.$

19. $\sqrt{\frac{5}{18}}+2\sqrt{\frac{32}{5}}-\sqrt{\frac{10}{9}}=\frac{1}{6}\sqrt{10}+\frac{8}{5}\sqrt{10}-\frac{1}{3}\sqrt{10}=\frac{43}{30}\sqrt{10}.$

20. $\sqrt[3]{56}+2\sqrt[3]{189}=2\sqrt[3]{7}+6\sqrt[3]{7}=8\sqrt[3]{7}.$

21. $2\sqrt[3]{320}-\sqrt{50}=8\sqrt[3]{5}-5\sqrt{2}.$

22. $\sqrt[4]{25}-\sqrt{20}=\sqrt{5}-2\sqrt{5}=-\sqrt{5}.$

23. $\sqrt[4]{9}+\sqrt{12}=\sqrt[4]{3^2}+2\sqrt{3}=\sqrt{3}+2\sqrt{3}=3\sqrt{3}.$

24. $\sqrt[4]{32}+5\sqrt[4]{162}=2\sqrt[4]{2}+15\sqrt[4]{2}=17\sqrt[4]{2}.$

25. $2x\sqrt[3]{54x}-3\sqrt[3]{16x^4}+\sqrt[6]{4x^2}=6x\sqrt[3]{2x}-6x\sqrt[3]{2x}+\sqrt[3]{2x}=\sqrt[3]{2x}.$

26. $\sqrt[3]{81x^7}+x\sqrt[3]{375x^4}-\sqrt[12]{16x^4}=3x^2\sqrt[3]{3x}+5x^2\sqrt[3]{3x}-\sqrt[3]{2x}$
$=8x^2\sqrt[3]{3x}-\sqrt[3]{2x}.$

27. $\sqrt{a^3bc}-a\sqrt{abc}+ac\sqrt{\frac{b}{ac}}=a\sqrt{abc}-a\sqrt{abc}+\sqrt{abc}=\sqrt{abc}.$

28. $rs\sqrt[3]{rs}+\sqrt[3]{\frac{1}{r^2s^2}}-2\sqrt[3]{r^4s^4}=rs\sqrt[3]{rs}+\frac{1}{rs}\sqrt[3]{rs}-2rs\sqrt[3]{rs}$
$=\frac{\sqrt[3]{rs}}{rs}-rs\sqrt[3]{rs}.$

29. $\sqrt{a^3+4a^2+4a}-\sqrt{a^3}-\frac{2}{a^3}\sqrt{a^7}=(a+2)\sqrt{a}-a\sqrt{a}-2\sqrt{a}=0.$

30. $\sqrt{3x^2-18x+27}-\sqrt{27(x^2+2x+1)}=(x-3)\sqrt{3}-(3x+3)\sqrt{3}$
$=-2x\sqrt{3}-6\sqrt{3}.$

Page 262

1. $\sqrt{5}\cdot\sqrt{6}=\sqrt{30}.$
2. $\sqrt{7}\cdot\sqrt{7}=7.$
3. $2\sqrt{3}\cdot 3\sqrt{2}=6\sqrt{6}.$
4. $3\sqrt{2}\cdot 5\sqrt{2}=30.$
5. $\sqrt{2}\cdot\sqrt{8}=4.$
6. $\sqrt{3}\cdot\sqrt{27}=9.$
7. $5^{\frac{1}{2}}\cdot 20^{\frac{1}{2}}=(100)^{\frac{1}{2}}=10.$
8. $18^{\frac{1}{2}}\cdot 8^{\frac{1}{2}}=(144)^{\frac{1}{2}}=12.$
9. $\sqrt{\frac{12}{25}}\cdot\sqrt{75}=\sqrt{36}=6.$
10. $\sqrt{\frac{2}{3}}\cdot\sqrt{\frac{27}{8}}=\frac{3}{2}.$
11. $\sqrt{11}\cdot\sqrt{\frac{1}{11}}=1.$
12. $a^{\frac{1}{2}}\cdot(bc)^{\frac{1}{2}}=\sqrt{abc}.$
13. $R\sqrt{2}\cdot R\sqrt{3}=R^2\sqrt{6}.$
14. $\left(\sqrt{\frac{R}{2}-\sqrt{2}}\right)^2=\frac{R}{2}-\sqrt{2}.$
15. $\frac{R}{2}\sqrt{2}\cdot R\sqrt{3}=\frac{R^2}{2}\sqrt{6}.$
16. $2\sqrt{x}\cdot\sqrt{4x^3}=4x^2.$
17. $2\sqrt{rs}\cdot 7\sqrt{r^3s^3t^2}=14r^2s^2t.$
18. $\sqrt{75a}\cdot(45a)^{\frac{1}{2}}=15a\sqrt{15}.$
19. $\sqrt{2u}\cdot\sqrt{4v}\cdot\sqrt{6uv}=4uv\sqrt{3}.$
20. $5\sqrt{3m}\cdot 5\sqrt{3m}=75m.$
21. $(3\sqrt{3x})^2=27x.$
22. $\sqrt[3]{16}\cdot\sqrt[3]{4}=4.$
23. $\sqrt[3]{4}\cdot\sqrt[3]{12}=2\sqrt[3]{6}.$
24. $(100)^{\frac{1}{3}}\cdot(30)^{\frac{1}{3}}=(3000)^{\frac{1}{3}}=10\sqrt[3]{3}.$

25. $\sqrt[4]{8}\cdot\sqrt[4]{32}=\sqrt[4]{256}=4.$

26. $5\sqrt[3]{2a}\cdot 3\sqrt[3]{16a}=30\sqrt[3]{4a^2}.$

27. $\left(\sqrt{R-\sqrt{2}}\right)^2=R-\sqrt{2}.$

28. $\left(\frac{R}{2}\sqrt{2-\sqrt{5}}\right)^2=\frac{R^2}{2}-\frac{R^2}{4}\sqrt{5}.$

29. $\sqrt{\frac{a}{x}}\cdot\sqrt{\frac{4x}{a}}=2.$

30. $\sqrt{\frac{5x}{a}}\cdot\sqrt{\frac{a^3}{5x^2}}=\frac{a}{x}\sqrt{x}.$

32. $7\sqrt{3}=\sqrt{49}\sqrt{3}=\sqrt{147}.$

33. $10\sqrt{5}=\sqrt{100}\sqrt{5}=\sqrt{500}.$

34. $8\sqrt{\tfrac{1}{2}}=\sqrt{64}\sqrt{\tfrac{1}{2}}=\sqrt{32}.$

35. $12\sqrt{\tfrac{5}{3}}=\sqrt{144}\cdot\sqrt{\tfrac{5}{3}}=\sqrt{240}.$

36. $a\sqrt{x}=\sqrt{a^2}\sqrt{x}=\sqrt{a^2x}.$

37. $2a\sqrt{c}=\sqrt{4a^2}\sqrt{c}=\sqrt{4a^2c}.$

38. $\sqrt[3]{27}\cdot\sqrt[3]{2}=\sqrt[3]{54}.$

39. $5\sqrt[3]{2}=\sqrt[3]{125}\sqrt[3]{2}=\sqrt[3]{250}.$

40. $2\sqrt[3]{3}=\sqrt[3]{8}\sqrt[3]{3}=\sqrt[3]{24}.$

41. $2\sqrt[3]{4}=\sqrt[3]{8}\cdot\sqrt[3]{4}=\sqrt[3]{32}.$

42. $3\sqrt[3]{\tfrac{1}{3}}=\sqrt[3]{27}\sqrt[3]{\tfrac{1}{3}}=\sqrt[3]{9}.$

43. $a\sqrt{\frac{1}{a}}=\sqrt[3]{a^3}\sqrt[3]{\frac{1}{a}}=\sqrt[3]{a^2}.$

44. $(a+x)\sqrt[3]{\frac{1}{a+x}}=\sqrt[3]{(a+x)^3}\sqrt[3]{\frac{1}{a+x}}=\sqrt[3]{(a+x)^2}.$

Page 263

1.
$$\begin{array}{r}
\sqrt{2}-3\\
3\sqrt{2}+5\\
\hline
+6-9\sqrt{2}\qquad\\
+5\sqrt{2}-15\\
\hline
-4\sqrt{2}-\ \ 9
\end{array}$$

2.
$$\begin{array}{r}
2\sqrt{5}-4\\
3\sqrt{5}+3\\
\hline
30-12\sqrt{5}\qquad\\
+\ 6\sqrt{5}-12\\
\hline
18-\ 6\sqrt{5}\qquad
\end{array}$$

3.
$$\begin{array}{r}
4\sqrt{7}-3\\
4\sqrt{7}-3\\
\hline
112-12\sqrt{7}\qquad\\
-12\sqrt{7}+9\\
\hline
121-24\sqrt{7}\qquad
\end{array}$$

4.
$$\begin{array}{r}
2\sqrt{7}-3\sqrt{2}\\
2\sqrt{7}-3\sqrt{2}\\
\hline
28-\ 6\sqrt{14}\qquad\\
-\ 6\sqrt{14}+18\\
\hline
46-12\sqrt{14}\qquad
\end{array}$$

5.
$$\begin{array}{r}
3\sqrt{2}+\sqrt{3}\\
-\sqrt{2}+2\sqrt{3}\\
\hline
-6-\ \sqrt{6}\qquad\\
+6\sqrt{6}+6\\
\hline
5\sqrt{6}\qquad
\end{array}$$

6.
$$\begin{array}{r}
2\sqrt{5}-3\sqrt{2}\\
3\sqrt{5}+2\sqrt{2}\\
\hline
30-9\sqrt{10}\qquad\\
+4\sqrt{10}-12\\
\hline
18-5\sqrt{10}\qquad
\end{array}$$

7.
$$\begin{array}{lllll}
\sqrt{5}-\sqrt{3}+\sqrt{2} & & & & \\
\sqrt{5}-\sqrt{3}+\sqrt{2} & & & & \\
\hline
 & 5- & \sqrt{15}+ & \sqrt{10} & \\
+ & 3- & \sqrt{15} & & -\sqrt{6}\\
+ & 2 & & +\ \sqrt{10} & -\sqrt{6}\\
\hline
 & 10- & 2\sqrt{15}+ & 2\sqrt{10} & -2\sqrt{6}
\end{array}$$

8.
$$\begin{array}{llll}
3\sqrt{2}+2\sqrt{3}+\sqrt{30} & & & \\
\sqrt{2}+\ \sqrt{3}-\sqrt{5} & & & \\
\hline
6+2\sqrt{6}+2\sqrt{15} & & & \\
6+3\sqrt{6} & & & +3\sqrt{10}\\
\quad-5\sqrt{6} & -2\sqrt{15} & & -3\sqrt{10}\\
\hline
12 & & &
\end{array}$$

9.
$$\begin{array}{r}
R\sqrt{2}-2\qquad\qquad\\
R\sqrt{2}-3\qquad\qquad\\
\hline
2R^2-2R\sqrt{2}\qquad\\
-3R\sqrt{2}+6\\
\hline
2R^2-5R\sqrt{2}+6
\end{array}$$

10.
$$\begin{array}{r}
R\sqrt{3}-R\sqrt{2}\\
R\sqrt{3}+2R\sqrt{2}\\
\hline
3R^2-\ R^2\sqrt{6}\\
-4R^2+2R^2\sqrt{6}\\
\hline
-\ R^2+\ R^2\sqrt{6}
\end{array}$$

11.

$$\begin{array}{r}
R - \frac{R}{2}\sqrt{2} \\
R - \frac{R}{2}\sqrt{2} \\
\hline
R^2 - \frac{R^2}{2}\sqrt{2} \qquad\qquad \\
- \frac{R^2}{2}\sqrt{2} + \frac{R^2}{2} \\
\hline
\frac{3R^2}{2} - R^2\sqrt{2} \qquad\qquad
\end{array}$$

12.

$$\begin{array}{r}
\frac{R}{2} + \frac{R}{2}\sqrt{3} \\
\frac{R}{2} + \frac{R}{2}\sqrt{3} \\
\hline
\frac{R^2}{4} + \frac{R^2}{4}\sqrt{3} \qquad\qquad \\
+ \frac{R^2}{4}\sqrt{3} + \frac{3R^2}{4} \\
\hline
R^2 + \frac{R^2}{2}\sqrt{3} \qquad\qquad
\end{array}$$

13. $\frac{R}{2}(\sqrt{5} - 1)^2 = \frac{R}{2}(6 - 2\sqrt{5}) = 3R - R\sqrt{5}.$

14. $\left(\frac{R}{2}\sqrt{5} - \frac{R}{2}\right)^2 = \frac{3R^2}{2} - \frac{R^2}{2}\sqrt{5}.$

$R^2 - \left(\frac{3R^2}{2} - \frac{R^2}{2}\sqrt{5}\right) = -\frac{R^2}{2} + \frac{R^2}{2}\sqrt{5}.$

15. $R^2 - \left(\frac{R}{2} - \frac{R}{2}\sqrt{2}\right)^2 = R^2 - \left(\frac{R^2}{4} - \frac{R^2}{2}\sqrt{2} + \frac{R^2}{2}\right) = \frac{R^2}{4} + \frac{R^2}{2}\sqrt{2}$

16. $\sqrt{R^2 - \left(\frac{R}{2} - \frac{R}{2}\sqrt{3}\right)^2} = \sqrt{R^2 - \left(\frac{R^2}{4} - \frac{R^2}{2}\sqrt{3} + \frac{3R^2}{4}\right)}$

$= \sqrt{\frac{R^2}{2}\sqrt{3}} = \frac{R}{2}\sqrt{2\sqrt{3}}.$

17. $\sqrt{R^2 - \left(\frac{R\sqrt{5} - R}{4}\right)^2} = \sqrt{R^2 - \frac{5R^2 - 2R^2\sqrt{5} + R^2}{16}}$

$= \sqrt{\frac{10R^2 + 2R^2\sqrt{5}}{16}} = \frac{R}{4}\sqrt{10 + 2\sqrt{5}}.$

18. $l = \sqrt{R^2 - \left(R - \frac{R}{2}\sqrt{3}\right)^2} = \sqrt{\cancel{R^2} - \cancel{R^2} + R^2\sqrt{3} - \frac{3R^2}{4}}$

$= \frac{R}{2}\sqrt{4\sqrt{3} - 3}.$

19. $l = \sqrt{R^2 - \left[\frac{R}{2}(\sqrt{5} - 1)\right]^2} = \sqrt{R^2 - \frac{5R^2}{4} + \frac{2R^2\sqrt{5}}{4} - \frac{R^2}{4}}$

$= \frac{R}{2}\sqrt{2\sqrt{5} - 2}.$

20. $h=\sqrt{\left(R\sqrt{2+\sqrt{2}}\right)^2+\left(\frac{R}{2}\sqrt{2-\sqrt{2}}\right)^2}$

$=\sqrt{2R^2+R^2\sqrt{2}+\frac{2R^2}{4}-\frac{R^2}{4}\sqrt{2}}$

$=\frac{R}{2}\sqrt{10+3\sqrt{2}}.$

$\text{Area}=\frac{1}{2}\left(R\sqrt{2+\sqrt{2}}\right)\left(\frac{R}{2}\sqrt{2-\sqrt{2}}\right)=\frac{R^2}{4}\sqrt{4-2}=\frac{R^2}{4}\sqrt{2}.$

21. $(2+2\sqrt{3})^2-4(2+2\sqrt{3})=4+8\sqrt{3}+12-8-8\sqrt{3}=8.$

$2+2\sqrt{3}$ is a root.

22. $(\frac{7}{2}-\frac{1}{2}\sqrt{41})^2-7(\frac{7}{2}-\frac{1}{2}\sqrt{41})+2$

$=\frac{49}{4}-\frac{7}{2}\sqrt{41}+\frac{41}{4}-\frac{49}{2}+\frac{7}{2}\sqrt{41}+2$

$=\frac{90}{4}-\frac{98}{4}+2=0.$ Yes.

Page 264

1. $\sqrt{2x-7}=3.$

$2x-7=9.$

$2x=16.$

$x=8.$

Check. $\sqrt{16-7}=3.$

$3=3.$

2. $\sqrt{3x-5}=5.$

$3x-5=25.$

$3x=30.$

$x=10.$

Check. $\sqrt{30-5}=5.$

$5=5.$

3. $\sqrt{x-6}=\sqrt{5}.$

$x-6=5.$

$x=11.$

Check. $\sqrt{11-6}=\sqrt{5}.$

$\sqrt{5}=\sqrt{5}.$

4. $\sqrt{2x+3}=\sqrt{7}.$

$2x+3=7.$

$2x=4.$

$x=2.$

Check. $\sqrt{4+3}=\sqrt{7}.$

$\sqrt{7}=\sqrt{7}.$

5. $2\sqrt{x-1}=\sqrt{6}.$

$4x-4=6.$

$4x=10.$

$x=\frac{5}{2}.$

Check. $2\sqrt{\frac{5}{2}-1}=\sqrt{6}.$

$2\sqrt{\frac{3}{2}}=\sqrt{6}.$

$\sqrt{4\cdot\frac{3}{2}}=\sqrt{6}.$

$\sqrt{6}=\sqrt{6}.$

6. $\sqrt{x-3}-6=0.$

$\sqrt{x-3}=6.$

$x-3=36.$

$x=39.$

Check. $\sqrt{39-3}-6=0.$

$\sqrt{36}-6=0.$

$6-6=0.$

7. $2\sqrt{3x+1}-8=0.$

$12x+4=64.$

$12x=60.$

$x=5.$

Check. $2\sqrt{15+1}-8=0.$

$8-8=0.$

8. $2\sqrt{5x-4}+3=11.$
$5x-4=16.$
$5x=20.$
$x=4.$
Check. $2\sqrt{20-4}+3=11.$
$2\cdot 4+3=11.$
$11=11.$

9. $\sqrt[3]{x-1}=2.$
$x-1=8.$
$x=9.$
Check. $\sqrt[3]{9-1}=2.$
$\sqrt[3]{8}=2.$

10. $\sqrt[3]{2x+1}=3.$
$2x+1=27.$
$2x=26.$
$x=13.$
Check. $\sqrt[3]{26+1}=3.$
$3=3.$

11. $\sqrt[3]{4x-6}=4.$
$4x-6=64.$
$x=\frac{35}{2}.$
Check. $\sqrt[3]{70-6}=4.$
$4=4.$

12. $\sqrt[3]{2x-1}+5=0.$
$2x-1=-125.$
$x=-62.$
Check. $\sqrt[3]{-124-1}+5=0.$
$\sqrt[3]{-125}+5=0.$
$-5+5=0.$

13. $3\sqrt[3]{2x+7}=6.$
$2x+7=8.$
$x=\frac{1}{2}.$
Check. $3\sqrt[3]{1+7}=6.$
$6=6.$

14. $2\sqrt[3]{3x-9}+4=0.$
$3x-9=-8.$
$x=\frac{1}{3}.$
Check. $2\sqrt[3]{1-9}+4=0.$
$-4+4=0.$

15. $3\sqrt{x-2}=\sqrt{2x+3}.$
$9x-18=2x+3.$
$7x=21.$
$x=3.$
Check. $3\sqrt{3-2}=\sqrt{6+3}.$
$3=3.$

16. $2\sqrt{x+3}=\sqrt{x+18}.$
$4x+12=x+18.$
$3x=6.$
$x=2.$
Check. $2\sqrt{2+3}=\sqrt{2+18}.$
$2\sqrt{5}=2\sqrt{5}.$

17. $3\sqrt{2x-6}=2\sqrt{x+4}.$
$18x-54=4x+16.$
$x=5.$
Check. $3\sqrt{10-6}=2\sqrt{5+4}.$
$6=6.$

18. $\sqrt{2x-1}-\sqrt{2-x}=0.$
$2x-1=2-x.$
$x=1.$
Check. $\sqrt{2-1}-\sqrt{2-1}=0.$
$1-1=0.$

19. $\sqrt{4x-3}-\sqrt{3x+5}=0.$
$4x-3=3x+5.$
$x=8.$
Check. $\sqrt{32-3}-\sqrt{24+5}=0.$
$\sqrt{29}-\sqrt{29}=0.$

20. $\sqrt{6x-5}-\sqrt{5x+7}=0.$

$6x-5=5x+7.$

$x=12.$

Check. $\sqrt{72-5}-\sqrt{60+7}=0.$

$\sqrt{67}-\sqrt{67}=0.$

21. $2\sqrt{3x-3}-3\sqrt{2+x}=0.$

$12x-12=18+9x.$

$x=10.$

Check. $2\sqrt{30-3}-3\sqrt{2+10}=0.$

$6\sqrt{3}-6\sqrt{3}=0.$

22. From Hint $x+3=4.$

$x=1.$

Check. $3-\sqrt{1+3}=\sqrt{1}.$

$3-2=1.$

$1=1.$

23. $5-\sqrt{2x+5}=\sqrt{2x}.$

$25-10\sqrt{2x+5}+2x+5=2x.$

$\sqrt{2x+5}=3.$

$2x+5=9.$

$x=2.$

Check. $5-\sqrt{4+5}=\sqrt{4}.$

$5-3=2.$

24. $7-\sqrt{3x}=\sqrt{3x+7}.$

$49-14\sqrt{3x}+3x=3x+7.$

$\sqrt{3x}=3.$

$x=3.$

Check. $7-\sqrt{9}=\sqrt{9+7}.$

$7-3=4.$

25. $\sqrt{3x-5}+\sqrt{3x+7}=6.$

$3x-5=3x+7-12\sqrt{3x+7}+36.$

$\sqrt{3x+7}=4.$

$3x+7=16.$

$x=3.$

Check. $\sqrt{9-5}+\sqrt{9+7}=6.$

$2+4=6.$

26. $\sqrt{x}+\sqrt{2}=\sqrt{x+2}.$

$x+2\sqrt{2x}+2=x+2.$

$x=0.$

Check. $0+\sqrt{2}-\sqrt{2}=0.$

27. $\sqrt{x+1}-\sqrt{2x-3}=\sqrt{3x-2}.$

$x+1-2\sqrt{(x+1)(2x-3)}+2x-3=3x-2.$

$\sqrt{(x+1)(2x-3)}=0.$

$x+1=0$, and $x=-1.$

$2x-3=0$, and $x=\frac{3}{2}.$

Check. $\sqrt{-1+1}-\sqrt{-2-3}=\sqrt{-3-2}.$

$-\sqrt{-5}=\sqrt{-5}.$

$\therefore\ -1$ is *not* a root.

$\sqrt{\frac{3}{2}+1}-\sqrt{3-3}=\sqrt{\frac{9}{2}-2}.$

$\sqrt{\frac{5}{2}}-0=\sqrt{\frac{5}{2}}.$

$\therefore\ \frac{3}{2}$ is a root.

28. $$\sqrt{x-3}+\sqrt{2x+4}=\sqrt{3x+1}.$$
$$x-3+2\sqrt{(x-3)(2x+4)}+2x+4=3x+1.$$
$$\sqrt{(x-3)(2x+4)}=0.$$
$$\therefore\ x-3=0,\text{ and }x=3.$$
$$2x+4=0,\text{ and }x=-2.$$

Check. $$\sqrt{3-3}+\sqrt{6+4}=\sqrt{9+1}.$$
$$0+\sqrt{10}=\sqrt{10}.$$
$$\therefore\ x=3\text{ is a root.}$$
$$\sqrt{-2-3}+\sqrt{-4+4}=\sqrt{-6+1}.$$
$$\sqrt{-5}+0=\sqrt{-5}.$$
$$\therefore\ x=-2\text{ is a root.}$$

29. $$\sqrt{x+3}+\sqrt{x-4}=\sqrt{4x-3}.$$
$$x+3+2\sqrt{(x+3)(x-4)}+x-4=4x-3.$$
$$\sqrt{(x+3)(x-4)}=x-1.$$
$$x^2-x-12=x^2-2x+1.$$
$$\therefore\ x=13.$$

Check. $$\sqrt{13+3}+\sqrt{13-4}=\sqrt{52-3}.$$
$$4+3=7.$$

30. $$\sqrt{x+4}+\sqrt{x-3}=\sqrt{4x-3}.$$
$$x+4+2\sqrt{x^2+x-12}+x-3=4x-3.$$
$$\sqrt{x^2+x-12}=x-2.$$
$$x^2+x-12=x^2-4x+4.$$
$$5x=16.$$
$$x=\tfrac{16}{5}.$$

Check. $$\sqrt{\tfrac{16}{5}+4}+\sqrt{\tfrac{16}{5}-3}=\sqrt{\tfrac{64}{5}-3}.$$
$$6\sqrt{\tfrac{1}{5}}+\sqrt{\tfrac{1}{5}}=7\sqrt{\tfrac{1}{5}}.$$

31. $$\frac{\sqrt{x-3}}{\sqrt{x+1}}=\frac{\sqrt{x-4}}{\sqrt{x-2}}.$$
$$x^2-5x+6=x^2-3x-4.$$
$$x=5.$$

Check. $$\frac{\sqrt{5-3}}{\sqrt{5+1}}=\frac{\sqrt{5-4}}{\sqrt{5-2}}.$$
$$\sqrt{\tfrac{2}{6}}=\sqrt{\tfrac{1}{3}}.$$

32. $$\frac{\sqrt{x-3}}{\sqrt{x+2}}=\frac{\sqrt{x-5}}{\sqrt{x-1}}.$$
$$x^2-4x+3=x^2-3x-10.$$
$$x=13.$$

Check. $$\frac{\sqrt{13-3}}{\sqrt{13+2}}=\frac{\sqrt{13-5}}{\sqrt{13-1}}.$$
$$\frac{\sqrt{10}}{\sqrt{15}}=\frac{\sqrt{8}}{\sqrt{12}},$$
or $$\sqrt{\tfrac{2}{3}}=\sqrt{\tfrac{2}{3}}.$$

Page 267

1. $\sqrt{10} \div \sqrt{2} = \dfrac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}.$

2. $\sqrt{18} \div \sqrt{3} = \dfrac{\sqrt{18}}{\sqrt{3}} = \sqrt{6}.$

4. $\sqrt{7} \div \sqrt{3} = \dfrac{\sqrt{7}\sqrt{3}}{\sqrt{3}\sqrt{3}} = \dfrac{1}{3}\sqrt{21}.$

5. $2\sqrt{6} \div 3\sqrt{2} = \dfrac{2\sqrt{6}}{3\sqrt{2}} = \dfrac{2}{3}\sqrt{3}.$

6. $\sqrt{10} \div \sqrt{3} = \dfrac{\sqrt{10}\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \dfrac{1}{3}\sqrt{30}.$

7. $2\sqrt{7} \div \sqrt{2} = \dfrac{2\sqrt{7}\sqrt{2}}{\sqrt{2}\sqrt{2}} = \sqrt{14}.$

8. $\sqrt{6} \div \sqrt{18} = \dfrac{\sqrt{6}}{\sqrt{18}} = \sqrt{\dfrac{1}{3}} = \dfrac{1}{3}\sqrt{3}.$

9. $6 \div 2\sqrt{2} = \dfrac{6\sqrt{2}}{2\sqrt{2}\sqrt{2}} = \dfrac{3}{2}\sqrt{2}.$

10. $8\sqrt{15} \div 4\sqrt{5} = \dfrac{8\sqrt{15}}{\sqrt{5}} = 2\sqrt{3}.$

11. $3\sqrt{2} \div 15\sqrt{8} = \dfrac{3\sqrt{2}}{15\sqrt{8}} = \dfrac{1}{5\sqrt{4}} = \frac{1}{10}.$

12. $\sqrt{6} - \sqrt{18} \div \sqrt{2} = \sqrt{6} - \dfrac{\sqrt{18}}{\sqrt{2}} = \sqrt{6} - 3.$

13. $(\sqrt{12} - \sqrt{24}) \div 3\sqrt{3} = \dfrac{\sqrt{12}}{3\sqrt{3}} - \dfrac{\sqrt{24}}{3\sqrt{3}} = \dfrac{2}{3} - \dfrac{2}{3}\sqrt{2}.$

14. $(\sqrt{6} - \sqrt{9} + 18) \div 2\sqrt{2} = \dfrac{\sqrt{6}}{2\sqrt{2}} + \dfrac{15\sqrt{2}}{2\sqrt{2}\sqrt{2}} = \dfrac{1}{2}\sqrt{3} + \dfrac{15}{4}\sqrt{2}.$

16. $9 \div (3 - \sqrt{2}) = \dfrac{9(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \dfrac{27 + 9\sqrt{2}}{9-2} = \dfrac{27}{7} + \dfrac{9}{7}\sqrt{2}.$

17. $\dfrac{\sqrt{5}}{\sqrt{5}-\sqrt{2}} = \dfrac{\sqrt{5}(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})} = \dfrac{5+\sqrt{10}}{5-2} = \dfrac{5}{3} + \dfrac{1}{3}\sqrt{10}.$

18. $\dfrac{2\sqrt{3}}{\sqrt{3}+\sqrt{2}} = \dfrac{2\sqrt{3}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = 6 - 2\sqrt{6}.$

19. $\dfrac{4\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \dfrac{(4\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} = \dfrac{23 + 5\sqrt{15}}{2}.$

20. $\dfrac{\sqrt{3}+\sqrt{7}}{\sqrt{7}-2} = \dfrac{(\sqrt{3}+\sqrt{7})(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)} = \dfrac{7+\sqrt{21}+2\sqrt{3}+2\sqrt{7}}{3}.$

21. $5 + 3\sqrt{2} = 5 + 3(1.4142) = 5 + 4.242 = 9.242.$

22. $7 - 3\sqrt{5} = 7 - 3(2.23607) = 7 - 67082 = .291.$

23. $\dfrac{7+\sqrt{6}}{3} = \dfrac{7 + 2.449}{3} = 3.149.$

24. $\dfrac{\sqrt{2}}{\sqrt{3}} = \dfrac{1}{3}\sqrt{6} = \dfrac{1}{3}(2.449) = .816.$

25. $\dfrac{\sqrt{5}+1}{\sqrt{5}}=1+\dfrac{1}{5}\sqrt{5}=1+\dfrac{1}{5}(2.236)=1.447.$

26. $\dfrac{2}{\sqrt{3}-\sqrt{2}}=\dfrac{2\sqrt{3}+2\sqrt{2}}{3-2}=2(1.7320)+2(1.4142)=6.292.$

27. $3\sqrt{8}-\sqrt{7}=3(2.8284)-2.6457=5.839.$

28. $\sqrt{3-\sqrt{2}}=\sqrt{3-1.414213}$
$=1.259.$

29. $\dfrac{\sqrt{a}}{\sqrt{x}}=\dfrac{\sqrt{a}\sqrt{x}}{\sqrt{x}\sqrt{x}}=\dfrac{1}{x}\sqrt{ax}.$

30. $\dfrac{\sqrt{3a^2}}{\sqrt{ax^2}}=\dfrac{1}{x}\sqrt{3a}.$

31. $\dfrac{2\sqrt{ax}}{3\sqrt{bx}}=\dfrac{2}{3}\sqrt{\dfrac{a}{b}}=\dfrac{2}{3b}\sqrt{ab}.$

Page 272

1. $x^2-4x-32=0.$
Transposing, $x^2-4x=32.$
Adding $(-\frac{4}{2})^2=4$, $x^2-4x+4=36.$
Extracting the square root, $x-2=\pm 6.$
$x=8,-4.$

2. $x^2-2x-15=0.$
Transposing, $x^2-2x=15.$
Completing the square, $x^2-2x+1=16.$
Extracting the square root, $x-1=\pm 4.$
$x=5,-3.$

3. $x^2=18.$
Extracting the square root, $x=\pm 4.24.$

4. $x^2-25=0.$
Transposing, $x^2=25.$
Extracting the square root, $x=\pm 5.$

5. $x^2-7x-18=0.$
Transposing, $x^2-7x=18.$
Completing the square, $x^2-7x+\frac{49}{4}=18+\frac{49}{4}=\frac{121}{4}.$
Extracting the square root, $x-\frac{7}{2}=\pm\frac{11}{2}.$
$x=9,-2.$

6. $x(x+4)-3(x+4)=0.$
Removing parentheses and combining, $x^2+x-12=0.$
$x^2+x=12.$
$x^2+x+\frac{1}{4}=12+\frac{1}{4}=\frac{49}{4}.$
$x+\frac{1}{2}=\pm\frac{7}{2}.$
$x=3,-4.$

7. $y+6=y^2.$
$y^2-y=6.$
Completing the square,
$y^2-y+\frac{1}{4}=6+\frac{1}{4}.$
$y-\frac{1}{2}=\pm\frac{5}{2}.$
$y=3,-2.$

8. $4x^2 - 4x - 3 = 0.$
$$x^2 - x - \tfrac{3}{4} = 0.$$
$$x^2 - x = \tfrac{3}{4}.$$
$$x^2 - x + \tfrac{1}{4} = 1.$$
$$x - \tfrac{1}{2} = \pm 1.$$
$$x = \tfrac{3}{2}, -\tfrac{1}{2}.$$

9. $z^2 - 5\tfrac{1}{2}z + 2\tfrac{1}{2} = 0.$
$$z^2 - \tfrac{11}{2}z = -\tfrac{5}{2}.$$
$$z^2 - \tfrac{11}{2}z + (-\tfrac{11}{4})^2 = -\tfrac{5}{2} + \tfrac{121}{16}$$
$$= \tfrac{81}{16}.$$
$$z - \tfrac{11}{4} = \pm \tfrac{9}{4}.$$
$$z = 5, \tfrac{1}{2}.$$

10. $\tfrac{5}{4} - x^2 = 2x.$
$$x^2 + 2x = \tfrac{5}{4}.$$
$$x^2 + 2x + 1 = \tfrac{9}{4}.$$
$$x + 1 = \pm \tfrac{3}{2}.$$
$$x = \tfrac{1}{2}, -\tfrac{5}{2}.$$

11. $2x^2 - 9x + 4 = 0.$
$$x^2 - \tfrac{9}{2}x + 2 = 0.$$
$$x^2 - \tfrac{9}{2}x = -2.$$
$$x^2 - \tfrac{9}{2}x + (-\tfrac{9}{4})^2 = -2 + \tfrac{81}{16}$$
$$= \tfrac{49}{16}.$$
$$x - \tfrac{9}{4} = \pm \tfrac{7}{4}.$$
$$x = 4, \tfrac{1}{2}.$$

12. $2t^2 - 3t = 9.$
$$t^2 - \tfrac{3}{2}t = \tfrac{9}{2}.$$
$$t^2 - \tfrac{3}{2}t + (-\tfrac{3}{4})^2 = \tfrac{81}{16}.$$
$$t - \tfrac{3}{4} = \pm \tfrac{9}{4}.$$
$$t = 3, -\tfrac{3}{2}.$$

13. $3x^2 - 5 = 70.$
$$3x^2 = 75.$$
$$x^2 = 25.$$
$$x = \pm 5.$$

14. $x^2 + 8 = 4x^2 - 40.$
$$3x^2 = 48.$$
$$x^2 = 16.$$
$$x = \pm 4.$$

15. $5x = 6x^2 - 14.$
$$14 = 6x^2 - 5x.$$
$$\tfrac{7}{3} = x^2 - \tfrac{5}{6}x.$$
$$\tfrac{7}{3} + \tfrac{25}{144} = \tfrac{361}{144}$$
$$= x^2 - \tfrac{5}{6}x + (-\tfrac{5}{12})^2.$$
$$\pm \tfrac{19}{12} = x - \tfrac{5}{12}.$$
$$x = 2, -\tfrac{7}{6}.$$

16. $x^2 - \dfrac{7x}{12} - 1 = 0.$
$$x^2 - \tfrac{7}{12}x = 1.$$
$$x^2 - \tfrac{7}{12}x + (-\tfrac{7}{24})^2 = 1 + \tfrac{49}{576}$$
$$= \tfrac{625}{576}.$$
$$x - \tfrac{7}{24} = \pm \tfrac{25}{24}.$$
$$x = \tfrac{4}{3}, -\tfrac{3}{4}.$$

17. $6t^2 - 13t + 6 = 0.$
$$t^2 - \tfrac{13}{6}t = -1.$$
$$t^2 - \tfrac{13}{6}t + (-\tfrac{13}{12})^2 = -1 + \tfrac{169}{144}$$
$$= \tfrac{25}{144}.$$
$$t - \tfrac{13}{12} = \pm \tfrac{5}{12}.$$
$$t = \tfrac{3}{2}, \tfrac{2}{3}.$$

18. $42 + 2x^2 = -19x.$
$$x^2 + \tfrac{19}{2}x = -21.$$
$$x^2 + \tfrac{19}{2}x + (\tfrac{19}{4})^2 = -21 + \tfrac{361}{16}$$
$$= \tfrac{25}{16}.$$
$$x + \tfrac{19}{4} = \pm \tfrac{5}{4}.$$
$$x = -6, -\tfrac{7}{2}.$$

19. Writing the double sign before both members would be equivalent to writing the two equations
$$x + 2 = \pm 5, \qquad (1)$$
$$-(x + 2) = \pm 5. \qquad (2)$$
But (2) may be multiplied in both members by -1, giving
$$x + 2 = \mp 5. \qquad (3)$$
Now (3) plainly is equivalent to (1), and because of the way in which it was obtained it is equivalent to (2). Hence (2) and (1) are equivalent, and no root is retained by attaching the double sign to both members.

21. $p^2+10p+17=0.$
$$p^2+10p=-17.$$
$$p^2+10p+25=8.$$
$$p+5=\pm 2\sqrt{2}.$$
$$p=-5\pm 2\sqrt{2}.$$

Extracting, $\sqrt{2}=1.4142.$

Hence $p=-5\pm 2.8284.$
$$p=-7.828,$$
$$-2.172.$$

22. $x^2+2x-4=0.$
$$x^2+2x=4.$$
$$x^2+2x+1=5.$$
$$x+1=\pm\sqrt{5}.$$
$$x=-1\pm\sqrt{5}.$$

Extracting, $\sqrt{5}=2.236.$

Hence $x=-1\pm 2.236$
$$=1.236,$$
$$-3.236.$$

23. $x^2-2x-3\frac{1}{2}=0.$
$$x^2-2x=\tfrac{7}{2}.$$
$$x^2-2x+1=\tfrac{9}{2}.$$
$$x-1=\pm\tfrac{3}{2}\sqrt{2}.$$

(See Ex. 21) $x=1\pm\frac{3}{2}(1.4142)$
$$=1\pm 2.1213.$$
$$x=3.121,$$
$$-1.121.$$

24. $3x^2-6x+1=0.$
$$x^2-2x=-\tfrac{1}{3}.$$
$$x^2-2x+1=\tfrac{2}{3}.$$
$$x-1=\pm\tfrac{1}{3}\sqrt{6}.$$

Extracting, $\sqrt{6}=2.449.$

Hence $x=1\pm .816$
$$=1.816,\ .184.$$

25. $9x=5x^2-2.$
$$2=5x^2-9x.$$
$$x^2-\tfrac{9}{5}x=\tfrac{2}{5}.$$
$$x^2-\tfrac{9}{5}x+(-\tfrac{9}{10})^2=\tfrac{2}{5}+\tfrac{81}{100}$$
$$=\tfrac{121}{100}.$$
$$x-\tfrac{9}{10}=\pm\tfrac{11}{10}.$$
$$x=2,\ -\tfrac{1}{5}.$$

26. $1-4v^2=2v.$
$$1=4v^2+2v.$$
$$v^2+\tfrac{1}{2}v=\tfrac{1}{4}.$$
$$v^2+\tfrac{1}{2}v+\tfrac{1}{16}=\tfrac{1}{4}+\tfrac{1}{16}=\tfrac{5}{16}.$$

(See Ex. 22) $v+\frac{1}{4}=\pm\frac{1}{4}\sqrt{5}$
$$=\pm\tfrac{1}{4}(2.236).$$
$$v=-\tfrac{1}{4}\pm .559$$
$$=.309,\ -.809.$$

27. $x^2-6\sqrt{3}x=-9.$
$$x^2-6\sqrt{3}x+(-3\sqrt{3})^2=-9+27=18.$$
$$x-3\sqrt{3}=\pm 3\sqrt{2}.$$
$$x=3(\sqrt{3}\pm\sqrt{2}).$$

But $\sqrt{3}=1.7321,$
$$\sqrt{2}=1.4142.$$
$$x=3(3.1463),$$
$$3(.3179)$$
$$=9.439,\ .954.$$

28. $q^2-3q\sqrt{2}+4=0.$
$$q^2-3\sqrt{2}q+(-\tfrac{3}{2}\sqrt{2})^2=-4+\tfrac{9}{2}=\tfrac{1}{2}.$$
$$q-\tfrac{3}{2}\sqrt{2}=\pm\tfrac{1}{2}\sqrt{2}.$$

(See Ex. 21) $q=(\frac{3}{2}\pm\frac{1}{2})\sqrt{2}$
$$=2.828,\ 1.414.$$

29. $x^2+5=6-4x-3x^2.$
$$4x^2+4x=1.$$
$$x^2+x=\tfrac{1}{4}.$$
$$x^2+x+\tfrac{1}{4}=\tfrac{1}{2}.$$

(See Ex. 21) $x+\frac{1}{2}=\pm\dfrac{1}{\sqrt{2}}=\pm\dfrac{\sqrt{2}}{2}$
$$=\pm .707.$$
$$x=-\tfrac{1}{2}\pm .707$$
$$=.207,\ -1.207.$$

30.
$$3+\frac{1}{3p}=3p.$$
$$9p+1=9p^2.$$
$$1=9p^2-9p.$$
$$p^2-p=\tfrac{1}{9}.$$
$$p^2-p+\tfrac{1}{4}=\tfrac{1}{9}+\tfrac{1}{4}=\tfrac{13}{36}.$$
$$p-\tfrac{1}{2}=\pm\tfrac{1}{6}\sqrt{13}$$
$$=\pm\tfrac{1}{6}(3.605).$$
$$p=.5\pm.601$$
$$=1.101,\ -.101.$$

32.
$$x^4-5x^2+4=0.$$
$$x^4-5x^2=-4.$$
$$x^4-5x^2+(-\tfrac{5}{2})^2=-4+\tfrac{25}{4}=\tfrac{9}{4}.$$
$$x^2-\tfrac{5}{2}=\pm\tfrac{3}{2}.$$
$$x^2=4,\ 1.$$
$$x=\pm 2,\ \pm 1.$$

33.
$$9x^4-13x^2+4=0.$$
$$x^4-\tfrac{13}{9}x^2=-\tfrac{4}{9}.$$
$$x^4-\tfrac{13}{9}x^2+(-\tfrac{13}{18})^2=-\tfrac{4}{9}+\tfrac{169}{324}.$$
$$=\tfrac{25}{324}.$$
$$x^2-\tfrac{13}{18}=\pm\tfrac{5}{18}.$$
$$x^2=1,\ \tfrac{4}{9}.$$
$$x=\pm 1,\pm\tfrac{2}{3}.$$

34.
$$9x^4-82x^2+9=0.$$
$$x^4-\tfrac{82}{9}x^2=-1.$$
$$x^4-\tfrac{82}{9}x^2+(-\tfrac{41}{9})^2=-1+\tfrac{1681}{81}$$
$$=\tfrac{1600}{81}.$$
$$x^2-\tfrac{41}{9}=\pm\tfrac{40}{9}.$$
$$x^2=9,\ \tfrac{1}{9}.$$
$$x=\pm 3,\ \pm\tfrac{1}{3}.$$

35.
$$x^4-10x^2+24=0.$$
$$x^4-10x^2=-24.$$
$$x^4-10x^2+25=1.$$
$$x^2-5=\pm 1.$$
$$x^2=4,\ 6.$$
$$x=\pm 2,\ \pm\sqrt{6}.$$

36.
$$4x^4-13x^2+3=0.$$
$$x^4-\tfrac{13}{4}x^2=-\tfrac{3}{4}.$$
$$x^4-\tfrac{13}{4}x^2+(\tfrac{13}{8})^2=-\tfrac{3}{4}+\tfrac{169}{64}$$
$$=\tfrac{121}{64}.$$
$$x^2-\tfrac{13}{8}=\pm\tfrac{11}{8}.$$
$$x^2=3,\ \tfrac{1}{4}.$$
$$x=\pm\sqrt{3},\ \pm\tfrac{1}{2}.$$

37.
$$16x^4-81x^2+5=0.$$
$$x^4-\tfrac{81}{16}x^2=-\tfrac{5}{16}.$$
$$x^4-\tfrac{81}{16}x^2+(-\tfrac{81}{32})^2=-\tfrac{5}{16}+\tfrac{6561}{1024}$$
$$=\tfrac{6241}{1024}.$$
$$x^2-\tfrac{81}{32}=\pm\tfrac{79}{32}.$$
$$x^2=5,\ \tfrac{1}{16}.$$
$$x=\pm\sqrt{5},\ \pm\tfrac{1}{4}.$$

Page 275

1.
$$12x^2+7x=5.$$
$$x^2+\tfrac{7}{12}x=\tfrac{5}{12}.$$
$$x^2+\tfrac{7}{12}x+(\tfrac{7}{24})^2=\tfrac{5}{12}+\tfrac{49}{576}$$
$$=\tfrac{289}{576}.$$
$$x+\tfrac{7}{24}=\pm\tfrac{17}{24}.$$
$$x=\tfrac{5}{12},\ -1.$$

2.
$$(5x-2)(x-6)=(2x-12)(2x-4).$$
$$5x^2-32x+12=4x^2-32x+48.$$
$$x^2=36.$$
$$x=\pm 6.$$

3.
$$4x^2+15x=-9.$$
$$x^2+\tfrac{15}{4}x=-\tfrac{9}{4}.$$
$$x^2+\tfrac{15}{4}x+(\tfrac{15}{8})^2=-\tfrac{9}{4}+\tfrac{225}{64}=\tfrac{81}{64}.$$
$$x+\tfrac{15}{8}=\pm\tfrac{9}{8}.$$
$$x=-3,\ -\tfrac{3}{4}.$$

4.
$$x+1=20x^2.$$
$$20x^2-x=1.$$
$$x^2-\tfrac{1}{20}x=\tfrac{1}{20}.$$
$$x^2-\tfrac{1}{20}x+(-\tfrac{1}{40})^2=\tfrac{1}{1600}+\tfrac{1}{20}$$
$$=\tfrac{81}{1600}.$$
$$x-\tfrac{1}{40}=\pm\tfrac{9}{40}.$$
$$x=\tfrac{1}{4},\ -\tfrac{1}{5}.$$

5. $25y^2 - 20y + 12 = 0.$

$$y^2 - \tfrac{4}{5}y = \tfrac{12}{25}.$$

$$y^2 - \tfrac{4}{5}y + (-\tfrac{2}{5})^2 = \frac{12+4}{25} = \frac{16}{25}.$$

$$y - \tfrac{2}{5} = \pm\tfrac{4}{5}.$$

$$y = \tfrac{6}{5}, -\tfrac{2}{5}.$$

6. $6x + x^2 + 3 = 2 - 3x^2 + 10x.$

$$4x^2 - 4x + 1 = 0.$$

$$(2x-1)^2 = 0.$$

$$x = \tfrac{1}{2}, \tfrac{1}{2}.$$

7. $(3x+6)(x-3) = (2x+1)(x-2).$

$$3x^2 - 3x - 18 = 2x^2 - 3x - 2.$$

$$x^2 = 16.$$

$$x = \pm 4.$$

8. $\dfrac{x}{3} + 4 - \dfrac{15}{x} = 0.$

$$x^2 + 12x - 45 = 0.$$

$$x^2 + 12x = 45.$$

$$x^2 + 12x + 36 = 81.$$

$$x + 6 = \pm 9.$$

$$x = 3, -15.$$

9. $\dfrac{1}{3} + \dfrac{a}{9} - \dfrac{2}{a} = 0.$

$$3a + a^2 - 18 = 0.$$

$$a^2 + 3a = 18.$$

$$a^2 + 3a + \tfrac{9}{4} = 18 + \tfrac{9}{4} = \tfrac{81}{4}.$$

$$a + \tfrac{3}{2} = \pm\tfrac{9}{2}.$$

$$a = 3, -6.$$

10. $\dfrac{3x}{2} + \dfrac{1}{2} = \dfrac{1}{3x}.$

$$9x^2 + 3x = 2.$$

$$x^2 + \tfrac{1}{3}x = \tfrac{2}{9}.$$

$$x^2 + \tfrac{1}{3}x + \tfrac{1}{36} = \tfrac{2}{9} + \tfrac{1}{36} = \tfrac{9}{36}$$

$$= \tfrac{1}{4}.$$

$$x + \tfrac{1}{6} = \pm\tfrac{1}{2} = \pm\tfrac{3}{6}.$$

$$x = \tfrac{1}{3}, -\tfrac{2}{3}.$$

11. $x = \dfrac{3}{x+2}.$

$$x^2 + 2x = 3.$$

$$x^2 + 2x + 1 = 4.$$

$$x + 1 = \pm 2.$$

$$x = -3, 1.$$

12. $\dfrac{x^2}{x-5} + \dfrac{5}{2} = 0.$

$$2x^2 + 5x - 25 = 0.$$

$$x^2 + \tfrac{5}{2}x = \tfrac{25}{2}.$$

$$x^2 + \tfrac{5}{2}x + (\tfrac{5}{4})^2 = \tfrac{25}{2} + \tfrac{25}{16}$$

$$= \tfrac{225}{16}.$$

$$x + \tfrac{5}{4} = \pm\tfrac{15}{4}.$$

$$x = \tfrac{5}{2}, -5.$$

13. $\dfrac{2}{t-2} - \dfrac{3t}{2} = 0.$

$$4 - 3t^2 + 6t = 0.$$

$$3t^2 - 6t = 4.$$

$$t^2 - 2t = \tfrac{4}{3}.$$

$$t^2 - 2t + 1 = \tfrac{7}{3}.$$

$$t - 1 = \pm\tfrac{1}{3}\sqrt{21}.$$

$$t = 1 \pm \tfrac{1}{3}\sqrt{21}.$$

14. $\dfrac{x}{5-x} + \dfrac{5x}{12} = \dfrac{3}{2}.$

$$12x + 5x(5-x) = 18(5-x).$$

Whence

$$5x^2 - 55x + 90 = 0.$$

$$x^2 - 11x + 18 = 0.$$

$$x^2 - 11x = -18.$$

$$x^2 - 11x + \tfrac{121}{4} = -18 + \tfrac{121}{4}$$

$$= \tfrac{49}{4}.$$

$$x - \tfrac{11}{2} = \pm\tfrac{7}{2}.$$

$$x = 9, 2.$$

15. $\dfrac{x}{x-2} - \dfrac{5}{2} = \dfrac{2-x}{x}.$

$$2x^2 - 5x(x-2) = 2(2-x)(x-2).$$

Whence $x^2 - 2x - 8 = 0.$

$$x^2 - 2x = 8.$$

$$x^2 - 2x + 1 = 9.$$

$$x - 1 = \pm 3.$$

$$x = 4, -2.$$

16. $$\frac{2}{s-2}-\frac{1}{s+2}=-\frac{15}{8}.$$
$$16(s+2)-8(s-2)=-15(s^2-4).$$
$$15s^2+8s-12=0.$$
$$s^2+\tfrac{8}{15}s=\tfrac{4}{5}.$$
$$s^2+\tfrac{8}{15}s+(\tfrac{4}{15})^2=\tfrac{4}{5}+\tfrac{16}{225}=\tfrac{196}{225}.$$
$$s+\tfrac{4}{15}=\pm\tfrac{14}{15}.$$
$$s=\tfrac{2}{3},-\tfrac{6}{5}.$$

17. $$\frac{3+x}{4+x}-\frac{1}{12}=\frac{5-x}{6-x}.$$
$$12(3+x)(6-x)-(4+x)(6-x)=12(5-x)(4+x).$$
Whence $$x^2+22x-48=0.$$
$$x^2+22x=48.$$
$$x^2+22x+121=169.$$
$$x+11=\pm 13.$$
$$x=2,-24.$$

18. $$\frac{1+x}{2+x}-\frac{1}{6}=\frac{3-x}{5-x}.$$
$$6(1+x)(5-x)-(2+x)(5-x)=6(3-x)(2+x).$$
Whence $$x^2+15x-16=0.$$
$$x^2+15x=16.$$
$$x^2+15x+(\tfrac{15}{2})^2=16+\tfrac{225}{4}=\tfrac{289}{4}.$$
$$x+\tfrac{15}{2}=\pm\tfrac{17}{2}.$$
$$x=1,-16.$$

19. $$\frac{x+2}{x-4}+\frac{x-1}{x+5}=-1.$$
$$(x+2)(x+5)+(x-1)(x-4)=-(x-4)(x+5).$$
Whence $$3x^2+3x-6=0.$$
$$x^2+x=2.$$
$$x^2+x+\tfrac{1}{4}=\tfrac{9}{4}.$$
$$x+\tfrac{1}{2}=\pm\tfrac{3}{2}.$$
$$x=1,-2.$$

20. $$\frac{x-2}{x+2}-\frac{x-1}{x+1}=6.$$
$$(x-2)(x+1)-(x-1)(x+2)=6(x+2)(x+1).$$
Whence $$6x^2+20x+12=0.$$
$$x^2+\tfrac{10}{3}x=-2.$$

$$x^2 + \tfrac{10}{3}x + (\tfrac{5}{3})^2 = -2 + \tfrac{25}{9} = \tfrac{7}{9}.$$
$$x + \tfrac{5}{3} = \pm \tfrac{1}{3}\sqrt{7}.$$
$$x = -\tfrac{5}{3} \pm \tfrac{1}{3}\sqrt{7}.$$

21. $(3x - 2)(x - 3) = (x + 1)(x + 4).$

Whence $2x^2 - 16x + 2 = 0.$

$$x^2 - 8x = -1.$$
$$x^2 - 8x + 16 = 15.$$
$$x - 4 = \pm\sqrt{15}.$$
$$x = 4 \pm \sqrt{15}.$$

22. $x^4 - 10x^2 + 16 = 0.$

$$x^4 - 10x^2 = -16.$$
$$x^4 - 10x^2 + 25 = 9.$$
$$x^2 - 5 = \pm 3.$$
$$x^2 = 8,\ 2.$$
$$x = \pm 2\sqrt{2},\ \pm\sqrt{2}.$$

23. $x^4 - 8x^2 + 15 = 0.$

$$x^4 - 8x^2 = -15.$$
$$x^4 - 8x^2 + 16 = 1.$$
$$x^2 - 4 = \pm 1.$$
$$x^2 = 3,\ 5.$$
$$x = \pm\sqrt{3},\ \pm\sqrt{5}.$$

24. $6x^4 - 19x^2 + 15 = 0.$

$$x^4 - \tfrac{19}{6}x^2 = -\tfrac{5}{2}.$$
$$x^4 - \tfrac{19}{6}x^2 + (-\tfrac{19}{12})^2 = -\tfrac{5}{2} + \tfrac{361}{144}$$
$$= \tfrac{1}{144}.$$
$$x^2 - \tfrac{19}{12} = \pm\tfrac{1}{12}.$$
$$x^2 = \tfrac{5}{3},\ \tfrac{3}{2}.$$
$$x = \pm\tfrac{1}{3}\sqrt{15},\ \pm\tfrac{1}{2}\sqrt{6}.$$

25. $8x^4 - 18x^2 + 9 = 0.$

$$x^4 - \tfrac{9}{4}x^2 = -\tfrac{9}{8}.$$
$$x^4 - \tfrac{9}{4}x^2 + (-\tfrac{9}{8})^2 = \tfrac{81}{64} - \tfrac{9}{8} = \tfrac{9}{64}.$$
$$x^2 - \tfrac{9}{8} = \pm\tfrac{3}{8}.$$
$$x^2 = \tfrac{3}{2},\ \tfrac{3}{4}.$$
$$x = \pm\tfrac{1}{2}\sqrt{6},\ \pm\tfrac{1}{2}\sqrt{3}.$$

Page 276

1. Let $x =$ the number.

Then $x^2 + 2x = 8.$

Whence $x = 2,\ -4.$

2. Let $x =$ the number.

Then $2x^2 - x = 28.$

Whence $x = 4,\ -\tfrac{7}{2}.$

3. Let x and $x + 2$ represent the numbers.

Then $x(x + 2) = 675.$

Whence $x = 25,\ -27.$

Therefore $x = 25,\ x + 2 = 27,$

or $x = -27,\ x + 2 = -25.$

4. Let x, $x + 2$, and $x + 4$ represent the numbers.

Then $\quad 3x + 6 = \frac{1}{2}(x^2 + 2x).$

Whence $\quad x = 6, -2.$

Therefore $\quad x = 6, x + 2 = 8, x + 4 = 10,$

or $\quad x = -2, x + 2 = 0, x + 4 = 2.$

5. Let $\quad x =$ the width of the field in rods.

Then $\quad x + 14 =$ the length in rods,

and $\quad x(x + 14) = 20 \cdot 160.$

Whence $\quad x = 50$, root -64 rejected,

and $\quad x + 14 = 64.$

6. Let $\quad x =$ the number.

Then $$x + \frac{1}{x} = \frac{13}{6}.$$

Whence $\quad x = \frac{2}{3}, \frac{3}{2}.$

7. Let $\quad x =$ the altitude of the field in rods.

Then $\quad x + 36 =$ the base in rods,

and $$\frac{x(x + 36)}{2} = \frac{9}{2}(160).$$

Whence $\quad x = 24$, root -60 rejected,

and $\quad x + 36 = 60.$

8. Let $\quad x =$ the side of the smaller field in rods.

Then $\quad x + 16 =$ the side of the larger field in rods,

and $\quad x^2 + (x + 16)^2 = (40)(160).$

Whence $\quad x = 48$, root -64 rejected,

and $\quad x + 16 = 64.$

9. Let $\quad x =$ the longer leg of the triangle in feet.

Then $\quad x - 31 =$ the shorter leg in feet,

and $\quad x^2 + (x - 31)^2 = 41^2 = 1681.$

Whence $\quad x = 40$, root -9 rejected,

and $\quad x - 31 = 9.$

10. Let $\quad x =$ the longer leg of the triangle.

Then $\quad \frac{3}{4}x =$ the shorter leg,

and $\quad x^2 + (\frac{3}{4}x)^2 = 900.$

Whence $\quad x = 24$, root -24 rejected,

and $\quad \frac{3}{4}x = 18.$

11. Let $x =$ the side of the square in inches.

Then $\dfrac{x}{12} =$ the side in feet,

and $$4x = \left(\frac{x}{12}\right)^2.$$

Whence $x = 576$, root 0 rejected.

12. Let x and $x + 1$ represent the dimensions.

Then the longest line is the diagonal.

Therefore $$x^2 + (x+1)^2 = (x+2)^2.$$

Whence $x = 3$, root -1 rejected,

and $x + 1 = 4$.

13. Let x and $x + 1$ represent the edges of the bins in yards.

Then $$(x+1)^3 - x^3 = 127.$$

Whence $x = 6$, root -7 rejected,

and $x + 1 = 7$.

14. Let $x =$ the greater rate in miles per hour.

Then $x - 6 =$ the less rate in miles per hour,

and $$\frac{252}{x-6} = \frac{252}{x} + 1.$$

Whence $x = 42$, root -36 rejected,

and $x - 6 = 36$.

15. Let $x =$ the rate going in miles per hour.

Then $x + 3 =$ the rate returning in miles per hour,

and $$\frac{90}{x} + \frac{90}{x+3} = 11.$$

Whence $x = 15$, root $-\frac{18}{11}$ rejected,

and $x + 3 = 18$.

16. Let $x =$ the width in inches.

Then $x + 2 =$ the length in inches,

$x - 2 =$ the width of the printed portion,

and $x =$ its length.

Then $$x(x-2) = \tfrac{1}{2}x(x+2).$$

Whence $x = 6$, root 0 rejected,

and $x + 2 = 8$.

17. Let $x =$ the original price of one orange in cents.

Then $12x + 10 =$ the new price per dozen in cents,

and $\frac{12x+10}{12} = \frac{6x+5}{6} =$ the new price of one orange in cents.

Therefore
$$\frac{100}{x} - 6 = \frac{100}{\frac{6x+5}{6}},$$

or
$$\frac{100}{x} - 6 = \frac{600}{6x+5}.$$

Whence $x = 3\frac{1}{3}$, root $-\frac{25}{6}$ rejected,

and $12x = 40.$ *Ans.* $= 40$.

Hence the original price was 40 cents per dozen.

18. Let $x =$ number of minutes by slower pump alone.

Then $x - 32 =$ number of minutes by faster pump alone,

and
$$\frac{1}{x} + \frac{1}{x-32} = \frac{1}{30}.$$

Whence $x = 12, 80.$

If $x = 12$, $x - 32 = -20$, which must be rejected.

Therefore $x = 80,$

and $x - 32 = 48.$

19. In the equation $s = 16t^2$

we have given $s = 1600.$

Therefore $16t^2 = 1600.$

Whence $t = 10$, root -10 rejected.

20. We know that $s = 16t^2$,

and since $t =$ the time in falling in seconds,

we have $6\frac{1}{2} - t =$ the number of seconds for the sound to return.

But $1152 =$ the rate of the sound in feet per second.

Therefore
$$s = 1152\left(\tfrac{13}{2} - t\right) = 1152\left(\frac{13-2t}{2}\right)$$
$$= 576(13 - 2t).$$

Equating the two expressions for s,

$$16t^2 = 576(13 - 2t),$$
$$t^2 = 36(13 - 2t).$$
$$t^2 + 72t - 468 = 0,$$
$$(t + 78)(t - 6) = 0.$$

Whence $t = 6$, root -78 rejected.

But if $t = 6,$

then $s = 16 \cdot 6^2 = 576$ feet.

Page 278

1. $$\begin{aligned} x^2 + 2bx &= 3b^2. \\ x^2 + 2bx + b^2 &= 4b^2. \\ x + b &= \pm 2b. \\ x &= b,\ -3b. \end{aligned}$$

2. $$\begin{aligned} x^2 + 4cx &= 5c^2. \\ x^2 + 4cx + 4c^2 &= 9c^2. \\ x + 2c &= \pm 3c. \\ x &= c,\ -5c. \end{aligned}$$

3. $$\begin{aligned} x^2 - 8cx - 9c^2 &= 0. \\ x^2 - 8cx + 16c^2 &= 25c^2. \\ x - 4c &= \pm 5c. \\ x &= -c,\ 9c. \end{aligned}$$

4. $$\begin{aligned} x^2 + ax - 2a^2 &= 0. \\ x^2 + ax &= 2a^2. \\ x^2 + ax + \frac{a^2}{4} &= \frac{9a^2}{4}. \\ x + \frac{a}{2} &= \pm \frac{3a}{2}. \\ x &= a,\ -2a. \end{aligned}$$

5. $$\begin{aligned} x^2 - 2ax &= 1 - a^2. \\ x^2 - 2ax + a^2 &= 1. \\ x - a &= \pm 1. \\ x &= a \pm 1. \end{aligned}$$

6. $$\begin{aligned} x^2 - a^2 &= 0. \\ x^2 &= a^2. \\ x &= \pm a. \end{aligned}$$

7. $$\begin{aligned} a^2x^2 &= b^2. \\ x^2 &= \frac{b^2}{a^2}. \\ x &= \pm \frac{b}{a}. \end{aligned}$$

8. $$\begin{aligned} x^2 + bx &= 2b^2. \\ x^2 + bx + \frac{b^2}{4} &= \frac{9b^2}{4}. \\ x + \frac{b}{2} &= \pm \frac{3b}{2}. \\ x &= b,\ -2b. \end{aligned}$$

9. $$\begin{aligned} x^2 - 12a &= 3ax - 4x. \\ x^2 + (4 - 3a)x &= 12a. \\ x^2 + (4 - 3a)x + \left(\frac{4 - 3a}{2}\right)^2 &= 12a + \frac{16 - 24a + 9a^2}{4} \\ &= \frac{16 + 24a + 9a^2}{4}. \\ x + \frac{4 - 3a}{2} &= \pm \frac{4 + 3a}{2}. \\ x &= 3a,\ -4. \end{aligned}$$

10. $$\begin{aligned} 2x^2 - 3hx - 2h^2 &= 0. \\ x^2 - \frac{3h}{2}x &= h^2. \\ x^2 - \frac{3h}{2}x + \left(-\frac{3h}{4}\right)^2 &= h^2 + \frac{9}{16}h^2 = \frac{25}{16}h^2. \\ x - \frac{3h}{4} &= \pm \frac{5h}{4}. \\ x &= 2h,\ -\frac{h}{2}. \end{aligned}$$

11.

$$10x^2 = 3a^2 - ax.$$

$$x^2 + \frac{a}{10}x = \frac{3a^2}{10}.$$

$$x^2 + \frac{a}{10}x + \frac{a^2}{400} = \frac{3a^2}{10} + \frac{a^2}{400} = \frac{121a^2}{400}.$$

$$x + \frac{a}{20} = \pm\frac{11a}{20}.$$

$$x = \frac{a}{2}, -\frac{3}{5}a.$$

12.

$$2x^2 - 5hx + 2h^2 = 0.$$

$$x^2 - \frac{5hx}{2} = -h^2.$$

$$x^2 - \frac{5hx}{2} + \left(-\frac{5h}{4}\right)^2 = -h^2 + \frac{25h^2}{16} = \frac{9h^2}{16}.$$

$$x - \frac{5h}{4} = \pm\frac{3h}{4}.$$

$$x = 2h, \frac{h}{2}.$$

13.

$$ax^2 - 5x - 2ax + 10 = 0.$$

$$x^2 - \frac{5+2a}{a}x = -\frac{10}{a}.$$

$$x^2 - \frac{5+2a}{a}x + \left(-\frac{5+2a}{2a}\right)^2 = -\frac{10}{a} + \frac{25+20a+4a^2}{4a^2} = \frac{25-20a+4a^2}{4a^2}.$$

$$x - \frac{5+2a}{2a} = \pm\frac{5-2a}{2a}.$$

$$x = \frac{5}{a}, 2.$$

14.

$$cx^2 + 2x = 3cx + 6.$$

$$x^2 + \frac{2-3c}{c}x = \frac{6}{c}.$$

$$x^2 + \frac{2-3c}{c}x + \left(\frac{2-3c}{2c}\right)^2 = \frac{6}{c} + \frac{4-12c+9c^2}{4c^2} = \frac{4+12c+9c^2}{4c^2}.$$

$$x + \frac{2-3c}{2c} = \pm\frac{2+3c}{2c}.$$

$$x = 3, -\frac{2}{c}.$$

15.

$$ax^2 + a = a^2x + x.$$
$$ax^2 - (a^2 + 1)x = -a.$$
$$x^2 - \frac{a^2+1}{a}x = -1.$$
$$x^2 - \frac{a^2+1}{a}x + \left(-\frac{a^2+1}{2a}\right)^2 = -1 + \frac{a^4 + 2a^2 + 1}{4a^2} = \frac{a^4 - 2a^2 + 1}{4a^2}.$$
$$x - \frac{a^2+1}{2a} = \pm\frac{a^2-1}{2a}.$$
$$x = a, \frac{1}{a}.$$

16.

$$cx^2 + ax = 2cx + 2a.$$
$$x^2 + \frac{a-2c}{c}x = \frac{2a}{c}.$$
$$x^2 + \frac{a-2c}{c}x + \left(\frac{a-2c}{2c}\right)^2 = \frac{2a}{c} + \frac{a^2 - 4ac + 4c^2}{4c^2} = \frac{a^2 + 4ac + 4c^2}{4c^2}.$$
$$x + \frac{a-2c}{2c} = \pm\frac{a+2c}{2c}.$$
$$x = 2, -\frac{a}{c}.$$

Page 280

1.

$$x^2 + y = 8, \quad (1)$$
$$2x + y = 5. \quad (2)$$

From (2), $y = 5 - 2x.$ (3)

Then in (1), $x^2 + 5 - 2x = 8.$

Whence $x = 3, -1,$

and in (3), $y = -1, 7.$

2.

$$x + y = 5, \quad (1)$$
$$x^2 + y^2 = 13. \quad (2)$$

From (1), $y = 5 - x.$ (3)

Then in (2), $x^2 + (5 - x)^2 = 13.$

Whence $x = 2, 3,$

and in (3), $y = 3, 2.$

3.

$$x^2 + y^2 = 25, \quad (1)$$
$$3x - 4y = 0. \quad (2)$$

From (2), $y = \frac{3}{4}x.$ (3)

Then in (1), $x^2 + \frac{9}{16}x^2 = 25.$
Whence $x = 4, -4,$
and in (3), $y = 3, -3.$

4. $x^2 + y^2 = 25,$ (1)
$x - 3y = 5.$ (2)
From (2), $x = 3y + 5.$ (3)
Then in (1), $(3y + 5)^2 + y^2 = 25.$
Whence $y = 0, -3,$
and in (3), $x = 5, -4.$

5. $xy = 12,$ (1)
$2x + y - 10 = 0.$ (2)
From (2), $y = 10 - 2x.$ (3)
Then in (1), $x(10 - 2x) = 12.$
Whence $x = 2, 3,$
and in (3), $y = 6, 4.$

6. $x^2 + 3xy = 25,$ (1)
$2x + y = 10.$ (2)
From (2), $y = 10 - 2x.$ (3)
Then in (1), $x^2 + 3x(10 - 2x) = 25.$
Whence $x = 5, 1,$
and in (3), $y = 0, 8.$

7. $4p + 5q = 6,$ (1)
$pq + 2 = 0.$ (2)
From (1), $p = \frac{6 - 5q}{4}.$ (3)
In (2), $\left(\frac{6 - 5q}{4}\right)q + 2 = 0.$
Whence $q = 2, -\frac{4}{5},$
and in (3), $p = -1, \frac{5}{2}.$

8. $4x - 3y = 30,$ (1)
$xy + 12 = 0.$ (2)
From (1), $y = \frac{4x - 30}{3}.$ (3)
In (2), $x\left(\frac{4x - 30}{3}\right) + 12 = 0.$
Whence $x = 6, \frac{3}{2},$
and in (3), $y = -2, -8.$

9. $$3A + 2B = 5, \qquad (1)$$
$$AB + 3 = 6A. \qquad (2)$$

From (1), $$B = \frac{5 - 3A}{2}. \qquad (3)$$

In (2), $$A\left(\frac{5 - 3A}{2}\right) + 3 = 6A.$$

Whence $$A = -3, \tfrac{2}{3},$$

and in (3), $$B = 7, \tfrac{3}{2}.$$

10. $$x^2 + xy = 5, \qquad (1)$$
$$x - y = 9. \qquad (2)$$

From (2), $$y = x - 9. \qquad (3)$$

In (1), $$x^2 + x(x - 9) = 5.$$

Whence $$x = 5, -\tfrac{1}{2},$$

and in (3), $$y = -4, -\tfrac{19}{2}.$$

11. $$x^2 + y^2 + y = 46, \qquad (1)$$
$$2x + y = 10. \qquad (2)$$

From (2), $$y = 10 - 2x. \qquad (3)$$

In (1), $$x^2 + (10 - 2x)^2 + (10 - 2x) = 46.$$

Whence $$x = 2, \tfrac{32}{5},$$

and in (3), $$y = 6, -\tfrac{14}{5}.$$

12. $$x^2 + 3xy + y^2 = 22, \qquad (1)$$
$$2x = y. \qquad (2)$$

Then (1) becomes $$x^2 + 6x^2 + 4x^2 = 22.$$

Whence $$x = \sqrt{2}, -\sqrt{2},$$

and in (2), $$y = 2\sqrt{2}, -2\sqrt{2}.$$

13. $$x^2 + y^2 - 4x - 2y = 20, \qquad (1)$$
$$3x - y = 10. \qquad (2)$$

From (2), $$y = 3x - 10. \qquad (3)$$

In (1), $$x^2 + (3x - 10)^2 - 4x - 2(3x - 10) = 20.$$

Whence $$x = 2, 5,$$

and in (3), $$y = -4, 5.$$

Page 281

1. $$x - y = 3,$$
$$x^2 - y^2 = 45.$$

Whence $$x = 9;\ y = 6.$$

2. $$x + y = 18,$$
$$x^2 + y^2 = 170.$$

Whence $$x = 7, 11;$$
$$y = 11, 7.$$

3. Let $x =$ the length of the field in rods,
and $y =$ the width in rods.
Then $x - y = 39,$
and $xy = 1600.$ (See Ex. 5, p. 276.)
Whence $x = 64,$ root -25 rejected;
$y = 25.$

4. Let x and y represent the sides in feet.
Then $4x - 4y = 32,$
and $x^2 - y^2 = 208.$
Whence $x = 17;$
$y = 9.$

5. Let $x =$ the length of the rectangle in feet,
and $y =$ the width in feet.
Then $2x + 2y = 92,$
and $xy = 504.$
Whence $x = 28, 18;$
$y = 18, 28.$

6. Let $y =$ the altitude of the triangle in inches,
and $x =$ the base in inches.
Then $x - y = 5,$
and $\frac{xy}{2} = \frac{25}{24} \cdot 144 = 150.$
Whence $y = 15,$ root -20 rejected;
$x = 20.$

7. As in Ex. 5, $2x + 2y = 3c,$
$$xy = \frac{c^2}{2}.$$
Whence $x = c, \frac{c}{2};$
$$y = \frac{c}{2}, c.$$

8. We are asked whether the system
$$x - y = 3, \quad (1)$$
$$x^2 + y^2 = 115 \quad (2)$$
has a solution in positive integers.

From (1), $y = x - 3.$
In (2), $x^2 + (x - 3)^2 = 115.$
Whence $x^2 - 3x - 53 = 0.$
Solving, $x = \frac{3}{2} \pm \frac{1}{2}\sqrt{221}.$
Since neither of these values is an integer, the answer is "No."

9. Let $x =$ the tens' digit,
and $y =$ the units' digit.
Then $(10x + y)(x + y) = 576,$
and $10x + y + 3(x + y) = 10y + x.$
Whence $x = 4$, root -4 rejected;
$y = 8.$ Ans. $= 48.$

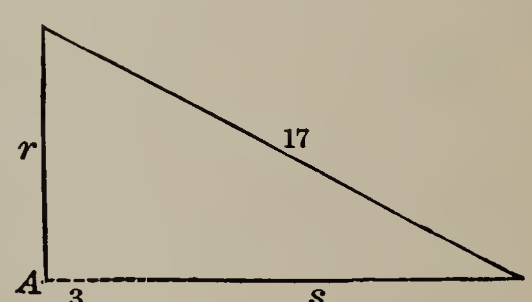
10. Let $p =$ the principal in dollars,
and $r =$ the rate per cent.

Then $\dfrac{pr}{100} = 48, \qquad (1)$

and $\dfrac{(p + 200)(r - 1)}{100} = 50. \qquad (2)$

From (1), $pr = 4800. \qquad (3)$
From (2), $pr + 200r - p = 5200. \qquad (4)$
(4) – (3), $200r - p = 400, \qquad (5)$
and $p = 200r - 400. \qquad (6)$
Then (3) becomes $(200r - 400)r = 4800.$
Whence $r^2 - 2r = 24,$
and $r = 6$, root -4 rejected.
Then in (6) $p = 800.$

11. Let $r =$ the rate in miles per hour of the slower,
and $s =$ the rate in miles per hour of the faster.
In one hour they travel r and s miles respectively.



Hence $r^2 + (s + 3)^2 = 17^2,$
$s = \frac{3}{2}r.$
Whence
$r = 8$, root $-\frac{140}{13}$ rejected;
$s = 12.$

Page 282

1. See p. 13.
2. $6 - 2 + 8 - 7 = 5.$
3. $8 + 12 \div 3 - 5 = 8 + 4 - 5 = 7.$
4. $8 \cdot 6 \div 4 = 12.$
5. $24 \div 2 \div 3 = 4.$
6. $36 \div 4 \cdot 3 - 8 + 2 = 27 - 8 + 2 = 21.$
7. $(19 - 3 \cdot 5)(8 - 5) \div (14 \div 7) = 4 \cdot 3 \div 2 = 6.$
8. $50 - 4(20 - 4 \cdot 2) \div 3 + 2 \cdot 7 = 50 - 4 \cdot 12 \div 3 + 14 = 50 - 16 + 14 = 48.$
9. $12 - 8 \div 2 + 10 \cdot 3 - 6 + 8 \cdot 2 = 12 - 4 + 30 - 6 + 16 = 48.$
10. $(28 \div 14 \cdot 24 \div 8 - 3 + 10) \times (30 \div 3 \div 5 - 2) = (6 - 3 + 10)(0) = 0.$
11. $(16 \div 32 \times 48 \div 8 - 8 + 3) + (42 \div 6 \cdot 7 - 42 - 6) \cdot 6$
$= (3 - 8 + 3) + (49 - 42 - 6)6 = -2 + 6 = 4.$

Page 283

1. See p. 23.
2. See p. 22
3. See p. 51.
4. See p. 33.
5. See pp. 33 and 35.
6. See p. 51.
7. $12 - 8 + 4 - 3 + 6 = 11$.
8. $4a + 3a - 7a + 6a - 2a = 4a$.

9.
$$\begin{array}{r} 5a - 3c + 6 \\ 2a - 6c + 11 \\ 4a - c - 9 \\ \hline 11a - 10c + 8 \end{array}$$

10.
$$\begin{array}{r} 3s - 4t + 6 \\ 6s - 7t - 8 \\ s + t + 10 \\ \hline 10s - 10t + 8 \end{array}$$

11.
$$\begin{array}{r} a^2 - 3a + 1 \\ -2a^2 - 7a + 6 \\ 3a^2 + 5a - 4 \\ \hline 2a^2 - 5a + 3 \end{array}$$

12. $6a - 4a = 2a$.

13. $8a^3 - 15a^3$
$= -7a^3$.

14.
$$\begin{array}{r} 4x + 3 \\ 8x + 6 \\ \hline -4x - 3 \end{array}$$

15.
$$\begin{array}{r} 7x^2 - 10 \\ 5x^2 + 20 \\ \hline 2x^2 - 30 \end{array}$$

16.
$$\begin{array}{r} 5x - 6 \\ 2x + 8 \\ \hline 3x - 14 \end{array}$$

17.
$$\begin{array}{r} x^2 - 5x + 6 \\ 2x^2 + 8x - 10 \\ \hline -x^2 - 13x + 16 \end{array}$$

18.
$$\begin{array}{r} a^2 - 4ac - 3c^2 \\ 4a^2 + 10ac - c^2 \\ \hline -3a^2 - 14ac - 2c^2 \end{array}$$

19.
$$\begin{array}{r} 3a - 2b - 6c \\ 4a + 6b - 7c - 2 \\ \hline -a - 8b + c + 2 \end{array}$$

20.
$$\begin{array}{r} 3a^2 - 2c^2 - 6ac \\ 5a^2 + 4c^2 - 3ac \\ \hline -2a^2 - 6c^2 - 3ac \end{array}$$

21.
$$\begin{array}{r} x - 3y^2 + z - 4ac + 7ax \\ 4x - y^2 + 9ac - 5ax + 8 \\ \hline -3x - 2y^2 + z - 13ac + 12ax - 8 \end{array}$$

22.
$$\begin{array}{r} a^3 - c + 3x - a^2m - 8ac \\ 4a^3 - 8x + 4a^2m - 10ac + m \\ \hline -3a^3 - c + 11x - 5a^2m + 2ac - m \end{array}$$

23.
$$\begin{array}{r} 6x^2 - 11x + 8 \\ 3x^2 - 5x + 2 \\ \hline 3x^2 - 6x + 6 \end{array}$$

24.
$$\begin{array}{r} 10x^2 + 8cx - 9c^2 \\ 4x^2 - 3cx + c^2 \\ \hline 6x^2 + 11cx - 10c^2 \end{array}$$

25.
$$\begin{array}{r} 7a^2 - 10ab + 6b^2 \\ 4a^2 - 2ab + b^2 \\ \hline 3a^2 - 8ab + 5b^2 \end{array}$$

26.
$$\begin{array}{r} 9x^2 - 10cx + 4 + c^2 \\ 8x^2 - 6cx + c^2 \\ \hline x^2 - 4cx + 4 \end{array}$$

27.
$$\begin{array}{r} t^2 - 4t - 9 \\ 3t^2 - 8t + 1 \\ \hline 4t^2 - 12t - 8 \\ -4t^2 + 7t - 1 \\ \hline 8t^2 - 19t - 7 \end{array}$$ (*Ans.*)

$$\begin{array}{r} 4t^2 + 3t - 6 \\ -8t^2 + 4t + 5 \\ \hline -4t^2 + 7t - 1 \end{array}$$

28.
$$\begin{array}{r} ax - ac - 4c^2 \\ -3ac + 4c^2 \\ \hline ax - 4ac \\ -5ax + 4ac - c^2 + a^2 \\ \hline 6ax - 8ac + c^2 - a^2 \end{array}$$ (*Ans.*)

$$\begin{array}{r} 4c^2 - 8ax + a^2 \\ -5c^2 + 3ax + 4ac \\ \hline -c^2 - 5ax + a^2 + 4ac \end{array}$$

Page 284

1. See index.

2. See p. 9.

3. See pp. 72–73.

4. See p. 75.

5. $b^4 + 4ab^3 - 6a^2b^2 - 4a^3b + a^4$.

6. $-5 + t - 2t^2 + t^3 - 3t^4$.

7. Yes. It makes for orderly arrangement and accuracy.

8. See p. 72.

9. $x^a \cdot x^b = x^{a+b}$.

10. See p. 76.

11. $(6x^2 - 3x + 4)3x = 18x^3 - 9x^2 + 12x$.

12.
$$\begin{array}{r}
3x - 5 \\
4x + 3 \\ \hline
12x^2 - 20x \phantom{{}-15} \\
+ 9x - 15 \\ \hline
12x^2 - 11x - 15
\end{array}$$

13.
$$\begin{array}{r}
2s - 3t \\
4s + 5t \\ \hline
8s^2 - 12st \phantom{{}-15t^2} \\
+ 10st - 15t^2 \\ \hline
8s^2 - 2st - 15t^2
\end{array}$$

14.
$$\begin{array}{r}
x^2 - x + 2 \\
3x - 4 \\ \hline
3x^3 - 3x^2 + 6x \phantom{{}-8} \\
- 4x^2 + 4x - 8 \\ \hline
3x^3 - 7x^2 + 10x - 8
\end{array}$$

15.
$$\begin{array}{r}
2x^2 - 5x + 3 \\
x^2 - 5x + 6 \\ \hline
2x^4 - 5x^3 + 3x^2 \phantom{{}-30x+18} \\
- 10x^3 + 25x^2 - 15x \phantom{{}+18} \\
+ 12x^2 - 30x + 18 \\ \hline
2x^4 - 15x^3 + 40x^2 - 45x + 18
\end{array}$$

16.
$$\begin{array}{r}
a^4 + 2a^2 - 4 \\
a^2 - 2a - 3 \\ \hline
a^6 + 2a^4 - 4a^2 \phantom{{}+8a+12} \\
- 2a^5 - 4a^3 + 8a \phantom{{}+12} \\
- 3a^4 - 6a^2 + 12 \\ \hline
a^6 - 2a^5 - a^4 - 4a^3 - 10a^2 + 8a + 12
\end{array}$$

17.
$$\begin{array}{r}
t^3 - 2t + 6 \\
t^4 - 3t^3 - t^2 \\ \hline
t^7 - 2t^5 + 6t^4 \phantom{{}-18t^3-6t^2} \\
- 3t^6 + 6t^4 - 18t^3 \phantom{{}-6t^2} \\
- t^5 + 2t^3 - 6t^2 \\ \hline
t^7 - 3t^6 - 3t^5 + 12t^4 - 16t^3 - 6t^2
\end{array}$$

18.
$$\begin{array}{r}
2s^2 - 3st + t^2 \\
s^2 + 5st - 4t^2 \\ \hline
2s^4 - 3s^3t + s^2t^2 \phantom{{}+12st^3-4t^4} \\
+ 10s^3t - 15s^2t^2 + 5st^3 \phantom{{}-4t^4} \\
- 8s^2t^2 + 12st^3 - 4t^4 \\ \hline
2s^4 + 7s^3t - 22s^2t^2 + 17st^3 - 4t^4
\end{array}$$

19.
$$\begin{array}{r}
a^2 - ac + c^2 \\
a^2 + ac + c^2 \\ \hline
a^4 - a^3c + a^2c^2 \phantom{{}-ac^3+c^4} \\
+ a^3c - a^2c^2 + ac^3 \phantom{{}+c^4} \\
+ a^2c^2 - ac^3 + c^4 \\ \hline
a^4 + a^2c^2 + c^4
\end{array}$$

20.
$$\begin{array}{r}
a^2 + 2ab + b^2 \\
a^2 - ab + b \\ \hline
a^4 + 2a^3b + a^2b^2 \phantom{{}-ab^3+a^2b+2ab^2+b^3} \\
- a^3b - 2a^2b^2 - ab^3 \phantom{{}+a^2b+2ab^2+b^3} \\
+ a^2b + 2ab^2 + b^3 \\ \hline
a^4 + a^3b - a^2b^2 - ab^3 + a^2b + 2ab^2 + b^3
\end{array}$$

21.
$$\begin{array}{rrrrr}
\multicolumn{5}{l}{4h^2 + 9hk + 6k^2} \\
\multicolumn{5}{l}{4h^2 - 9hk + 6k^2} \\ \hline
16h^4 & + 36h^3k & + 24h^2k^2 & & \\
 & - 36h^3k & - 81h^2k^2 & - 54hk^3 & \\
 & & + 24h^2k^2 & + 54hk^3 & + 36k^4 \\ \hline
16h^4 & & - 33h^2k^2 & & + 36k^4
\end{array}$$

22.
$$\begin{array}{rrrrr}
\multicolumn{5}{l}{2a^2 + 4ac^3 - 3c^6} \\
\multicolumn{5}{l}{2a^2 - 4ac^3 - 3c^6} \\ \hline
4a^4 & + 8a^3c^3 & - 6a^2c^6 & & \\
 & - 8a^3c^3 & - 16a^2c^6 & + 12ac^9 & \\
 & & - 6a^2c^6 & - 12ac^9 & + 9c^{12} \\ \hline
4a^4 & & - 28a^2c^6 & & + 9c^{12}
\end{array}$$

23.
$$\begin{array}{rrrrr}
\multicolumn{5}{l}{t^2 + t + 1} \\
\multicolumn{5}{l}{t^2 - t + 1} \\ \hline
t^4 & + t^3 & + t^2 & & \\
 & - t^3 & - t^2 & - t & \\
 & & + t^2 & + t & + 1 \\ \hline
t^4 & & + t^2 & & + 1 \\
t^4 & & - t^2 & & + 1
\end{array}$$

$$\begin{array}{rrrrr}
t^8 & + t^6 & + t^4 & & \\
 & - t^6 & - t^4 & - t^2 & \\
 & & + t^4 & + t^2 & + 1 \\ \hline
t^8 & & + t^4 & & + 1
\end{array}$$

24.
$$\begin{array}{rrrrrrrrrr}
\multicolumn{10}{l}{x^2 - 4x + 3xy + 12y + 9y^2 + 16} \\
\multicolumn{10}{l}{x - 3y + 4} \\ \hline
x^3 & - 4x^2 & + 3x^2y & + 12xy & + 9xy^2 & + 16x & & & & \\
 & & - 3x^2y & + 12xy & - 9xy^2 & & - 36y^2 & - 27y^3 & - 48y & \\
 & + 4x^2 & & + 12xy & & - 16x & + 36y^2 & & + 48y & + 64 \\ \hline
x^3 & & & + 36xy & & & & - 27y^3 & & + 64
\end{array}$$

25. $3x^2 + 2x + 5 = 3 \cdot 5^2 + 2 \cdot 5 + 5 = 75 + 10 + 5 = 90.$

26. $3t^2 - 4t + 8 = 3(-3)^2 - 4(-3) + 8 = +27 + 12 + 8 = 47.$

27. $9 - 8t + 5t^2 - 3t^3 = 9 - 16 + 20 - 24 = -11.$

28.
$$\begin{aligned}
2a^3 - 3a^2c + 4ac^2 - 3c^2 &= 2(3)^3 - 3 \cdot 3^2(-2) + 4(3)(-2)^2 - 3(-2)^2 \\
&= 54 + 54 + 48 - 12 \\
&= 144.
\end{aligned}$$

29.
$$\begin{aligned}
15(x - a) - 6(x + a) &= 3(5a - 3x). \\
15(2a - a) - 6(2a + a) &= 3(5a - 6a). \\
15a - 18a &= -3a. \\
-3a &= -3a. \text{ Yes.}
\end{aligned}$$

30.
$$\begin{aligned} ax(a+3)+a(10-a^2)&=x+3.\\ a(a-3)(a+3)+a(10-a^2)&=a-3+3.\\ a^3-9a+10a-a^3&=a.\\ a&=a.\quad \text{Yes.}\end{aligned}$$

31.
$$\frac{3}{x-1}-\frac{2x}{x+1}=\frac{3+5x-2x^2}{x^2-1}.$$
$$\frac{3}{0-1}-\frac{2\cdot 0}{0+1}=\frac{3+5\cdot 0-2\cdot 0}{0-1}.$$
$$-3=-3.\quad \text{Yes.}$$
$$\frac{3}{-2-1}-\frac{2\cdot -2}{-2+1}=\frac{3+5(-2)-2(-2)^2}{(-2)^2-1}.$$
$$-1-4=\frac{3-10-8}{4-1}.$$
$$-5=-5.\quad \text{Yes.}$$

Page 286

1. See pp. 87–88.
2. See pp. 71 and 88.
3. See p. 91.
4. See p. 93.

5.
$$\begin{array}{l|l} 2x^2-5x+3 & 2x-3\\ \cline{2-2} \underline{2x^2-3x} & x-1\\ \quad -2x & \\ \quad \underline{-2x+3} & \end{array}$$

6.
$$\begin{array}{l|l} 6x^2-13x+6 & -2x+3\\ \cline{2-2} \underline{6x^2-\ 9x} & -3x+2\\ \quad -4x & \\ \quad \underline{-4x+6} & \end{array}$$

7.
$$\begin{array}{l|l} 6a^2+23at-55t^2 & 3a-5t\\ \cline{2-2} \underline{6a^2-10at} & 2a+11t\\ \qquad 33at & \\ \qquad \underline{33at-55t^2} & \end{array}$$

8.
$$\begin{array}{l|l} 6a^3+\ 6a^2-26a-28 & 2a+4\\ \cline{2-2} \underline{6a^3+12a^2} & 3a^2-3a-7\\ \quad -\ 6a^2 & \\ \quad \underline{-\ 6a^2-12a} & \\ \qquad -14a & \\ \qquad \underline{-14a-28} & \end{array}$$

9.
$$\begin{array}{l|l} 6x^6-17x^5-\ 5x^4+25x^3 & -2x^3+5x^2\\ \cline{2-2} \underline{6x^6-15x^5} & -3x^3+x^2+5x\\ \quad -\ 2x^5 & \\ \quad \underline{-\ 2x^5+\ 5x^4} & \\ \qquad -10x^4 & \\ \qquad \underline{-10x^4+25x^3} & \end{array}$$

10.
$$\begin{array}{l|l} s^6 \qquad\qquad -7s^3 \qquad\qquad -8 & s^2+2s+4\\ \cline{2-2} \underline{s^6+2s^5+4s^4} & s^4-2s^3+s-2\\ \quad -2s^5-4s^4-7s^3 & \\ \quad \underline{-2s^5-4s^4-8s^3} & \\ \qquad\qquad s^3 & \\ \qquad\qquad \underline{s^3+2s^2+4s} & \\ \qquad\qquad\quad -2s^2-4s & \\ \qquad\qquad\quad \underline{-2s^2-4s-8} & \end{array}$$

11.
$$\begin{array}{l|l}
x^3 \qquad\qquad - 5a^2x + 2a^3 & x^2 + 2ax - a^2 \\ \cline{2-2}
\underline{x^3 + 2ax^2 - a^2x} & x - 2a \\ \cline{2-2}
\quad - 2ax^2 - 4a^2x & \\
\quad \underline{- 2ax^2 - 4a^2x + 2a^3} &
\end{array}$$

12.
$$\begin{array}{l|l}
2t^4 - 7t^3 - 12t^2 + 11t - 2 & -2t^2 - 3t + 1 \\ \cline{2-2}
\underline{2t^4 + 3t^3 - t^2} & -t^2 + 5t - 2 \\ \cline{2-2}
\quad -10t^3 - 11t^2 & \\
\quad \underline{-10t^3 - 15t^2 + 5t} & \\
\qquad\qquad 4t^2 + 6t & \\
\qquad\qquad \underline{4t^2 + 6t - 2} &
\end{array}$$

13.
$$\begin{array}{l|l}
32x^4 - 104x^3 + 92x^2 - 2x - 60 & -4x^2 + 6x + 5 \\ \cline{2-2}
\underline{32x^4 - 48x^3 - 40x^2} & -8x^2 + 14x - 12 \\ \cline{2-2}
\quad - 56x^3 + 132x^2 & \\
\quad \underline{- 56x^3 + 84x^2 + 70x} & \\
\qquad\qquad 48x^2 - 72x & \\
\qquad\qquad \underline{48x^2 - 72x - 60} &
\end{array}$$

14.
$$\begin{array}{l|l}
a^2 - 2ab \qquad + b^2 \qquad - 9x^2 & -a + b + 3x \\ \cline{2-2}
\underline{a^2 - ab - 3ax} & -a + b - 3x \\ \cline{2-2}
\quad - ab + 3ax + b^2 & \\
\quad \underline{- ab \qquad + b^2 + 3bx} & \\
\qquad\qquad 3ax \quad - 3bx & \\
\qquad\qquad \underline{3ax \quad - 3bx - 9x^2} &
\end{array}$$

15.
$$\begin{array}{l|l}
-4c^2 + 4bc \qquad - b^2 \qquad + 1 & 2c - b - 1 \\ \cline{2-2}
\underline{-4c^2 + 2bc + 2c} & -2c + b - 1 \\ \cline{2-2}
\qquad 2bc - 2c - b^2 & \\
\qquad \underline{2bc \qquad - b^2 - b} & \\
\qquad\quad -2c \qquad + b & \\
\qquad\quad \underline{-2c \qquad + b + 1} &
\end{array}$$

16.
$$\begin{array}{l|l}
-3a^9 \qquad\qquad + 3ab^4 - 18ab^2 + 27a & a^4 - b^2 + 3 \\ \cline{2-2}
\underline{-3a^9 + 3a^5b^2 - 9a^5} & -3a^5 - 3ab^2 + 9a \\ \cline{2-2}
\quad - 3a^5b^2 + 9a^5 & \\
\quad \underline{- 3a^5b^2 \qquad + 3ab^4 - 9ab^2} & \\
\qquad\qquad 9a^5 \qquad - 9ab^2 & \\
\qquad\qquad \underline{9a^5 \qquad - 9ab^2 + 27a} &
\end{array}$$

17.

$$\begin{array}{l|l}
3x^5 - x^4 - 8x^3 + 9x^2 - 5x + 2 & x^2 - 2x + 1 \\ \cline{2-2}
\underline{3x^5 - 6x^4 + 3x^3} & 3x^3 + 5x^2 - x + 2 \\
\quad 5x^4 - 11x^3 & \\
\quad \underline{5x^4 - 10x^3 + 5x^2} & \\
\qquad - x^3 + 4x^2 & \\
\qquad \underline{- x^3 + 2x^2 - x} & \\
\qquad\quad 2x^2 - 4x & \\
\qquad\quad \underline{2x^2 - 4x + 2} &
\end{array}$$

18.

$$\begin{array}{l|l}
b^2 - 6b - 4ab + 12a + 4a^2 + 9 & b - 3 - 2a \\ \cline{2-2}
\underline{b^2 - 3b - 2ab} & b - 3 - 2a \\
\quad - 3b - 2ab & \\
\quad \underline{- 3b + 6a + 9} & \\
\qquad - 2ab + 6a + 4a^2 & \\
\qquad \underline{- 2ab + 6a + 4a^2} &
\end{array}$$

19.

$$\begin{array}{l|l}
x^3 - 30xy + 8y^3 + 125 & x + 2y + 5 \\ \cline{2-2}
\underline{x^3 + 2x^2y + 5x^2} & x^2 - 2xy - 5x + 4y^2 - 10y + 25 \\
\quad - 2x^2y - 5x^2 - 30xy & \\
\quad \underline{- 2x^2y - 4xy^2 - 10xy} & \\
\qquad - 5x^2 + 4xy^2 - 20xy & \\
\qquad \underline{- 5x^2 - 10xy - 25x} & \\
\qquad\quad 4xy^2 - 10xy + 25x & \\
\qquad\quad \underline{4xy^2 + 8y^3 + 20y^2} & \\
\qquad\qquad - 10xy + 25x - 20y^2 & \\
\qquad\qquad \underline{- 10xy - 20y^2 - 50y} & \\
\qquad\qquad\quad 25x + 50y & \\
\qquad\qquad\quad \underline{25x + 50y + 125} &
\end{array}$$

20.
$$
\begin{array}{l|l}
x^3 - 3xyz + y^3 + z^3 & x+y+z \\
\hline
\underline{x^3 + x^2y + x^2z} & x^2 - xy - xz + z^2 - yz + y^2 \\
\quad - x^2y - x^2z - 3xyz & \\
\quad \underline{- x^2y - xyz - xy^2} & \\
\qquad - x^2z - 2xyz + xy^2 & \\
\qquad \underline{- x^2z - xz^2 - xyz} & \\
\qquad\quad xz^2 - xyz + xy^2 & \\
\qquad\quad \underline{xz^2 + yz^2 + z^3} & \\
\qquad\quad - xyz + xy^2 - yz^2 + y^3 & \\
\qquad\quad \underline{- xyz - y^2z - yz^2} & \\
\qquad\qquad xy^2 + y^2z + y^3 & \\
\qquad\qquad \underline{xy^2 + y^2z + y^3} &
\end{array}
$$

Page 287

1. $a + (a - b) - (a - 3b) = a + a - b - a + 3b = a + 2b.$
2. $a - (a - c) + (2c - 3a) = a - a + c + 2c - 3a = 3c - 3a.$
3. $3b - [(a - b) - (c - a)] = 3b - (a - b) + (c - a) = 3b - a + b + c - a = -2a + 4b + c.$
4. $2c - 2(a - c) + 3(c - a) = 2c - 2a + 2c + 3c - 3a = 7c - 5a.$
5. $x - y + 3(x - y) - 4(2x - y) = x - y + 3x - 3y - 8x + 4y = -4x.$
6. $4x - a + [-(3c - x) - (2a - 3x)] = 4x - a - (3c - x) - (2a - 3x) = 4x - a - 3c + x - 2a + 3x$
$= 8x - 3a - 3c.$
7. $a - [-(a - 3) + (3c - 2a) - 5a] + 6c = a + (a - 3) - (3c - 2a) + 5a + 6c$
$= a + a - 3 - 3c + 2a + 5a + 6c = 9a + 3c - 3.$
8. $x - 3 - (a - 2x) - [2(a - x - 5) - 3(6 - 2a)] = x - 3 - a + 2x - [2a - 2x - 10 - 18 + 6a]$
$= x - 3 - a + 2x - 8a + 2x + 28$
$= 5x - 9a + 25.$

9. $(2t^2 - 3t) - 2t(3 + t) = 2t^2 - 3t - 6t - 2t^2 = -9t.$

10. $(a + b)a - (a - b)b + 3ab = a^2 + ab - ab + b^2 + 3ab = a^2 + 3ab + b^2.$

11. $4x^2 - 3a^2 - (a - 2x)(3x - a) = 4x^2 - 3a^2 - (-a^2 + 5ax - 6x^2)$
$= 4x^2 - 3a^2 + a^2 - 5ax + 6x^2$
$= 10x^2 - 2a^2 - 5ax.$

12. $(x - 4)(x - 3) - (x - 3)(x + 2) = x^2 - 7x + 12 - (x^2 - x - 6)$
$= x^2 - 7x + 12 - x^2 + x + 6$
$= 18 - 6x.$

13. $3a(a - b) - (a + b)(a - b) = 3a^2 - 3ab - (a^2 - b^2)$
$= 3a^2 - 3ab - a^2 + b^2$
$= 2a^2 - 3ab + b^2.$

14. $(x - 3)(x - 4) - (x - 5)(x + 3) = x^2 - 7x + 12 - (x^2 - 2x - 15)$
$= x^2 - 7x + 12 - x^2 + 2x + 15$
$= 27 - 5x.$

15. $6x - [-(a - c) + (3c - 4a)] = 6x + (a - c) - (3c - 4a)$
$= 6x + a - c - 3c + 4a$
$= 6x + 5a - 4c.$

16. $7c - [(3c - 4) - 6 - (4x - 3a - c)] = 7c - (3c - 4) + 6 + (4x - 3a - c)$
$= 7c - 3c + 4 + 6 + 4x - 3a - c$
$= 3c + 4x - 3a + 10.$

17. $4x - 2(x - 3) - 3[x - 3(4 - 2x) + 8]$
$= 4x - 2x + 6 - 3x + 9(4 - 2x) - 24$
$= 4x - 2x + 6 - 3x + 36 - 18x - 24$
$= 18 - 19x.$

18. $3x - 2[1 - 3(2x - 3 - a) - 5\{a - (3x - 2a) - 4\}]$
$= 3x - 2 + 6(2x - 3 - a) + 10\{a - (3x - 2a) - 4\}$
$= 3x - 2 + 12x - 18 - 6a + 10a - 30x + 20a - 40$
$= -15x + 24a - 60.$

19. (*a*) $x + (-y - 3).$
(*b*) $x - (y + 3).$

20. (*a*) $2x + (y - 8).$
(*b*) $2x - (8 - y).$

21. (*a*) $7x + (10 - 4y).$
(*b*) $7x - (4y - 10).$

22. (*a*) $x + (y + z).$
(*b*) $x - (-y - z).$

23. (*a*) $x + (4z - 3y).$
(*b*) $x - (3y - 4z).$

24. (*a*) $3x + (4y - 7z).$
(*b*) $3x - (7z - 4y).$

25. (*a*) $2x + (5y - z).$
(*b*) $2x - (z - 5y).$

26. (*a*) $3x + (6z - 8y).$
(*b*) $3x - (8y - 6z).$

27. (*a*) $x + (-y - z).$
(*b*) $x - (y + z).$

Page 291

1. $$5x + 1 = 2x + 7.$$
$$5x - 2x = 7 - 1.$$
$$3x = 6.$$
$$x = 2.$$

2. $$1 + x + 5x + 17 = 0.$$
$$6x = -18.$$
$$x = -3.$$

3. $$15x = 3(4x - 5).$$
$$15x = 12x - 15.$$
$$15x - 12x = -15.$$
$$3x = -15.$$
$$x = -5.$$

4. $$2(x + 2) - (x + 5) = 0.$$
$$2x + 4 - x - 5 = 0.$$
$$x = 1.$$

5. $$3(2x - 7) - 2(5 - 2x) + 1 = 0.$$
$$6x - 21 - 10 + 4x + 1 = 0.$$
$$10x = 30.$$
$$x = 3.$$

6. $$3(x + 2) - 5(2x - 3) = 0.$$
$$3x + 6 - 10x + 15 = 0.$$
$$-7x = -21.$$
$$x = 3.$$

7. $$4(4x - 1) + 3 - 2(3 + x) = 0.$$
$$16x - 4 + 3 - 6 - 2x = 0.$$
$$14x = 7.$$
$$x = \tfrac{1}{2}.$$

8. $$4(x - 2) + 3(2 - x) - 3x = 6(x + 1).$$
$$4x - 8 + 6 - 3x - 3x = 6x + 6.$$
$$-2x - 6x = 6 + 2.$$
$$-8x = 8.$$
$$x = -1.$$

9. $$(x + 1)(x - 2) = x^2 + 3.$$
$$x^2 - x - 2 = x^2 + 3.$$
$$-x = 5.$$
$$x = -5.$$

10. $$(4x - 3)(2x - 5) - (4x - 7)(2x - 1) = 0.$$
$$8x^2 - 26x + 15 - 8x^2 + 18x - 7 = 0.$$
$$-8x = -8.$$
$$x = 1.$$

11. $$(x + 2)^2 + 48 = (x - 4)^2.$$
$$x^2 + 4x + 4 + 48 = x^2 - 8x + 16.$$
$$4x + 8x = 16 - 52.$$
$$12x = -36.$$
$$x = -3.$$

12. $$(x + 4)^2 - (2 - x)^2 = 84.$$
$$x^2 + 8x + 16 - 4 + 4x - x^2 = 84.$$
$$12x = 72.$$
$$x = 6.$$

13. $(x+3)(6x+5)-(2x+4)(3x-8)=38.$
$$6x^2+23x+15-6x^2+4x+32=38.$$
$$27x=38-47.$$
$$27x=-9.$$
$$x=-\tfrac{1}{3}.$$

14. $(y-4)(6-y)+(y+2)(y-4)=0.$
$$-y^2+10y-24+y^2-2y-8=0.$$
$$8y=32.$$
$$y=4.$$

15. $(x+4)(x+3)=(x+2)(x+1)+42.$
$$x^2+7x+12=x^2+3x+2+42.$$
$$4x=32.$$
$$x=8.$$

16. $5-(7-3)=1.$

17. See p. 57.

18. See p. 55.

19. $x-2a=4a-x.$
$$2x=6a.$$
$$x=3a.$$

20. $8s-x=x-4r.$
$$-2x=-8s-4r.$$
$$x=4s+2r.$$

21. $ax-2ab=4ab-ax.$
$$2ax=6ab.$$
$$x=3b.$$

22. $m(x-5a)-3m(a-x)=0.$
$$mx-5am-3am+3mx=0.$$
$$4mx=8am.$$
$$x=2a.$$

23. $x(b+a)=ab+a^2.$
$$x(b+a)=a(b+a).$$
$$x=a.$$

24. $x(a-c)=a^2-c^2.$
$$x=a+c.$$

25. $ax-2cx=a^2-4c^2.$
$$(a-2c)x=(a-2c)(a+2c).$$
$$x=a+2c.$$

26. $2ax-4a^2+4a=1+x.$
$$2ax-x=4a^2-4a+1.$$
$$(2a-1)x=(2a-1)^2.$$
$$x=2a-1.$$

27. $ax+1-a^3=x.$
$$ax-x=a^3-1.$$
$$(a-1)x=(a-1)(a^2+a+1).$$
$$x=a^2+a+1.$$

28. $(x+a)(x+b)=x^2+2a^2+3ab.$
$$x^2+(a+b)x+ab=x^2+2a^2+3ab.$$
$$(a+b)x=2a^2+2ab.$$
$$x=2a.$$

29. $(a-c)(x-m)=(m-c)(x-a).$
$$ax-cx-am+cm=mx-cx-am+ac.$$
$$ax-mx=ac-cm.$$
$$(a-m)x=c(a-m).$$
$$x=c.$$

30.
$$a^2x - a^3 + 3ax = 3 - 10a + x.$$
$$(a^2 + 3a - 1)x = a^3 - 10a + 3.$$
$$x = a - 3.$$

31.
$$c(1 + x) + m(x + 1) - x(m + c + 1) = 0.$$
$$c + cx + mx + m - mx - cx - x = 0.$$
$$-x = -c - m.$$
$$x = c + m.$$

32.
$$x - 4 = 50 - 5x;\ 2x + 47 = 17.$$

Page 293

1. Let $x =$ the less number.

Then $5x =$ the greater number.
$$x + 5x = 102.$$
$$6x = 102.$$
$$x = 17.$$
$$5x = 85.$$

2. Let $x =$ one of the numbers.

Then $54 - x =$ the other number.
$$10x = 2(54 - x).$$
$$10x = 108 - 2x.$$
$$12x = 108.$$
$$x = 9.$$
$$54 - x = 45.$$

3. Let $x =$ the least number.

Then $x + 2 =$ the next consecutive even number,

and $x + 4 =$ the following consecutive even number.
$$x + x + 2 + x + 4 = 1044.$$
$$3x = 1038.$$
$$x = 346.$$
$$x + 2 = 348.$$
$$x + 4 = 350.$$

4. Let $x =$ the least number.

Then $x + 1, x + 2, x + 3 =$ the three following consecutive numbers.
$$x(x + 2) + 201 = (x + 1)(x + 3).$$
$$x^2 + 2x + 201 = x^2 + 4x + 3.$$
$$2x = 198.$$
$$x = 99,\ x + 1 = 100,\ x + 2 = 101,\ x + 3 = 102.$$

5. Let $x =$ the number of years in age of younger pupil.
Then $x + 4 =$ the number of years in age of older pupil.

$$(x + 4) - 8 = 2(x - 8).$$
$$x - 4 = 2x - 16.$$
$$x = 12.$$
$$x + 4 = 16.$$

6. Let $x =$ the number of years required.

$$48 + x = 2(18 + x).$$
$$48 + x = 36 + 2x.$$
$$x = 12.$$

7. Let $x =$ the number of years in age of younger man.
Then $3x =$ the number of years in age of older man.

$$3x - 15 = 6(x - 15).$$
$$3x - 15 = 6x - 90.$$
$$3x = 75.$$
$$x = 25.$$
$$3x = 75.$$

8. Let $x =$ the number of years in the age of B.
Then $2x =$ the number of years in the age of A,
and $x - 8 =$ the number of years in the age of D,
and $3(x - 8) =$ the number of years in the age of C.

$$x + 10 + 2x + 10 + 3(x - 8) + 10 + x - 8 + 10 = 113.$$
$$7x = 105.$$

$x = 15$, or B's age in years.
$2x = 30$, or A's age in years.
$x - 8 = 7$, or D's age in years.
$3(x - 8) = 21$, or C's age in years.

9. Let $x =$ the number of feet in the side of the square.

$$(x + 12)^2 - x^2 = 816.$$
$$x^2 + 24x + 144 - x^2 = 816.$$
$$24x = 672.$$
$$x = 28.$$

10. Let $x =$ the number of inches in the width of the picture.
Then $x + 4 =$ the number of inches in the length of the picture.

$$(x + 4)(x + 8) = x(x + 4) + 192.$$

$$x^2 + 12x + 32 = x^2 + 4x + 192.$$
$$8x = 160.$$
$$x = 20.$$
$$x + 4 = 24.$$

11. Let $x =$ the number of inches in the width of the supposed rectangle.

Then $2x + 5 =$ the number of inches in the length of the supposed rectangle.

Then $2x + 2(2x + 5)$ should equal 112 inches.

Equating, $2x + 2(2x + 5) = 112.$
$$6x = 102.$$
$$x = 17.$$
$2x + 5 = 39.$ Such a rectangle is possible.

12. Let $x =$ the number of dollars.

Then $x + 6 =$ the number of quarters,

and $2(x + 6) =$ the number of dimes.
$$100x + 25(x + 6) + 20(x + 6) = 1575.$$
$$145x = 1305.$$
$$x = 9.$$
$$x + 6 = 15.$$
$$2(x + 6) = 30.$$

13. Let $x =$ the number of dimes.

Then $x + 7 =$ the number of quarters,

and $5x - 3 =$ the number of nickels.
$$x + x + 7 + 5x - 3 = 109.$$
$$7x = 105.$$
$$x = 15,\ x + 7 = 22,\ 5x - 3 = 72.$$

Amount in dimes = \$1.50.
Amount in quarters = \$5.50.
Amount in nickels = \$3.60.
Total amount = \$10.60.

14. Let $x =$ the tens' digit.

Then $14 - x =$ the units' digit,

and $10x + (14 - x) =$ the number.
$$10x + (14 - x) = 10(14 - x) + x + 36.$$
$$10x + 14 - x = 140 - 10x + x + 36.$$
$$18x = 162.$$
$$x = 9.$$
$14 - x = 5.$ The number $= 95.$

15. Let $x =$ the hundreds' digit.

Then $x + 2 =$ the tens' digit,

and $x + 4 =$ the units' digit.

$$x + x + 2 + x + 4 = 15.$$
$$3x = 9.$$
$$x = 3.$$
$$x + 2 = 5.$$
$x + 4 = 7.$ The number is 357.

16. Let $r =$ the number of inches in the radius of smaller circle.

Then $r + 14 =$ the number of inches in the radius of larger circle.

$$\pi r^2 + 1232 = \pi(r + 14)^2.$$
$$\pi r^2 + 1232 = \pi r^2 + 28\pi r + 196\pi.$$
$$28\pi r = 1232 - 196\pi.$$
$$88r = 616.$$
$$r = 7.$$

17. Let $x =$ the number of inches increase in the circumference.

$$2\pi \cdot 8 + x = 2\pi \cdot (8 + 2).$$
$$x = 20\pi - 16\pi = 4\pi = \tfrac{88}{7}.$$

Let $y =$ the number of square inches increase in area.

$$\pi \cdot 8^2 + y = \pi(8 + 2)^2.$$
$$y = 100\pi - 64\pi = 36\pi = \tfrac{792}{7}.$$

18. The first equation becomes

$$2\pi R + x = 2\pi(R + 2).$$

Whence $x = 4\pi.$

The second equation becomes

$$\pi R^2 + y = \pi(R + 2)^2.$$

Whence $y = \pi(4R + 4) = 2\pi(2R + 2).$

The solution leads one to observe that whatever number is added to the length of the radius increases the circumference by 2π times that number, and that it increases the area by π times the sum of twice the radius and the number, all multiplied by that number.

19. Let $R =$ the number of feet in the earth's radius,

and let $x =$ the number of feet distant the hoop will be.

$$2\pi R + 1 = 2\pi(R + x).$$
$$2\pi x = 1.$$
$$x = \frac{1}{2\pi} = \frac{7}{44} \text{ feet, or 1.9 inches.}$$

20. Here $\frac{3}{4}$ = the number of feet in the radius of the sphere.

Let x = the number of feet distant the hoop will be.

$$2\pi(\tfrac{3}{4}) + 1 = 2\pi(\tfrac{3}{4} + x).$$

$$x = \frac{1}{2\pi} = \frac{7}{44} \text{ feet, or } 1.9 \text{ inches.}$$

21. Let x = the number of feet in the height of the pole.

Then $x + 5$ = the number of feet in the length of the rope.

$$(x + 5)^2 = x^2 + 25^2.$$

$$10x = 600.$$

$$x = 60.$$

Page 296

1. $2(a + b) + x(a + b) = (a + b)(2 + x)$.

2. $3(b + 5) + a(b + 5) = (b + 5)(3 + a)$.

3. $a(c - d) - b(c - d) = (c - d)(a - b)$.

4. $2(x - y) - x(x - y) = (x - y)(2 - x)$.

5. $h(m + 3n) - 2k(m + 3n) = (m + 3n)(h - 2k)$.

6. $x(r - s) + y(s - r) = (r - s)(x - y)$.

7. $5r(3m - 2n) - 2s(2n - 3m) = (3m - 2n)(5r + 2s)$.

8. $ac + 2cx + 3ay + 6xy = c(a + 2x) + 3y(a + 2x) = (a + 2x)(c + 3y)$.

9. $cd - 3cf + 2d - 6f = c(d - 3f) + 2(d - 3f) = (d - 3f)(c + 2)$.

10. $r^2s + 2rs - 3r^2t - 6rt = r[(rs + 2s) - (3rt + 6t)]$
$= r[s(r + 2) - 3t(r + 2)] = r(r + 2)(s - 3t)$.

11. $x^3 - 3xy^2 - 3x^2y + 9y^3 = x(x^2 - 3y^2) - 3y(x^2 - 3y^2)$
$= (x^2 - 3y^2)(x - 3y)$.

12. $ax + ay + bx + by + cx + cy = a(x + y) + b(x + y) + c(x + y)$
$= (x + y)(a + b + c)$.

13. $mr - 2r + ms - 2s + mt - 2t = r(m - 2) + s(m - 2) + t(m - 2)$
$= (m - 2)(r + s + t)$.

14. See p. 118.

Page 297

1. $a^2 - x^2 = (a - x)(a + x)$.

2. $x^2 - 9 = (x - 3)(x + 3)$.

3. $16 - x^2 = (4 - x)(4 + x)$.

4. $a^2 - 16b^2 = (a - 4b)(a + 4b)$.

5. $16x^2 - 25y^2 = (4x - 5y)(4x + 5y)$.

6. $25a^2 - 36b^2c^2 = (5a - 6bc)(5a + 6bc)$.

7. $a^4 - 25b^4 = (a^2 - 5b^2)(a^2 + 5b^2)$.

8. $x^4y^4 - 9 = (x^2y^2 - 3)(x^2y^2 + 3)$.

9. $a^4 - 16 = (a^2 + 4)(a^2 - 4) = (a^2 + 4)(a - 2)(a + 2)$.

10. $a^4 - b^4 = (a^2 + b^2)(a^2 - b^2) = (a^2 + b^2)(a + b)(a - b)$.

11. $x^4 - 1 = (x^2 + 1)(x^2 - 1) = (x^2 + 1)(x + 1)(x - 1)$.

12. $x^8 - 1 = (x^4 + 1)(x^4 - 1)$
$= (x^4 + 1)(x^2 + 1)(x^2 - 1)$
$= (x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$.

13. $16 - a^4 = (4 + a^2)(4 - a^2) = (4 + a^2)(2 + a)(2 - a)$.

14. $625 - x^4 = (25 + x^2)(25 - x^2) = (25 + x^2)(5 + x)(5 - x)$.

15. $81 - c^4 = (9 + c^2)(9 - c^2) = (9 + c^2)(3 + c)(3 - c)$.

16. See p. 124.

17. $a^4 - x^8 = (a^2 + x^4)(a + x^2)(a - x^2)$.

18. $(a - 2)^2 - c^2 = (a - 2 + c)(a - 2 - c)$.

19. $16(x - y)^2 - z^4 = (4x - 4y + z^2)(4x - 4y - z^2)$.

20. $9(x - 3y)^2 - 16z^4 = (3x - 9y + 4z^2)(3x - 9y - 4z^2)$.

21. $a^2 - (b + c)^2 = (a + b + c)(a - b - c)$.

22. $4r^2s^2 - (r - s)^4 = (2rs + r^2 - 2rs + s^2)(2rs - r^2 + 2rs - s^2)$
$= (r^2 + s^2)(4rs - r^2 - s^2)$.

23. $9m^2 - 4(n + 3)^4 = (3m + 2n^2 + 12n + 18)(3m - 2n^2 - 12n - 18)$.

24. $(a - c)^2 - (d + e)^2 = (a - c + d + e)(a - c - d - e)$.

25. $a(r + 2s)^2 - a(x - y)^2 = a(r + 2s + x - y)(r + 2s - x + y)$.

26. $4a(a - x)^2 - 9a(c - 2d)^4$
$= a(2a - 2x + 3c^2 - 12cd + 12d^2)(2a - 2x - 3c^2 + 12cd - 12d^2)$.

27. $m^2 + 2mn + n^2 - (x^2 - 2xy + y^2) = (m + n)^2 - (x - y)^2$
$= (m + n + x - y)(m + n - x + y)$.

28. $x^2 - 2xy + y^2 - a^2 - 2ab - b^2 = (x - y)^2 - (a + b)^2$
$= (x - y + a + b)(x - y - a - b)$.

29. $a^2 - 22a + 121 - b^2 + 20bc - 100c^2 = (a - 11)^2 - (b - 10c)^2$
$= (a - 11 + b - 10c)(a - 11 - b + 10c)$.

30. $4c^2 - a^2 - 2ab - b^2 = 4c^2 - (a + b)^2 = (2c + a + b)(2c - a - b)$.

31. $49x^4 - 49y^2 - 14y - 1 = 49x^4 - (7y + 1)^2$
$= (7x^2 + 7y + 1)(7x^2 - 7y - 1)$.

32. $a^2 - b^2 - (a - b) = (a + b)(a - b) - (a - b) = (a - b)(a + b - 1)$.

33. $r^2 - rs - (r^2 - s^2) = r(r - s) - (r + s)(r - s)$
$= (r - s)(r - r - s)$
$= (r - s)(-s)$
$= s(s - r)$.

34. $m + n - m^2 + n^2 = (m + n) - (m^2 - n^2)$
$= (m + n)(1 - m + n)$.

35. $r^2 - r - 3s - 9s^2 = (r^2 - 9s^2) - (r + 3s)$
$= (r + 3s)(r - 3s - 1)$.

Page 298

1. $x^2 + 5x + 6 = (x + 2)(x + 3).$
2. $x^2 + 7x + 12 = (x + 3)(x + 4).$
3. $d^2 + 11d + 18 = (d + 9)(d + 2).$
4. $c^2 - 2c - 35 = (c - 7)(c + 5).$
5. $r^2 - r - 90 = (r - 10)(r + 9).$
6. $9 - 10x + x^2 = (9 - x)(1 - x).$
7. $r^2 - 4rs + 3s^2 = (r - 3s)(r - s).$
8. $m^2 - 7mn + 10n^2 = (m - 5n)(m - 2n).$
9. $1 + 2n - 24n^2 = (1 + 6n)(1 - 4n).$
10. $(a + b)^2 - 2(a + b) - 8 = (a + b - 4)(a + b + 2).$
11. $(x - y)^2 + 4(x - y) + 3 = (x - y + 3)(x - y + 1).$
12. $a^{2n} + 12a^n + 35 = (a^n + 7)(a^n + 5).$
13. $c^{2a} - 7c^a - 18 = (c^a - 9)(c^a + 2).$
14. See p. 128.
15. $2x^2 + 5x + 2 = (x + 2)(2x + 1).$
16. $4a^2 + 7a + 3 = (4a + 3)(a + 1).$
17. $3x^2 + 13x + 12 = (3x + 4)(x + 3).$
18. $3x^2 + 17x + 10 = (3x + 2)(x + 5).$
19. $4x^2 + 8x + 3 = (2x + 3)(2x + 1).$
20. $2b^2 - 5b + 2 = (2b - 1)(b - 2).$
21. $6c^2 + 7c + 2 = (3c + 2)(2c + 1).$
22. $3x^2 - 11x + 8 = (3x - 8)(x - 1).$
23. $10x^2 - 29x + 10 = (5x - 2)(2x - 5).$
24. $12x^2 - 11xy + 2y^2 = (3x - 2y)(4x - y).$
25. $2a^2 + 3a - 2 = (2a - 1)(a + 2).$
26. $2a^2 - a - 15 = (2a + 5)(a - 3).$
27. $9a^2 + 3a - 2 = (3a + 2)(3a - 1).$
28. $12r^2 + 10r - 12 = 2(3r - 2)(2r + 3).$
29. $6a^4 - 11a^3b - 7a^2b^2 = a^2(3a - 7b)(2a + b).$
30. $6x^3 + 10x^2y - 4xy^2 = 2x(3x - y)(x + 2y).$
31. $2x^2 + 5xy - 12y^2 = (2x - 3y)(x + 4y).$
32. $6a^{2n} - 7a^n + 2 = (3a^n - 2)(2a^n - 1).$
33. $3a^{2n} - 10a^n - 8 = (3a^n + 2)(a^n - 4).$
34. $10a^{2n} - a^n - 3 = (5a^n - 3)(2a^n + 1).$

Page 299

1. $x^4 + x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4 - x^2y^2 = (x^2 + y^2)^2 - (xy)^2$
   $= (x^2 + y^2 + xy)(x^2 + y^2 - xy).$
2. $c^4 + c^2d^2 + d^4 = c^4 + 2c^2d^2 + d^4 - c^2d^2 = (c^2 + d^2 + cd)(c^2 + d^2 - cd).$
3. $a^4 + a^2b^4 + b^8 = a^4 + 2a^2b^4 + b^8 - a^2b^4$
   $= (a^2 + b^4 + ab^2)(a^2 + b^4 - ab^2).$

4. $a^4 + 3a^2b^2 + 4b^4 = a^4 + 4a^2b^2 + 4b^4 - a^2b^2$
$= (a^2 + 2b^2 + ab)(a^2 + 2b^2 - ab)$.

5. $m^4 + m^2 + 1 = m^4 + 2m^2 + 1 - m^2 = (m^2 + 1 - m)(m^2 + 1 + m)$.

6. $x^4 + 5x^2 + 9 = x^4 + 6x^2 + 9 - x^2 = (x^2 + 3 + x)(x^2 + 3 - x)$.

7. $c^4 + 4c^2 + 16 = c^4 + 8c^2 + 16 - 4c^2 = (c^2 + 4 + 2c)(c^2 + 4 - 2c)$.

8. $25a^4 - 19a^2 + 1 = 25a^4 - 10a^2 + 1 - 9a^2$
$= (5a^2 - 1 + 3a)(5a^2 - 1 - 3a)$.

9. $25x^4 - 11x^2 + 1 = 25x^4 - 10x^2 + 1 - x^2$
$= (5x^2 - 1 + x)(5x^2 - 1 - x)$.

10. $4r^4 - 44r^2s^2 + 49s^4 = 4r^4 - 28r^2s^2 + 49s^4 - 16r^2s^2$
$= (2r^2 - 7s^2 + 4rs)(2r^2 - 7s^2 - 4rs)$.

11. $25x^4 - 19x^2 + 9 = 25x^4 + 30x^2 + 9 - 49x^2$
$= (5x^2 + 3 + 7x)(5x^2 + 3 - 7x)$.

12. $9ax^8 - 28ax^4y^4 + 4ay^8 = a(9x^8 - 12x^4y^4 + 4y^8 - 16x^4y^4)$
$= a(3x^4 - 2y^4 + 4x^2y^2)(3x^4 - 2y^4 - 4x^2y^2)$.

13. $4a^8x + 3a^4b^4x + 9b^8x = x(4a^8 + 12a^4b^4 + 9b^8 - 9a^4b^4)$
$= x(2a^4 + 3b^4 + 3a^2b^2)(2a^4 + 3b^4 - 3a^2b^2)$.

14. $4a^4 + 1 = 4a^4 + 4a^2 + 1 - 4a^2 = (2a^2 + 1 - 2a)(2a^2 + 1 + 2a)$.

15. $c^4 + 4d^4 = c^4 + 4c^2d^2 + 4d^4 - 4c^2d^2$
$= (c^2 + 2d^2 + 2cd)(c^2 + 2d^2 - 2cd)$.

16. $64a^4x^2 + x^2 = x^2(64a^4 + 16a^2 + 1 - 16a^2)$
$= x^2(8a^2 + 1 + 4a)(8a^2 + 1 - 4a)$.

17. $4a^{4n} + b^{4n} = 4a^{4n} + 4a^{2n}b^{2n} + b^{4n} - 4a^{2n}b^{2n}$
$= (2a^{2n} + b^{2n} + 2a^nb^n)(2a^{2n} + b^{2n} - 2a^nb^n)$.

18. $x^{4m} + 4y^{4n} = x^{4m} + 4x^{2m}y^{2n} + 4y^{4n} - 4x^{2m}y^{2n}$
$= (x^{2m} + 2y^{2n} + 2x^my^n)(x^{2m} + 2y^{2n} - 2x^my^n)$.

Page 300

1. $a^3 + d^3 = (a + d)(a^2 - ad + d^2)$.

2. $a^3 + 2^3 = (a + 2)(a^2 - 2a + 4)$.

3. $d^3 + 8 = (d + 2)(d^2 - 2d + 4)$.

4. $x^3 + 125 = (x + 5)(x^2 - 5x + 25)$.

5. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.

6. $x^3 - 2^3 = (x - 2)(x^2 + 2x + 4)$.

7. $m^3 - 64 = (m - 4)(m^2 + 4m + 16)$.

8. $m^3 - (2n)^3 = (m - 2n)(m^2 + 2mn + 4n^2)$.

9. $8x^3 - y^3 = (2x - y)(4x^2 + 2xy + y^2)$.

10. $125 - x^3 = (5 - x)(25 + 5x + x^2)$.

11. $a^3 + (b^2)^3 = (a + b^2)(a^2 - ab^2 + b^4)$.

12. $c^3 + d^6 = (c + d^2)(c^2 - cd^2 + d^4)$.

13. $m^3 - n^9 = (m - n^3)(m^2 + mn^3 + n^6)$.

14. $(2a)^3 - (3b)^3 = (2a - 3b)(4a^2 + 6ab + 9b^2)$.

15. $125x^3 + 8y^3 = (5x + 2y)(25x^2 - 10xy + 4y^2)$.

16. $(a + b)^3 + c^3 = (a + b + c)(a^2 + 2ab + b^2 - ac - bc + c^2)$.

17. $m^3 - n^3 - m + n = (m^3 - n^3) - (m - n) = (m - n)(m^2 + mn + n^2 - 1)$.

18. $x^3 - 8y^3 + x - 2y = (x - 2y)(x^2 + 2xy + 4y^2 + 1)$.

19. $r^3 - 8s^3 + r - 2s = (r - 2s)(r^2 + 2rs + 4s^2 + 1)$.

20. $a^{3n} + b^{3n} = (a^n + b^n)(a^{2n} - a^nb^n + b^{2n})$.

21. $a^{3m} - b^{3n} = (a^m - b^n)(a^{2m} + a^mb^n + b^{2n})$.

22. See p. 133.

Page 301

1.
$$
\begin{array}{l|l}
x^2 + bx \qquad\qquad + c & x - n \\ \hline
x^2 - nx & x + b + n \\ \cline{1-1}
\quad bx + nx & \\
\quad bx \qquad - bn & \\ \cline{1-1}
\qquad nx + bn & \\
\qquad nx - n^2 & \\ \cline{1-1}
\qquad\quad n^2 + bn + c = \text{Remainder.} &
\end{array}
$$

2.
$$
\begin{array}{l|l}
x^2 + bx \qquad\qquad + c & x - a \\ \hline
x^2 - ax & x + b + a \\ \hline
\quad bx + ax \qquad + c & \\
\quad bx \qquad - ab & \\ \cline{1-1}
\qquad ax + ab + c & \\
\qquad ax - a^2 & \\ \cline{1-1}
\qquad\quad a^2 + ab + c = \text{Remainder.} &
\end{array}
$$

3.
$$
\begin{array}{l|l}
x^3 + ax^2 + \qquad bx + c & x - n \\ \hline
x^3 - nx^2 & x^2 + (a + n)x + b + an + n^2 \\ \hline
(a + n)x^2 + \qquad bx + c & \\
(a + n)x^2 - (an + n^2)x & \\ \cline{1-1}
\qquad (b + an + n^2)x + c & \\
\qquad (b + an + n^2)x \quad - bn - an^2 - n^3 & \\ \cline{1-1}
\qquad\qquad c + bn + an^2 + n^3 = \text{Remainder.} &
\end{array}
$$

4. (*a*)
$$
\begin{array}{l|l}
x^3 + x^2 - 5x + 3 & x - 2 \\ \hline
x^3 - 2x^2 & x^2 + 3x + 1 \\ \cline{1-1}
\quad 3x^2 - 5x & \\
\quad 3x^2 - 6x & \\ \cline{1-1}
\qquad\quad x + 3 & \\
\qquad\quad x - 2 & \\ \cline{1-1}
\qquad\qquad 5 = \text{Remainder.} &
\end{array}
$$

(*b*) Substituting 2 for x, $x^3 + x^2 - 5x + 3 = 8 + 4 - 10 + 3 = 5$.

5. (*a*)
$$\begin{array}{llll}
x^3 \qquad\quad - x + 5 & \underline{|\,x - 3 \quad\;\;} \\
\underline{x^3 - 3x^2 \qquad\quad} & |\,x^2 + 3x + 8 \\
\quad\; 3x^2 - x & \\
\quad\; \underline{3x^2 - 9x \qquad} & \\
\qquad\qquad 8x + 5 & \\
\qquad\qquad \underline{8x - 24} & \\
\qquad\qquad\qquad 29 = \text{Remainder.} &
\end{array}$$

(*b*) Substituting 3 for x, $x^3 - x + 5 = 27 - 3 + 5 = 29$, Remainder.

6. Substituting 3 for x,

$x^3 + x^2 - 5x + 8 = 27 + 9 - 15 + 8 = 29$, Remainder.

7. Substituting -4 for x,

$x^3 - 3x - 15 = -64 + 12 - 15 = -67$, Remainder.

8. Substituting 5 for x,

$x^3 - 2x^2 - 100 = 125 - 50 - 100 = -25$, Remainder.

9. Substituting 3 for x,

$x^3 - 2x^2 - 2x - 3 = 27 - 18 - 6 - 3 = 0$, Remainder.

10. Substituting 2 for x,

$x^4 - 2x^3 + x - 2 = 16 - 16 + 2 - 2 = 0$, Remainder.

Page 303

1. $x^3 + x - 2 = (x - 1)(x^2 + x + 2)$.
Substituting 1 for x, $x^3 + x - 2 = 1 + 1 - 2 = 0$.
Hence $x - 1$ is a factor. Dividing by $x - 1$, the other factor is $x^2 + x + 2$.
2. $x^3 + 2x + 3 = (x + 1)(x^2 - x + 3)$.
3. $a^3 + a^2 - 36 = (a - 3)(a^2 + 4a + 12)$.
4. $x^3 + x - 10 = (x - 2)(x^2 + 2x + 5)$.
5. $d^3 + d^2 - 12 = (d - 2)(d^2 + 3d + 6)$.
6. $x^3 - x^2 + 4x - 4 = (x - 1)(x^2 + 4)$.
7. $x^3 - 2x^2 - 5x + 6 = (x - 1)(x + 2)(x - 3)$, for 1, -2, and 3 when substituted for x each gives the remainder 0.
8. $x^3 - x^2 - 4 = (x - 2)(x^2 + x + 2)$.
9. $x^4 - 7x^2 - 6x = x(x + 1)(x + 2)(x - 3)$.
10. $y^3 - y^2 - 9y + 9 = (y - 1)(y - 3)(y + 3)$.
11. $2x^3 - 2x^2 - x - 6 = (x - 2)(2x^2 + 2x + 3)$.
12. $x^3 - 7x^2 + 4x + 12 = (x + 1)(x - 2)(x - 6)$.
13. $a^4 - 6a^3 + 11a^2 - 6a = a(a - 1)(a - 2)(a - 3)$.
14. See solution to Example, p. 302.

Page 304

1. $x^5 + z^5 = (x + z)(x^4 - x^3z + x^2z^2 - xz^3 + z^4)$.
2. $x^5 + 1 = (x + 1)(x^4 - x^3 + x^2 - x + 1)$.
3. $a^5 + 2^5 = (a + 2)(a^4 - 2a^3 + 4a^2 - 8a + 16)$.
4. $x^5 + 32 = (x + 2)(x^4 - 2x^3 + 4x^2 - 8x + 16)$.
5. $(a^2)^5 + (b^2)^5 = (a^2 + b^2)(a^8 - a^6b^2 + a^4b^4 - a^2b^6 + b^8)$.
6. $x^5 - z^5 = (x - z)(x^4 + x^3z + x^2z^2 + xz^3 + z^4)$.
7. $a^5 - 2^5 = (a - 2)(a^4 + 2a^3 + 4a^2 + 8a + 16)$.
8. $a^5 - 32x^5 = (a - 2x)(a^4 + 2a^3x + 4a^2x^2 + 8ax^3 + 16x^4)$.
9. $(2x)^5 - 243y^5 = (2x - 3y)(16x^4 + 24x^3y + 36x^2y^2 + 54xy^3 + 81y^4)$.
10. $a^7 - x^7 = (a - x)(a^6 + a^5x + a^4x^2 + a^3x^3 + a^2x^4 + ax^5 + x^6)$.
11. $1 - r^7 = (1 - r)(1 + r + r^2 + r^3 + r^4 + r^5 + r^6)$.
12. $x^7 - 128 = (x - 2)(x^6 + 2x^5 + 4x^4 + 8x^3 + 16x^2 + 32x + 64)$.
13. $x^{10} + y^{15} = (x^2 + y^3)(x^8 - x^6y^3 + x^4y^6 - x^2y^9 + y^{12})$.
14. $a^{10} + 32x^{15} = (a^2 + 2x^3)(a^8 - 2a^6x^3 + 4a^4x^6 - 8a^2x^9 + 16x^{12})$.
15. $x^7 + a^7 = (x + a)(x^6 - x^5a + x^4a^2 - x^3a^3 + x^2a^4 - xa^5 + a^6)$.
16. $1 + r^7 = (1 + r)(1 - r + r^2 - r^3 + r^4 - r^5 + r^6)$.
17. $128x^7 + 1 = (2x + 1)(64x^6 - 32x^5 + 16x^4 - 8x^3 + 4x^2 - 2x + 1)$.
18. $1 - r^n = (1 - r)(1 + r + r^2 + r^3 + r^4 + \cdots + r^{n-1})$.
19. $1 + r^n = (1 + r)(1 - r + r^2 - r^3 + r^4 - r^5 \cdots + r^{n-1})$ if n is odd.
20. $x^6 - y^6 = (x^3 + y^3)(x^3 - y^3) = (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$.
21. $x^8 - y^8 = (x^4 + y^4)(x^4 - y^4) = (x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$.
22. $a^{10} - b^{10} = (a^5 + b^5)(a^5 - b^5)$
$$= (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)(a - b)$$
$$(a^4 + a^3b + a^2b^2 + ab^3 + b^4).$$
23. $a^{12} - b^{12} = (a^6 + b^6)(a^6 - b^6)$
$$= (a^2 + b^2)(a^4 - a^2b^2 + b^4)(a + b)(a^2 - ab + b^2)(a - b)(a^2 + ab + b^2).$$

Page 305

1. $6x^4 + 2x^3 + 2x^2 = 2x^2(3x^2 + x + 1)$.
2. $5a^3 + 2a^2 - 15a - 6 = a^2(5a + 2) - 3(5a + 2) = (a^2 - 3)(5a + 2)$.
3. $2c^3d - 8cd^3 = 2cd(c^2 - 4d^2) = 2cd(c + 2d)(c - 2d)$.
4. $3m^3 - 3m^2 - 18m = 3m(m^2 - m - 6) = 3m(m - 3)(m + 2)$.
5. $2x^2 + 3ax + a^2 = (2x + a)(x + a)$.
6. $x^4 - 7x^2y^2 + 9y^4 = x^4 - 6x^2y^2 + 9y^4 - x^2y^2$
$$= (x^2 - 3y^2 + xy)(x^2 - 3y^2 - xy).$$
7. $a^4c - ac^4 = ac(a^3 - c^3) = ac(a - c)(a^2 + ac + c^2)$.
8. $2x^7y - 2xy^7 = 2xy(x^6 - y^6)$
$$= 2xy(x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2).$$
9. $x^5 - 2x^4 - 9x^2 = x^2(x^3 - 2x^2 - 9) = x^2(x - 3)(x^2 + x + 3)$.
10. $18r^3 - 24r^2s + 8rs^2 = 2r(9r^2 - 12rs + 4s^2) = 2r(3r - 2s)^2$.

11. $45x^3y - 20xy^3 = 5xy(9x^2 - 4y^2) = 5xy(3x + 2y)(3x - 2y)$.

12. $2h^3k + 4h^2k^2 - 30hk^3 = 2hk(h^2 + 2hk - 15k^2) = 2hk(h + 5k)(h - 3k)$.

13. $x^4 + 7x^2 + 16 = x^4 + 8x^2 + 16 - x^2 = (x^2 + 4 - x)(x^2 + 4 + x)$.

14. $a^4d - 8ad^4 = ad(a^3 - 8d^3) = ad(a - 2d)(a^2 + 2ad + 4d^2)$.

15. $2a^7 + 64a^2 = 2a^2(a^5 + 32) = 2a^2(a + 2)(a^4 - 2a^3 + 4a^2 - 8a + 16)$.

16. $m^8n + mn^8 = mn(m^7 + n^7)$
$= mn(m + n)(m^6 - m^5n + m^4n^2 - m^3n^3 + m^2n^4 - mn^5 + n^6)$.

17. $x^3 + 4x - 5 = (x - 1)(x^2 + x + 5)$.

18. $ax^2 - 4a + 3x^2 - 12 = (a + 3)(x^2 - 4) = (a + 3)(x + 2)(x - 2)$.

19. $a^2c^2 - c^2 - a^2 + 1 = (a^2 - 1)(c^2 - 1) = (a + 1)(a - 1)(c + 1)(c - 1)$.

20. $x^4 - 13x^2 + 36 = (x^2 - 9)(x^2 - 4)$
$= (x + 3)(x - 3)(x + 2)(x - 2)$.

21. $a^5 - a^2b^3 - a^3b^2 + b^5 = a^2(a^3 - b^3) - b^2(a^3 - b^3)$
$= (a^2 - b^2)(a^3 - b^3)$
$= (a + b)(a - b)(a - b)(a^2 + ab + b^2)$.

22. $x^{12} - x^2 = x^2(x^{10} - 1)$
$= x^2(x^5 + 1)(x^5 - 1)$
$= x^2(x + 1)(x^4 - x^3 + x^2 - x + 1)(x - 1)(x^4 + x^3 + x^2 + x + 1)$.

23. $x^3 - 3x^2 - 4x + 12 = x^2(x - 3) - 4(x - 3)$
$= (x^2 - 4)(x - 3)$
$= (x + 2)(x - 2)(x - 3)$.

24. $(a + x)^2 + 10(a + x) + 25 = (a + x + 5)^2$.

25. $a^8 + 27a^2 = a^2(a^6 + 27) = a^2(a^2 + 3)(a^4 - 3a^2 + 9)$.

26. $(x + y)^2 - 6(x + y)z + 9z^2 = (x + y - 3z)^2$.

27. $(a - x)^2 - 16(m - n)^2 = (a - x + 4m - 4n)(a - x - 4m + 4n)$.

28. $a^4 - 11a^2 + 1 = a^4 - 2a^2 + 1 - 9a^2$
$= (a^2 - 1)^2 - 9a^2$
$= (a^2 - 1 - 3a)(a^2 - 1 + 3a)$.

29. $a^4 + ab^{15} = a(a^3 + b^{15}) = a(a + b^5)(a^2 - ab^5 + b^{10})$.

30. $4(a - b)^2 - 12(a - b)c + 9c^2 = (2a - 2b - 3c)^2$.

31. $a^2 - 10ab + 25b^2 - c^2 = (a - 5b)^2 - c^2 = (a - 5b + c)(a - 5b - c)$.

32. $x^2 - 2x(a - b) - 35(a - b)^2 = (x - 7a + 7b)(x + 5a - 5b)$.

33. $x^3 - 8x^2 + 17x - 10 = (x - 1)(x - 2)(x - 5)$.

34. $x^4 - x^2y^2 + x^3 - x^2y = x^2(x^2 - y^2 + x - y) = x^2(x - y)(x + y + 1)$.

35. $a^2 - b^2 + (a - b)^2 = (a - b)(a + b + a - b) = 2a(a - b)$.

36. $c^2 - 2cd + d^2 - 2(c - d) - 35 = (c - d)^2 - 2(c - d) - 35$
$= (c - d - 7)(c - d + 5)$.

37. $2(a + b)^2 + 3c(a + b) - 2c^2 = (2a + 2b - c)(a + b + 2c)$.

38. $a^3 + b^3 + 3a^2b + 3ab^2 = (a + b)^3$.

39. $32x^8 - x^3y^{10} = x^3(32x^5 - y^{10})$
$= x^3(2x - y^2)(16x^4 + 8x^3y^2 + 4x^2y^4 + 2xy^6 + y^8)$.

40. $c^2 + 4d^2 - x^2 - 4cd = (c - 2d)^2 - x^2 = (c - 2d + x)(c - 2d - x).$

41. $(a - 2x)c^2 + (2x - a)d^2 = (a - 2x)c^2 - (a - 2x)d^2$
$= (a - 2x)(c + d)(c - d).$

42. $6a^4 + 3a^3 - 3a^2 = 3a^2(2a^2 + a - 1) = 3a^2(2a - 1)(a + 1).$

43. $16a^4 + 7a^2 + 1 = 16a^4 + 8a^2 + 1 - a^2$
$= (4a^2 + 1 + a)(4a^2 + 1 - a).$

44. $x^6y^6 - 64 = (x^3y^3 + 8)(x^3y^3 - 8)$
$= (xy + 2)(x^2y^2 - 2xy + 4)(xy - 2)(x^2y^2 + 2xy + 4).$

45. $a^7 - a^2b^5 + a^2b - a^3 = a^2(a^5 - b^5 + b - a)$
$= a^2[(a^5 - b^5) - (a - b)]$
$= a^2(a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4 - 1).$

46. $a^4 + 8ab^3 + a^2 + 2ab = a[(a^3 + 8b^3) + (a + 2b)]$
$= a(a + 2b)(a^2 - 2ab + 4b^2 + 1).$

47. $9x^2 - 4y^2 - 3x - 2y = (9x^2 - 4y^2) - (3x + 2y)$
$= (3x + 2y)(3x - 2y - 1).$

48. $x^2 - 5(2x - 5) = x^2 - 10x + 25 = (x - 5)^2.$

49. $3a^2 - 14a(b - c) + 8(b - c)^2 = (3a - 2b + 2c)(a - 4b + 4c).$

50. $a^4x^4 + 4 = a^4x^4 + 4a^2x^2 + 4 - 4a^2x^2$
$= (a^2x^2 + 2 + 2ax)(a^2x^2 + 2 - 2ax).$

51. $x^8y^6 + x^2z^6 = x^2(x^6y^6 + z^6) = x^2(x^2y^2 + z^2)(x^4y^4 - x^2y^2z^2 + z^4).$

52. $x^3 - 10x - 3 = (x + 3)(x^2 - 3x - 1).$

53. $a^8 - 5a^4 + 4 = (a^4 - 1)(a^4 - 4)$
$= (a^2 + 1)(a + 1)(a - 1)(a^2 + 2)(a^2 - 2).$

54. $x - 1 + x^5 - x^3 = x^5 - x^3 + x - 1 = (x - 1)(x^4 + x^3 + 1).$

55. $x^3 + x - y - y^3 = (x^3 - y^3) + (x - y) = (x - y)(x^2 + xy + y^2 + 1).$

56. $x^2 + 2xy + y^2 - 25a^2 - 10a - 1 = (x + y)^2 - (5a + 1)^2$
$= (x + y + 5a + 1)(x + y - 5a - 1).$

57. $a^4 - b^4 - 2ab(a^2 - b^2) = (a^2 - b^2)(a^2 + b^2 - 2ab) = (a + b)(a - b)^3.$

58. $2x^3 + 3x^2 + x = x(2x^2 + 3x + 1) = x(2x + 1)(x + 1).$

59. $m^7 - 8m - 7m^4 = m(m^6 - 7m^3 - 8)$
$= m(m^3 - 8)(m^3 + 1)$
$= m(m - 2)(m^2 + 2m + 4)(m + 1)(m^2 - m + 1).$

60. $2x^4 - 10x^2 + 4x = 2x(x^3 - 5x + 2) = 2x(x - 2)(x^2 + 2x - 1).$

61. $a^3 + 1 + 3a^2 + 3a = (a^3 + 1) + 3a(a + 1)$
$= (a + 1)(a^2 - a + 1 + 3a)$
$= (a + 1)(a^2 + 2a + 1)$
$= (a + 1)^3.$

62. $12x^2 - 8x + x^4 - 6x^3 = x(x^3 - 6x^2 + 12x - 8)$
$= x(x - 2)(x^2 + 2x + 4 - 6x)$
$= x(x - 2)^3.$

63. $a^nx^2 + 2a^nx + a^n = a^n(x + 1)^2.$

64. $mr - ms - nr + ns = m(r - s) - n(r - s) = (r - s)(m - n).$

65. $h^{2m} - 2h^mk^n + k^{2n} = (h^m - k^n)^2.$

66. $a^{2m} - b^{2n} = (a^m + b^n)(a^m - b^n).$

67. $x^{2n} + (r + s)x^n + rs = (x^n + r)(x^n + s).$

68. $hrx^{2n} + hrx^n + hsx^n + hs = hrx^n(x^n + 1) + hs(x^n + 1)$
$= (hrx^n + hs)(x^n + 1)$
$= h(rx^n + s)(x^n + 1).$

69. $a^{4m} + a^{2m}b^{2n} + b^{4n} = a^{4m} + 2a^{2m}b^{2n} + b^{4n} - a^{2m}b^{2n}$
$= (a^{2m} + b^{2n} + a^mb^n)(a^{2m} + b^{2n} - a^mb^n).$

70. $a^{3m} + b^{3n} = (a^m + b^n)(a^{2m} - a^mb^n + b^{2n}).$

71. $a^{3m} - b^{3n} = (a^m - b^n)(a^{2m} + a^mb^n + b^{2n}).$

72. $a^{5m} + b^{5n} = (a^m + b^n)(a^{4m} - a^{3m}b^n + a^{2m}b^{2n} - a^mb^{3n} + b^{4n}).$

73. $a^{5m} - b^{5n} = (a^m - b^n)(a^{4m} + a^{3m}b^n + a^{2m}b^{2n} + a^mb^{3n} + b^{4n}).$

75.
$$cx + dx = c^2 + cd.$$
$$(c + d)x = c(c + d).$$
$$x = c.$$

76.
$$ma - mc + bc = ab.$$
$$m(a - c) = b(a - c).$$
$$m = b.$$

77. $5ay - 3by - 5a^2 + 3ab = 0.$
$$(5a - 3b)y = a(5a - 3b).$$
$$y = a.$$

78.
$$az - 3ad = bz - 3bd.$$
$$(a - b)z = 3d(a - b).$$
$$z = 3d.$$

79.
$$cx - 2dx = c^2 - 4d^2.$$
$$(c - 2d)x = (c - 2d)(c + 2d).$$
$$x = c + 2d.$$

80.
$$r(2a - 7c) = 4a^2 - 28ac + 49c^2.$$
$$r(2a - 7c) = (2a - 7c)^2.$$
$$r = 2a - 7c.$$

81.
$$m(3a - c) - 9ad = 2ec - 6ae - 3cd.$$
$$m(3a - c) = 9ad - 3cd - 6ae + 2ec.$$
$$m(3a - c) = 3d(3a - c) - 2e(3a - c).$$
$$m = 3d - 2e.$$

82.
$$ax - 3bx - a^2 = 3b^2 - 4ab.$$
$$(a - 3b)x = a^2 - 4ab + 3b^2.$$
$$(a - 3b)x = (a - 3b)(a - b).$$
$$x = a - b.$$

83. $$y + 21d^2 + 4d = 7dy + 1.$$
$$y(1 - 7d) = 1 - 4d - 21d^2.$$
$$= (1 - 7d)(1 + 3d).$$
$$y = 1 + 3d.$$

84. $$d(1 - 3a) + x + 3ac = c + 3ax.$$
$$(1 - 3a)x = c - 3ac - d(1 - 3a).$$
$$= c(1 - 3a) - d(1 - 3a).$$
$$x = c - d.$$

85. $$az + ae - 2ec + 2cd = 2cz + ad.$$
$$az - 2cz = ad - 2cd - ae + 2ec.$$
$$(a - 2c)z = d(a - 2c) - e(a - 2c).$$
$$z = d - e.$$

86. $$5x - 2cx - a(5 - 2c) = 5a + 5 - 2c - 2ac.$$
$$(5 - 2c)x = a(5 - 2c) + a(5 - 2c) + (5 - 2c).$$
$$x = 2a + 1.$$

87 $$a^2x - ax + x - a^3 - 1 = 0.$$
$$(a^2 - a + 1)x = a^3 + 1.$$
$$x = a + 1.$$

88. $$c^3 - c^2x - 2cdx - 8d^3 = 4d^2x.$$
$$(c^2 + 2cd + 4d^2)x = c^3 - 8d^3.$$
$$x = c - 2d.$$

89. $$m(a - 2)(a^2 + 4) = a^4 - 16.$$
$$m = a + 2.$$

90. $$16s - 8as + 4a^2s - 2a^3s + a^4s - 32 = a^5.$$
$$(a^4 - 2a^3 + 4a^2 - 8a + 16)s = a^5 + 32$$
$$s = a + 2.$$

Page 307

1. $$x^2 - 4 = 0.$$
$$(x + 2)(x - 2) = 0.$$
$$x = -2,\ 2.$$

2. $$x^2 - 5x = 0.$$
$$x(x - 5) = 0.$$
$$x = 0,\ 5.$$

3. $$x^2 - 4a^2 = 0.$$
$$(x + 2a)(x - 2a) = 0.$$
$$x = -2a,\ 2a.$$

4. $$y^3 = 64y.$$
$$y(y + 8)(y - 8) = 0.$$
$$y = 0,\ -8,\ 8.$$

5. $$x^2 + 5 = 6x.$$
$$x^2 - 6x + 5 = 0.$$
$$(x - 5)(x - 1) = 0.$$
$$x = 5,\ 1.$$

6. $$2x^2 + x = 6.$$
$$2x^2 + x - 6 = 0.$$
$$(2x - 3)(x + 2) = 0.$$
$$x = \tfrac{3}{2},\ -2.$$

7. $$m^2 - am - 2a^2 = 0.$$
$$(m - 2a)(m + a) = 0.$$
$$m = 2a,\ -a.$$

8.
$$y^3 - y + 2 = 2y^2.$$
$$y^3 - 2y^2 - y + 2 = 0.$$
$$(y^2 - 1)(y - 2) = 0.$$
$$(y + 1)(y - 1)(y - 2) = 0.$$
$$y = -1, +1, 2.$$

9.
$$x^2 - 9a^2 - x + 3a = 0.$$
$$(x - 3a)(x + 3a - 1) = 0.$$
$$x = 3a, 1 - 3a.$$

10.
$$x^3 - a^2x + 2ax^2 - 2a^3 = 0.$$
$$(x + 2a)(x + a)(x - a) = 0.$$
$$x = -2a, -a, a.$$

11.
$$y^3 - 7y - 6 = 0.$$
$$(y + 1)(y - 3)(y + 2) = 0.$$
$$y = -1, 3, -2.$$

12.
$$y^4 - 13y^2 + 36 = 0.$$
$$(y^2 - 9)(y^2 - 4) = 0.$$
$$(y - 3)(y + 3)(y - 2)(y + 2) = 0.$$
$$y = \pm 3, \pm 2.$$

13.
$$x^5 - 5x^3 = -4x.$$
$$x^5 - 5x^3 + 4x = 0.$$
$$x(x + 2)(x - 2)(x + 1)(x - 1) = 0.$$
$$x = 0, \pm 2, \pm 1.$$

14.
$$z^3 + 12z - 6z^2 - 8 = 0.$$
$$z^3 - 8 - 6z(z - 2) = 0.$$
$$(z - 2)(z^2 - 4z + 4) = 0.$$
$$(z - 2)^3 = 0.$$
$$z = 2, 2, 2.$$

15.
$$3x^3 + 72x + 33x^2 = 0.$$
$$3x(x^2 + 11x + 24) = 0.$$
$$3x(x + 8)(x + 3) = 0.$$
$$x = 0, -8, -3.$$

16.
$$8x^2 + 71x^3 - 9x^4 = 0.$$
$$x^2(9x + 1)(x - 8) = 0.$$
$$x = 0, 0, -\tfrac{1}{9}, 8.$$

17.
$$(2x - 3)^2 - (5x + 6)^2 = 0.$$
$$(2x - 3 + 5x + 6)(2x - 3 - 5x - 6) = 0.$$
$$(7x + 3)(-3x - 9) = 0.$$
$$x = -\tfrac{3}{7}, -3.$$

18.
$$(x - 3)^2 - 2(x - 3) = 8.$$
$$(x - 7)(x - 1) = 0.$$
$$x = 7, 1.$$

19.
$$x^3 + 50 - 25x - 2x^2 = 0.$$
$$x^2(x - 2) - 25(x - 2) = 0.$$
$$(x - 2)(x - 5)(x + 5) = 0.$$
$$x = 2, 5, -5.$$

20.
$$x^3 - ax^2 - 4a^2x + 4a^3 = 0.$$
$$(x - a)(x^2 - 4a^2) = 0.$$
$$x = a, \pm 2a.$$

21.
$$y^3 + 3y^2 + 3y + 1 = 0.$$
$$(y + 1)^3 = 0.$$
$$y = -1, -1, -1.$$

22.
$$acx^2 + bcx + adx + bd = 0.$$
$$(cx + d)(ax + b) = 0.$$
$$x = -\frac{d}{c}, -\frac{b}{a}.$$

23.
$$x^2 - 7x - 8 = x + 1.$$
$$x^2 - 8x - 9 = 0.$$
$$(x - 9)(x + 1) = 0.$$
$$x = 9, -1.$$

24.
$$3x^2 - 11x + 6 = 3x - 2.$$
$$3x^2 - 14x + 8 = 0.$$
$$(3x - 2)(x - 4) = 0.$$
$$x = \tfrac{2}{3}, 4.$$

25.
$$6x^2 + 7x - 3 = 3x - 1.$$
$$6x^2 + 4x - 2 = 0.$$
$$2(3x - 1)(x + 1) = 0.$$
$$x = \tfrac{1}{3}, -1.$$

26. See p. 140.

Page 309

1.
$$12 = 2^2 \cdot 3.$$
$$18 = 2 \cdot 3^2.$$
$$24 = 2^3 \cdot 3.$$
$$\text{H.C.F.} = 2 \cdot 3 = 6.$$

2.
$$15 = 3 \cdot 5.$$
$$25 = 5^2.$$
$$40 = 5 \cdot 2^3.$$
$$\text{H.C.F.} = 5.$$

3.
$$12\,a^2 = 2^2 \cdot 3\,a^2.$$
$$30\,a^5 = 2 \cdot 3 \cdot 5\,a^5.$$
$$36\,a^4 = 2^2 \cdot 3^2\,a^4.$$
$$\text{H.C.F.} = 2 \cdot 3\,a^2 = 6\,a^2.$$

4.
$$28\,a^2b^4 = 2^2 \cdot 7\,a^2b^4.$$
$$42\,ab^5 = 2 \cdot 3 \cdot 7\,ab^5.$$
$$70\,a^2b^3 = 2 \cdot 5 \cdot 7\,a^2b^3.$$
$$\text{H.C.F.} = 2 \cdot 7\,ab^3 = 14\,ab^3.$$

5.
$$66\,c^4x = 2 \cdot 3 \cdot 11\,c^4x.$$
$$132\,c^2x^3 = 2^2 \cdot 3 \cdot 11\,c^2x^3.$$
$$165\,c^4x^2 = 3 \cdot 5 \cdot 11\,c^4x^2.$$
$$\text{H.C.F.} = 3 \cdot 11\,c^2x = 33\,c^2x.$$

6.
$$a^2 + 2\,ab + b^2 = (a + b)^2.$$
$$a^2 - b^2 = (a + b)(a - b).$$
$$\text{H.C.F.} = a + b.$$

7.
$$3\,a^2 - 3\,b^2 = 3\,(a + b)(a - b).$$
$$9\,(a - b)^2 = 3^2\,(a - b)^2.$$
$$3\,a^3 - 3\,b^3 = 3\,(a - b)(a^2 + ab + b^2).$$
$$\text{H.C.F.} = 3\,(a - b).$$

8.
$$ax^2 - 2\,axy + ay^2 = a\,(x - y)^2.$$
$$a^2x^2 - a^2y^2 = a^2\,(x - y)(x + y).$$
$$2\,ax^3 - 2\,ay^3 = 2\,a\,(x - y)(x^2 + xy + y^2).$$
$$\text{H.C.F.} = a\,(x - y).$$

9.
$$2\,a^2m^2 - 2\,a^2n^2 = 2\,a^2\,(m + n)(m - n).$$
$$4\,am^2 - 12\,amn + 8\,an^2 = 2^2a\,(m - 2\,n)(m - n).$$
$$10\,am - 10\,an = 2 \cdot 5\,a\,(m - n).$$
$$\text{H.C.F.} = 2\,a\,(m - n).$$

10.
$$25\,x^3 - 25\,x^2y = 5^2\,x^2\,(x - y).$$
$$10\,x^3 - 20\,x^2y - 30\,xy^2 = 2 \cdot 5\,x\,(x + y)(x - 3\,y).$$
$$5\,x^4 - 5\,xy^3 = 5\,x\,(x - y)(x^2 + xy + y^2).$$
$$\text{H.C.F.} = 5\,x.$$

11.
$$24\,a^6 - 6\,a^4b^2 = 2 \cdot 3\,a^4\,(2\,a + b)(2\,a - b).$$
$$48\,a^6 + 24\,a^5b = 2^3 \cdot 3\,a^5\,(2\,a + b).$$
$$48\,a^5 - 48\,a^4b - 36\,a^3b^2 = 2^2 \cdot 3\,a^3\,(2\,a - 3\,b)(2\,a + b).$$
$$\text{H.C.F.} = 6\,a^3\,(2\,a + b).$$

12.
$$5\,x^7 - 160\,x^2 = 5\,x^2\,(x - 2)(x^4 + 2\,x^3 + 4\,x^2 + 8\,x + 16).$$
$$15\,x^5 - 60\,x^3 = 3 \cdot 5\,x^3\,(x + 2)(x - 2).$$
$$25\,x^7 - 200\,x^4 = 5^2x^4\,(x - 2)(x^2 + 2\,x + 4).$$
$$\text{H.C.F.} = 5\,x^2\,(x - 2).$$

13.
$$18\,a^4 - 2\,a^2b^2 = 2\,a^2(3\,a+b)(3\,a-b).$$
$$12\,a^6 - 8\,a^5b - 4\,a^4b^2 = 2^2a^4(3\,a+b)(a-b).$$
$$30\,a^4b^2 + 10\,a^3b^3 = 2\cdot 5\,a^3b^2(3\,a+b).$$
$$\text{H.C.F.} = 2\,a^2(3\,a+b).$$

14.
$$a^5 - 3\,a^4b + 2\,a^3b^2 = a^3(a-2\,b)(a-b).$$
$$a^5 - 2\,a^4b - a^3b^2 + 2\,a^2b^3 = a^2(a-2\,b)(a-b)(a+b).$$
$$a^7 - 5\,a^5b^2 + 4\,a^3b^4 = a^3(a-2\,b)(a+2\,b)(a-b)(a+b).$$
$$\text{H.C.F.} = a^2(a-2\,b)(a-b) = a^2(a^2-3\,ab+2\,b^2).$$

Page 310 (First set)

1. $\dfrac{2\,a^3b^2}{10\,a^2b^3} = \dfrac{a}{5\,b}.$

2. $\dfrac{15\,xy^3}{245\,x^3y} = \dfrac{3\,y^2}{49\,x^2}.$

3. $\dfrac{x^2-9}{2\,x^2+6\,x} = \dfrac{(x+3)(x-3)}{2\,x(x+3)} = \dfrac{x-3}{2\,x}.$

4. $\dfrac{5\,x^2+5\,xy}{25\,x^4-25\,x^2y^2} = \dfrac{5\,x(x+y)}{25\,x^2(x+y)(x-y)} = \dfrac{1}{5\,x(x-y)}.$

5. $\dfrac{48\,x^4-6\,xy^3}{32\,x^4-32\,x^3y+8\,x^2y^2} = \dfrac{6\,x(2\,x-y)(4\,x^2+2\,xy+y^2)}{8\,x^2(2\,x-y)(2\,x-y)}$
$= \dfrac{3(4\,x^2+2\,xy+y^2)}{4\,x(2\,x-y)}.$

6. $\dfrac{3\,x^3-375}{2\,x^2+10\,x+50} = \dfrac{3(x-5)(x^2+5\,x+25)}{2(x^2+5\,x+25)} = \dfrac{3(x-5)}{2}.$

7. $\dfrac{2\,c^2+2\,cd-4\,d^2}{5\,c^2-5\,d^2} = \dfrac{2(c+2\,d)(c-d)}{5(c+d)(c-d)} = \dfrac{2(c+2\,d)}{5(c+d)}.$

8. $\dfrac{2\,c^6-64\,cd^5}{16\,c^4-40\,c^3d+16\,c^2d^2} = \dfrac{2\,c(c-2\,d)(c^4+2\,c^3d+4\,c^2d^2+8\,cd^3+16\,d^4)}{8\,c^2(c-2\,d)(2\,c-d)}$
$= \dfrac{c^4+2\,c^3d+4\,c^2d^2+8\,cd^3+16\,d^4}{4\,c(2\,c-d)}.$

9. $\dfrac{ad-3\,ax+2\,cd-6\,cx}{ad-3\,ax-cd+3\,cx} = \dfrac{(a+2\,c)(d-3\,x)}{(a-c)(d-3\,x)} = \dfrac{a+2\,c}{a-c}.$

10. $\dfrac{4\,x^2+6\,x-40}{4\,x-15\,a+6\,ax-10} = \dfrac{2(2\,x-5)(x+4)}{(2\,x-5)(2+3\,a)} = \dfrac{2(x+4)}{2+3\,a}.$

11. $\dfrac{5\,x^4-40\,x}{3\,x^5-96} = \dfrac{5\,x(x-2)(x^2+2\,x+4)}{3(x-2)(x^4+2\,x^3+4\,x^2+8\,x+16)}$
$= \dfrac{5\,x(x^2+2\,x+4)}{3(x^4+2\,x^3+4\,x^2+8\,x+16)}.$

12. $\dfrac{12\,a^3+10\,a^2-12\,a}{4+9\,a^2-12\,a} = \dfrac{2\,a(3\,a-2)(2\,a+3)}{(3\,a-2)(3\,a-2)} = \dfrac{2\,a(2\,a+3)}{3\,a-2}.$

13. $$\frac{32x^2 - x^7}{32 + 16x + 4x^3 + 8x^2 + 2x^4} = \frac{x^2(2-x)(16 + 8x + 4x^2 + 2x^3 + x^4)}{2(16 + 8x + 4x^2 + 2x^3 + x^4)}$$
$$= \frac{x^2(2-x)}{2}.$$

14. $$\frac{(\frac{2}{3})^2 - 1}{\frac{2}{3} + 1} = \frac{(\frac{2}{3}+1)(\frac{2}{3}-1)}{(\frac{2}{3}+1)} = \frac{2}{3} - 1 = -\frac{1}{3}.$$

15. $$\frac{(\frac{3}{5})^3 - 1}{\frac{3}{5} - 1} = \frac{(\frac{3}{5}-1)[(\frac{3}{5})^2 + \frac{3}{5} + 1]}{\frac{3}{5} - 1} = \left(\frac{3}{5}\right)^2 + \frac{3}{5} + 1 = \frac{49}{25}.$$

16. $$\frac{(\frac{3}{2})^2 - (\frac{3}{2})(\frac{2}{5}) + (\frac{2}{5})^2}{(\frac{3}{2})^3 + (\frac{2}{5})^3} = \frac{(\frac{3}{2})^2 - (\frac{3}{2})(\frac{2}{5}) + (\frac{2}{5})^2}{[(\frac{3}{2}) + (\frac{2}{5})][(\frac{3}{2})^2 - (\frac{3}{2})(\frac{2}{5}) + (\frac{2}{5})^2]}$$
$$= \frac{1}{\frac{3}{2} + \frac{2}{5}} = \frac{1}{\frac{19}{10}} = \frac{10}{19}.$$

Page 310 (Second set)

1. $$\frac{2}{3x} = \frac{4x}{6x^2}, \quad \frac{5}{2x^2} = \frac{15}{6x^2}.$$

2. $$\frac{2x}{x-2} = \frac{2x(x+2)}{(x-2)(x+2)}, \quad \frac{3}{x+2} = \frac{3(x-2)}{(x-2)(x+2)}.$$

3. $$\frac{3}{a^2-9} = \frac{3}{a^2-9}, \quad \frac{2a}{a-3} = \frac{2a(a+3)}{a^2-9}.$$

4. $$\frac{2x-1}{x^2-6x+9} = \frac{2(2x-1)}{2(x-3)^2}, \quad \frac{-x+5}{-2x+6} = \frac{(x-5)(x-3)}{2(x-3)^2}.$$

5. $$\frac{x+1}{x^2-5x+6} = \frac{(x+1)(x+2)}{(x-2)(x-3)(x+2)}, \quad \frac{-x}{-x^2+4} = \frac{x(x-3)}{(x-2)(x+2)(x-3)}.$$

6. $$\frac{2a}{a+2} = \frac{2a(3a+1)}{(a+2)(3a+1)},$$
$$\frac{a+3}{3a+1} = \frac{(a+3)(a+2)}{(3a+1)(a+2)},$$
$$\frac{2a^2+5}{3a^2+7a+2} = \frac{2a^2+5}{(3a+1)(a+2)}.$$

7. $$\frac{a}{2x} = \frac{3a(x+2)}{6x(x+2)},$$
$$\frac{2a}{3x} = \frac{4a(x+2)}{6x(x+2)},$$
$$\frac{5a}{x^2+2x} = \frac{30a}{6x(x+2)}.$$

8. $$\frac{2x+1}{x^2-2x+4} = \frac{(2x+1)(x+2)}{(x+2)(x^2-2x+4)},$$
$$\frac{5}{x^3+8} = \frac{5}{(x+2)(x^2-2x+4)},$$
$$\frac{1}{x+2} = \frac{x^2-2x+4}{(x+2)(x^2-2x+4)}.$$

9. $\dfrac{2x}{x^3-8}=\dfrac{2x^2(x+2)}{x(x-2)(x+2)(x^2+2x+4)}$,

$\dfrac{5x}{2-x}=\dfrac{-5x^2(x+2)(x^2+2x+4)}{x(x-2)(x+2)(x^2+2x+4)}$,

$\dfrac{4}{4-x^2}=\dfrac{-4x(x^2+2x+4)}{x(x-2)(x+2)(x^2+2x+4)}$,

$\dfrac{3x+2}{x^3+2x^2+4x}=\dfrac{(3x+2)(x-2)(x+2)}{x(x-2)(x+2)(x^2+2x+4)}$.

10. $\dfrac{2x-9}{x^5-32}=\dfrac{x^2(2x-9)}{x^2(x^5-32)}$,

$\dfrac{-2}{-x^3+2x^2}=\dfrac{2}{x^2(x-2)}=\dfrac{2(x^4+2x^3+4x^2+8x+16)}{x^2(x^5-32)}$,

$\dfrac{x-2}{x^5+2x^4+4x^3+8x^2+16x}=\dfrac{x(x-2)^2}{x^2(x^5-32)}$.

Page 311

1. $\dfrac{2x}{5}+\dfrac{3x}{10}-\dfrac{2x}{15}=\dfrac{12x+9x-4x}{30}=\dfrac{17x}{30}$.

2. $\dfrac{2c}{3a}-\dfrac{5c^2}{6a^2}+\dfrac{c}{9a}=\dfrac{12ac-15c^2+2ac}{18a^2}=\dfrac{14ac-15c^2}{18a^2}$.

3. $\dfrac{2rs^3+r}{r^2s}-\dfrac{r-6s}{3rs^2}-\dfrac{s^2}{2r}=\dfrac{(12rs^4+6rs)-(2r^2-12rs)-3rs^4}{6r^2s^2}$

$=\dfrac{9rs^4+18rs-2r^2}{6r^2s^2}$

$=\dfrac{9s^4+18s-2r}{6rs^2}$.

4. $\dfrac{5a}{a-3}-\dfrac{2}{3}=\dfrac{15a-(2a-6)}{3(a-3)}=\dfrac{13a+6}{3(a-3)}$.

5. $\dfrac{c}{c-4}-\dfrac{2c}{c+3}=\dfrac{(c^2+3c)-(2c^2-8c)}{(c-4)(c+3)}=\dfrac{11c-c^2}{(c-4)(c+3)}$.

6. $\dfrac{a}{a-5}+\dfrac{2a}{a+4}-\dfrac{1}{2}=\dfrac{(2a^2+8a)+(4a^2-20a)-(a^2-a-20)}{2(a-5)(a+4)}$

$=\dfrac{5a^2-11a+20}{2(a-5)(a+4)}$.

7. $\dfrac{2x+1}{x^2-9}-\dfrac{4}{x-3}=\dfrac{(2x+1)-4(x+3)}{(x+3)(x-3)}=\dfrac{-2x-11}{x^2-9}=\dfrac{2x+11}{9-x^2}.$

8. $\dfrac{a-3}{a^2-4}-\dfrac{a-3}{2-a}+\dfrac{5}{a+2}=\dfrac{a-3}{a^2-4}+\dfrac{a-3}{a-2}+\dfrac{5}{a+2}$

$$=\frac{a-3+a^2-a-6+5a-10}{a^2-4}$$

$$=\frac{a^2+5a-19}{a^2-4}.$$

9. $\dfrac{2c-3}{c(c-2)}-\dfrac{c^2-7}{8-c^3}=\dfrac{2c-3}{c(c-2)}+\dfrac{c^2-7}{c^3-8}$

$$=\frac{2c^3+c^2+2c-12+c^3-7c}{c(c^3-8)}$$

$$=\frac{3c^3+c^2-5c-12}{c(c^3-8)}.$$

10. $\dfrac{m}{m-1}-\dfrac{2}{m}-\dfrac{5m-2}{m^2-1}=\dfrac{m^3+m^2-2m^2+2-5m^2+2m}{m(m^2-1)}$

$$=\frac{m^3-6m^2+2m+2}{m(m^2-1)}.$$

11. $2+\dfrac{5x}{x-7}=\dfrac{2x-14+5x}{x-7}=\dfrac{7x-14}{x-7}.$

12. $x+3-\dfrac{x^2}{x-3}=\dfrac{x^2-9-x^2}{x-3}=\dfrac{9}{3-x}.$

13. $x^2+2x+4-\dfrac{x^3+8}{x-2}=\dfrac{x^3-8-x^3-8}{x-2}=\dfrac{16}{2-x}.$

14. $a-\dfrac{a-6}{2a-3}-2=a-2-\dfrac{a-6}{2a-3}$

$$=\frac{2a^2-7a+6-a+6}{2a-3}$$

$$=\frac{2a^2-8a+12}{2a-3}.$$

15. $c^2-\dfrac{c^4+d^4}{c^2+cd+d^2}-cd+d^2=-\dfrac{c^4+d^4}{c^2+cd+d^2}+c^2-cd+d^2$

$$=\frac{-c^4-d^4+c^4+c^2d^2+d^4}{c^2+cd+d^2}$$

$$=\frac{c^2d^2}{c^2+cd+d^2}.$$

16. $\dfrac{x+2}{2x-1}-\dfrac{x}{1-x}-\dfrac{3x^2+2x-2}{2x^2-3x+1}=\dfrac{x+2}{2x-1}+\dfrac{x}{x-1}-\dfrac{3x^2+2x-2}{(2x-1)(x-1)}$

$$=\frac{x^2+x-2+2x^2-x-3x^2-2x+2}{(2x-1)(x-1)}$$

$$=-\frac{2x}{2x^2-3x+1}.$$

17. $\dfrac{a-3}{a^2-2a-3}-\dfrac{2-a}{a^2-3a+2}-\dfrac{1}{1-a^2}$

$$=\frac{a-3}{(a-3)(a+1)}+\frac{a-2}{(a-2)(a-1)}+\frac{1}{a^2-1}$$

$$=\frac{1}{a+1}+\frac{1}{a-1}+\frac{1}{a^2-1}$$

$$=\frac{a-1+a+1+1}{a^2-1}$$

$$=\frac{2a+1}{a^2-1}.$$

18. $\dfrac{b+a}{b-a}-2\left(\dfrac{b}{a}-\dfrac{b}{a-b}\right)=-\dfrac{a+b}{a-b}+\dfrac{2b^2}{a(a-b)}$

$$=\frac{-a^2-ab+2b^2}{a(a-b)}$$

$$=-\frac{a^2+ab-2b^2}{a(a-b)}$$

$$=-\frac{(a-b)(a+2b)}{a(a-b)}=-\frac{a+2b}{a}.$$

19. $b-2a\left(1-\dfrac{2c-b}{c-b}\right)=b-2a\left(\dfrac{-c}{c-b}\right)$

$$=b+\frac{2ac}{c-b}=\frac{bc-b^2+2ac}{c-b}.$$

20. $\dfrac{x^2-(m-n)^2}{(x+n)^2-m^2}+\dfrac{m^2-(x-n)^2}{(x+m)^2-n^2}+\dfrac{n^2-(x-m)^2}{(m+n)^2-x^2}$

$$=\frac{(x+m-n)(x-m+n)}{(x+n+m)(x+n-m)}+\frac{(m+x-n)(m-x+n)}{(x+m+n)(x+m-n)}$$

$$+\frac{(n+x-m)(n-x+m)}{(m+n+x)(m+n-x)}$$

$$=\frac{x+m-n+m-x+n+n+x-m}{m+n+x}$$

$$=\frac{x+m+n}{x+m+n}$$

$$=1.$$

21. See p. 159.

Page 312

1. $\frac{6a}{25x^2y} \cdot \frac{10xy^2}{4a^2x} = \frac{3y}{5ax^2}.$

2. $\frac{5c}{1} \cdot \frac{3d}{10cd^2} \cdot \frac{4c}{9d} = \frac{2c}{3d^2}.$

3. $4\frac{1}{2} \cdot \frac{2a^2}{3x^3} \cdot \frac{x^2}{15a} = \frac{9}{2} \cdot \frac{2a^2}{3x^3} \cdot \frac{x^2}{15a} = \frac{a}{5x}.$

4. $\frac{10a}{1} \cdot \frac{7x}{2a^4} \cdot \frac{a}{21x^3} = \frac{5}{3a^2x^2}.$

5. $\frac{a^2-4}{2a^2} \cdot \frac{4a}{2a-4} = \frac{(a+2)(a-2)}{2a^2} \cdot \frac{4a}{2(a-2)} = \frac{a+2}{a}.$

6. $\frac{a^2-1}{2a} \cdot \frac{6a^3}{a^2-2a+1} \cdot \frac{5a-5}{3(1-a)}$

$= \frac{(a+1)(a-1)}{2a} \cdot \frac{6a^3}{(a-1)(a-1)} \cdot \frac{5(a-1)}{3(1-a)}$

$= \frac{5a^2(a+1)}{1-a}.$

7. $\left(2a+\frac{1}{a}\right)\left(\frac{a^2}{4a^4-1}\right)\left(a-\frac{1}{2a}\right) = \frac{2a^2+1}{a} \cdot \frac{a^2}{(2a^2+1)(2a^2-1)} \cdot \frac{2a^2-1}{2a} = \frac{1}{2}.$

8. $\frac{35}{42} \div \frac{25}{30} = \frac{5 \cdot 7}{2 \cdot 3 \cdot 7} \cdot \frac{3 \cdot 2 \cdot 5}{5^2} = 1.$

9. $\frac{14x}{15ax^2} \div \frac{84a}{60a^2x^3} = \frac{2 \cdot 7 \cdot x}{3 \cdot 5 \cdot ax^2} \cdot \frac{2^2 \cdot 3 \cdot 5a^2x^3}{2^2 \cdot 3 \cdot 7a} = \frac{2x^2}{3}.$

10. $\frac{120c^2d}{42c^3x} \div \frac{100cd^3}{147c^2x^2} = \frac{2^3 \cdot 3 \cdot 5c^2d}{2 \cdot 3 \cdot 7c^3x} \cdot \frac{3 \cdot 7^2c^2x^2}{2^2 \cdot 5^2cd^3} = \frac{3 \cdot 7x}{5d^2} = \frac{21x}{5d^2}.$

11. See pp. 167 and 170.
12. See p. 167.
13. See p. 170.

14. $\frac{24x^2}{99xy^2} \cdot \frac{8y^2z}{22x^2y} \div \frac{48x^3z}{121x^2y^3} = \frac{24x^2}{99xy^2} \cdot \frac{8y^2z}{22x^2y} \cdot \frac{121x^2y^3}{48x^3z} = \frac{2y^2}{9x^2}.$

15. $\frac{15rs^2}{14s^2t} \div \frac{300rt^3}{98r^2s} \cdot \frac{160st^4}{28r^2s^3} = \frac{15rs^2}{14s^2t} \cdot \frac{98r^2s}{300rt^3} \cdot \frac{160st^4}{28r^2s^3} = \frac{2}{s}.$

16. $\frac{22m^2n^2}{35x^3} \div \frac{52m^5n}{125x^2} \div \frac{275n^2x}{39m^3x} = \frac{22m^2n^2}{35x^3} \cdot \frac{125x^2}{52m^5n} \cdot \frac{39m^3x}{275n^2x} = \frac{3}{14nx}.$

17. $\frac{5x^2}{x-y} \cdot \frac{(x-y)^2}{20x^2} \div \frac{1}{x(x-y)} = \frac{5x^2}{x-y} \cdot \frac{(x-y)^2}{20x^2} \cdot \frac{x(x-y)}{1} = \frac{x(x-y)^2}{4}.$

18. $\frac{x+3}{x^2-4} \cdot \frac{x^2-4x+4}{2x+6} \div \frac{x^2-2x}{x^2+2x} = \frac{x+3}{(x+2)(x-2)} \cdot \frac{(x-2)^2}{2(x+3)} \cdot \frac{x(x+2)}{x(x-2)} = \frac{1}{2}.$

19. $\frac{x^2+2x+4}{x^2-2x+4} \div \frac{x^3-8}{x^2+2x} \div \frac{x^2-2x}{x^3+8}$

$= \frac{(x^2+2x+4)}{(x^2-2x+4)} \cdot \frac{x(x+2)}{(x-2)(x^2+2x+4)} \cdot \frac{(x+2)(x^2-2x+4)}{x(x-2)}$

$= \frac{(x+2)^2}{(x-2)^2}.$

20. $\dfrac{2x^2+9x+9}{2x^2-9x+9} \div \dfrac{x^2-9}{4x^3-9x} \cdot \dfrac{x^2-6x+9}{2x^2-3x} = \dfrac{(2x+3)(x+3)}{(2x-3)(x-3)} \cdot \dfrac{x(2x+3)(2x-3)}{(x+3)(x-3)} \cdot \dfrac{(x-3)^2}{x(2x-3)} = \dfrac{(2x+3)^2}{2x-3}.$

21. $\dfrac{x^3-6x^2+11x-6}{x^4-1} \div \dfrac{x^2-5x+6}{x^2+2x+1} \cdot \dfrac{x^2+1}{x+1} = \dfrac{(x-1)(x-2)(x-3)}{(x^2+1)(x+1)(x-1)} \cdot \dfrac{(x+1)^2}{(x-3)(x-2)} \cdot \dfrac{x^2+1}{x+1} = 1.$

22. $\left(\dfrac{2x}{3}\right)^2 \cdot \left(\dfrac{5}{4x}\right)^3 \left(\dfrac{6x}{25}\right)^2 = \dfrac{2^2x^2}{3^2} \cdot \dfrac{5^3}{2^6x^3} \cdot \dfrac{2^2 \cdot 3^2x^2}{5^4} = \dfrac{x}{20}.$

23. $\left(\dfrac{3a}{2b}\right)^3 \div \left(\dfrac{3a^2x}{5b^2}\right) \div \dfrac{(10x)^3}{64} = \dfrac{3^3a^3}{2^3b^3} \cdot \dfrac{5b^2}{3a^2x} \cdot \dfrac{2^6}{2^3 \cdot 5^3x^3} = \dfrac{3^2a}{5^2bx^4} = \dfrac{9a}{25bx^4}.$

24. $\left(\dfrac{2a}{5x^2}\right)^3 - 3\left(\dfrac{2a}{5x^2}\right)^2 \left(\dfrac{5x^3}{4a^2}\right) + 3\left(\dfrac{2a}{5x^2}\right)\left(\dfrac{5x^3}{4a^2}\right)^2 = \dfrac{2^3a^3}{5^3x^6} - \dfrac{3 \cdot 2^2a^2 \cdot 5 \cdot x^3}{5^2x^4 \cdot 2^2a^2} + \dfrac{3 \cdot 2a \cdot 5^2x^6}{5x^2 \cdot 2^4a^4}$

$= \dfrac{8a^3}{125x^6} - \dfrac{3}{5x} + \dfrac{15x^4}{8a^3}$

$= \dfrac{64a^6 - 600a^3x^5 + 1875x^{10}}{1000a^3x^6}.$

25. $\left(\dfrac{a^2}{2b}\right)^6 + 6\left(\dfrac{a^2}{2b}\right)^5\left(\dfrac{2b}{a}\right) + 15\left(\dfrac{a^2}{2b}\right)^4\left(\dfrac{2b}{a}\right)^2 + 30\left(\dfrac{a^2}{2b}\right)^3\left(\dfrac{2b}{a}\right)^3 = \dfrac{a^{12}}{64b^6} + \dfrac{3a^9}{8b^4} + \dfrac{15a^6}{4b^2} + 30a^3$

$= \dfrac{a^{12} + 24a^9b^2 + 240a^6b^4 + 1920a^3b^6}{64b^6}.$

26. $\dfrac{1-a^3}{1+a^2} \div \left(1 - \dfrac{2a^6}{a^6+1}\right) \div \left(\dfrac{1-a^2+a^4}{a^3+1}\right)$

$= \dfrac{(1-a)(1+a+a^2)}{1+a^2} \cdot \dfrac{(1+a^2)(1-a^2+a^4)}{(1+a)(1-a+a^2)(1-a)(1+a+a^2)} \cdot \dfrac{(a+1)(1-a+a^2)}{1-a^2+a^4}$

$= 1.$

27. $$\frac{a}{a^2-4} \div \left(\frac{a}{3a^2+5a+2}\right)\left(3a-\frac{2}{a}-5\right)$$
$$= \frac{a}{(a+2)(a-2)} \cdot \frac{(3a+2)(a+1)}{a} \cdot \frac{(3a+1)(a-2)}{a}$$
$$= \frac{(3a+2)(a+1)(3a+1)}{a(a+2)}.$$

28. $$\frac{x+1+\dfrac{4}{x+3}}{x-5+\dfrac{12}{x+3}} = \frac{\dfrac{x^2+4x+7}{x+3}}{\dfrac{x^2-2x-3}{x+3}} = \frac{x^2+4x+7}{x^2-2x-3}.$$

29. $$\frac{2+\dfrac{x+6}{x-2}}{3+\dfrac{5}{x-2}} = \frac{\dfrac{3x+2}{x-2}}{\dfrac{3x-1}{x-2}} = \frac{3x+2}{3x-1}.$$

30. $$\frac{x-1-\dfrac{3}{x}}{x-\dfrac{x}{x-3}} = \frac{\dfrac{x^2-x-3}{x}}{\dfrac{x^2-x-3}{x-3}} = \frac{x-3}{x}.$$

31. $$\frac{\dfrac{a}{b}+1+\dfrac{b}{a}}{\dfrac{b^2}{a^2}+1+\dfrac{a^2}{b^2}} = \frac{\dfrac{a^2+ab+b^2}{ab}}{\dfrac{a^4+a^2b^2+b^4}{a^2b^2}} = \frac{ab(a^2+ab+b^2)}{(a^2+ab+b^2)(a^2-ab+b^2)} = \frac{ab}{a^2-ab+b^2}.$$

32. $$\frac{\dfrac{a}{a+1}+\dfrac{1-a}{a}}{\dfrac{a}{a+1}-\dfrac{1-a}{a}} = \frac{\dfrac{a^2+1-a^2}{a(a+1)}}{\dfrac{a^2-1+a^2}{a(a+1)}} = \frac{1}{2a^2-1}.$$

33. $$\frac{\dfrac{a-b}{a+b}+\dfrac{a+b}{a-b}}{1+\dfrac{2a^2b^2+2b^4}{a^4-b^4}} = \frac{\dfrac{2a^2+2b^2}{a^2-b^2}}{\dfrac{a^4+2a^2b^2+b^4}{a^4-b^4}} = 2.$$

34. $$\left(\frac{x+y}{x-y}+2\right) \div \left(\frac{x+y}{x-y}-2\right) = \frac{3x-y}{x-y} \cdot \frac{x-y}{3y-x} = \frac{3x-y}{3y-x}.$$

35. $$\frac{\dfrac{2}{b}-\dfrac{1}{a+b}+\dfrac{1}{a-b}}{\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}} = \frac{\dfrac{2a^2-2b^2-ab+b^2+ab+b^2}{b(a^2-b^2)}}{\dfrac{a^2+2ab+b^2-a^2+2ab-b^2}{a^2-b^2}} = \frac{2a^2}{b} \cdot \frac{1}{4ab} = \frac{a}{2b^2}.$$

36. $\dfrac{a^2+b^2}{a^2-b^2}-\dfrac{\dfrac{b}{a+b}}{1+\dfrac{2b-a}{a-b}}=\dfrac{a^2+b^2}{a^2-b^2}-\dfrac{a-b}{a+b}=\dfrac{a^2+b^2-(a-b)^2}{a^2-b^2}=\dfrac{2ab}{a^2-b^2}.$

37. $\dfrac{2}{\dfrac{1}{b^2}\left(\dfrac{1}{c}-\dfrac{b}{2a}\right)}-b^2c\left(1-\dfrac{bc}{a-\dfrac{bc}{2}}\right)=\dfrac{2}{\dfrac{1}{b^2c}-\dfrac{1}{2ab}}-b^2c\left(1-\dfrac{2bc}{2a-bc}\right)$

$$=\frac{2}{\dfrac{2a-bc}{2ab^2c}}-b^2c\left(\frac{2a-3bc}{2a-bc}\right)$$

$$=\frac{4ab^2c}{2a-bc}-\frac{2ab^2c-3b^3c^2}{2a-bc}$$

$$=\frac{2ab^2c+3b^3c^2}{2a-bc}.$$

38. $\dfrac{2b}{\dfrac{c}{2a}-2}-\dfrac{b}{ac}\left(\dfrac{4ab}{\dfrac{4b}{c}-\dfrac{b}{a}}-\dfrac{a}{\dfrac{1}{c}-\dfrac{1}{b}}\right)=\dfrac{2b}{\dfrac{c-4a}{2a}}-\dfrac{b}{ac}\left(\dfrac{4ab}{\dfrac{4ab-bc}{ac}}-\dfrac{a}{\dfrac{b-c}{bc}}\right)$

$$=\frac{4ab}{c-4a}-\frac{b}{ac}\left(\frac{4a^2bc}{b(4a-c)}-\frac{abc}{b-c}\right)$$

$$=\frac{4ab}{c-4a}-\frac{4ab}{4a-c}+\frac{b^2}{b-c}$$

$$=\frac{8ab}{c-4a}+\frac{b^2}{b-c}$$

$$=\frac{8ab^2-8abc+b^2c-4ab^2}{(c-4a)(b-c)}$$

$$=\frac{4ab^2-8abc+b^2c}{(c-4a)(b-c)}.$$

39. If $a=x-\dfrac{1}{x}$ and $b=x+\dfrac{1}{x}$, then

$$\frac{a^3-b^3}{a^2+b^2}=\frac{\left(x-\dfrac{1}{x}\right)^3-\left(x+\dfrac{1}{x}\right)^3}{\left(x-\dfrac{1}{x}\right)^2+\left(x+\dfrac{1}{x}\right)^2}=\frac{-6x-\dfrac{2}{x^3}}{2x^2+\dfrac{2}{x^2}}=-\frac{3x+\dfrac{1}{x^3}}{x^2+\dfrac{1}{x^2}}=-\frac{3x^4+1}{x(x^4+1)}.$$

40. $1-\dfrac{1}{1-\dfrac{1}{1+\frac{1}{2}}}=1-\dfrac{1}{1-\frac{2}{3}}=1-3=-2.$

41. $1-\cfrac{1}{1+\cfrac{2}{1-\frac{2}{3}}}=1-\dfrac{1}{1+6}=1-\dfrac{1}{7}=\dfrac{6}{7}.$

42. $1+\cfrac{1}{1-\cfrac{1}{1+\cfrac{1}{a}}}=1+\cfrac{1}{1-\cfrac{1}{\cfrac{a+1}{a}}}=1+\cfrac{1}{1-\cfrac{a}{a+1}}=1+\cfrac{1}{\cfrac{1}{a+1}}$

$=1+a+1$

$=a+2.$

Page 314

1. $\dfrac{5x}{2}-2=2x.$ (1)

(1) · 2, $5x-4=4x.$

$x=4.$

2. $\dfrac{2x}{5}+1=x-\dfrac{7}{5}.$ (1)

(1) · 5, $2x+5=5x-7.$

$-3x=-12.$

$x=4.$

3. $\dfrac{11x^2}{9}-\dfrac{2x}{3}=x^2.$ (1)

(1) · 9, $11x^2-6x=9x^2.$

$2x^2-6x=0.$

$x=0, 3.$

4. $\dfrac{3x}{2}-\dfrac{16}{3}=x-\dfrac{25}{6}.$ (1)

(1) · 6, $9x-32=6x-25.$

$3x=7.$

$x=\frac{7}{3}.$

5. $\dfrac{5x}{6}-\dfrac{21}{5}=\dfrac{5x-2}{10}.$ (1)

(1) · 30, $25x-126=15x-6.$

$10x=120.$

$x=12.$

6. $\dfrac{2(x-4)}{3}-1=\dfrac{x+5}{3}.$ (1)

(1) · 3, $2x-8-3=x+5.$

$x=16.$

7. $\dfrac{2x+1}{4}-\dfrac{1-2x}{3}=9\frac{1}{4}.$ (1)

(1) · 12,

$6x+3-4+8x=111.$

$14x=112.$

$x=8.$

8. $\dfrac{3}{4x}+\dfrac{7}{16}=\dfrac{4}{3x}.$ (1)

(1) · $48x$, $36+21x=64.$

$21x=28.$

$x=\frac{4}{3}.$

9. $3(x+2)-4(2x-3)+2=0.$

$3x+6-8x+12+2=0.$

$-5x=-20.$

$x=4.$

10. $\dfrac{x+5}{x-2}=\dfrac{10}{3}.$ (1)

(1) · $3(x-2)$,

$3x+15=10x-20.$

$-7x=-35.$

$x=5.$

11. $\dfrac{8}{7x+3}=\dfrac{3}{3x+1}.$ (1)

(1) · $(7x+3)(3x+1)$,

$24x+8=21x+9.$

$3x=1.$

$x=\frac{1}{3}.$

12. $$\frac{2x-7}{3x-4}+\frac{3}{2}=0. \qquad (1)$$

(1) · 2 (3 x − 4), $$4x-14+9x-12=0.$$
$$13x=26.$$
$$x=2.$$

13. $$(x+5)(x+1)-(x-3)(x-2)=10.$$
$$x^2+6x+5-x^2+5x-6=10.$$
$$11x=11.$$
$$x=1.$$

14. $$\frac{3}{2x-5}+\frac{1}{2x-1}=0. \qquad (1)$$

(1) · (2 x − 5) (2 x − 1), $$6x-3+2x-5=0.$$
$$x=1.$$

15. $$\frac{2x-5}{2x+7}=\frac{3x-14}{3x-2}. \qquad (1)$$

(1) · (2 x + 7) (3 x − 2), $$6x^2-19x+10=6x^2-7x-98.$$
$$-12x=-108.$$
$$x=9.$$

16. $$5x+3=\frac{(5x+7)(2x-3)}{2x}. \qquad (1)$$

(1) · 2 x $$10x^2+6x=10x^2-x-21.$$
$$x=-3.$$

17. $$\frac{1}{x}-\frac{1-x}{6-x^2}=0. \qquad (1)$$

(1) · x (6 − x²), $$6-x^2-x+x^2=0.$$
$$x=6.$$

18. $$\frac{x+3}{2x+1}=\frac{3x+4}{6x-2}. \qquad (1)$$

(1) · (2 x + 1) (6 x − 2), $$6x^2+16x-6=6x^2+11x+4.$$
$$x=2.$$

19. $$\frac{x-3}{x+7}=1-\frac{5}{x+1}. \qquad (1)$$

(1) · (x + 7) (x + 1), $$x^2-2x-3=x^2+8x+7-5x-35.$$
$$x=5.$$

20. $$\frac{3(1-8x)}{5}-\frac{1+8x}{8x-1}=\frac{1-34x}{5}. \qquad (1)$$

$(1)\cdot 5(8x-1)$,

$$-192x^2+48x-3-10x-80x^2=-272x^2+42x-1.$$
$$x=-\tfrac{1}{2}.$$

21. $$\frac{3}{2x+1}+\frac{2x+1}{1-2x}=1-\frac{8x^2}{4x^2-1}. \qquad (1)$$

$(1)\cdot(4x^2-1)$,

$$6x-3-4x^2-4x-1=4x^2-1-8x^2.$$
$$x=\tfrac{3}{2}.$$

22. $$\frac{1+2x}{2x-1}-\frac{3x+1}{x+1}=\frac{8+3x-4x^2}{2x^2+x-1}. \qquad (1)$$

$(1)\cdot(2x-1)(x+1)$,

$$2x^2+3x+1-6x^2+x+1=8+3x-4x^2.$$
$$x=6.$$

23. $$\frac{x}{a}-a=\frac{x}{b}-b. \qquad (1)$$

$(1)\cdot ab$, $$bx-a^2b=ax-ab^2.$$
$$bx-ax=a^2b-ab^2.$$
$$x=-ab.$$

24. $$\frac{x+a}{a}-\frac{x+b}{b}=b-a. \qquad (1)$$

$(1)\cdot ab$, $$bx+ab-ax-ab=ab^2-a^2b.$$
$$(b-a)x=ab(b-a).$$
$$x=ab.$$

25. $$\frac{x^2-x}{a-1}+\frac{x^2-x}{a+1}=2a(x-1). \qquad (1)$$

$(1)\cdot(a-1)(a+1)$, $$2ax^2-2a^3x=2a-2a^3.$$
$$x^2-1-a^2(x-1)=0.$$
$$(x-1)(x+1-a^2)=0.$$
$$x=a^2-1,\ 1.$$

26. $$\frac{x}{a+2}-\frac{x+16}{a-2}=4a. \qquad (1)$$

$(1)\cdot(a^2-4)$,

$$ax-2x-ax-2x-16a-32=4a^3-16a.$$
$$x=-a^3-8.$$

27. $$\frac{2x+1}{x+3}+\frac{3x-7}{2-x}=\frac{9-3x-x^2}{x^2+x-6}. \qquad (1)$$

$(1)\cdot(x^2+x-6)$,

$$2x^2-3x-2-3x^2-2x+21=9-3x-x^2.$$
$$x=5.$$

28. $$\frac{x-a}{2x-a}-\frac{3x-c}{6x-c}=0. \quad (1)$$

$(1)\cdot(2x-a)(6x-c)$,
$$6x^2-cx-6ax+ac=6x^2-3ax-2cx+ac.$$
$$cx-3ax=0.$$
$$(c-3a)x=0.$$
$$x=0.$$

29. $$11.3-\frac{5-2x}{.5}=2.3-(5-7x)+\frac{x+2}{.2}. \quad (1)$$

$(1)\cdot 10$, $$113-100+40x=23-50+70x+50x+100.$$
$$-80x=60.$$
$$x=-\tfrac{3}{4}.$$

30. $$\frac{3x-1}{.25}+\frac{x-4}{.5}=3(3x-14). \quad (1)$$

$(1)\cdot 1$, $$12x-4+2x-8=9x-42.$$
$$5x=-30.$$
$$x=-6.$$

Page 316

1. Let $x=$ the first part.
Then $\frac{11}{6}x=$ the second part,
and $\frac{25}{9}x=$ the third part.
$$x+\frac{11x}{6}+\frac{25x}{9}=1010.$$
$$18x+33x+50x=18{,}180.$$
$$x=180,$$
$$\tfrac{11}{6}x=330,$$
and $$\tfrac{25}{9}x=500.$$

2. Let $x=$ the divisor.
Then $$\frac{352}{x}=15+\frac{7}{x}.$$
$$352=15x+7.$$
$$x=23.$$

3. Let $x=$ the number to be subtracted.
Then $$\frac{32-x}{39-x}=\frac{2}{3}.$$
$$96-3x=78-2x.$$
$$x=18.$$

4. Let $x=$ one part.
Then $96-x=$ the other part.
$$56-\tfrac{2}{3}x=96-x-16.$$
$$x=72,$$
and $$96-x=24.$$

5. Let $x=$ the number of years.
$$x+12=\tfrac{6}{5}(x+8).$$
$$5x+60=6x+48.$$
$$x=12.$$

6. Let $x =$ the number.

Then $x + 1$ and $x + 2 =$ the next two consecutive numbers.

$$x^2 - 4 = \tfrac{2}{3}(x+1)(x+2).$$
$$3x^2 - 12 = 2x^2 + 6x + 4.$$
$$x^2 - 6x - 16 = 0.$$
$$x = 8, -2.$$

7. Let $x =$ the number of yards in the width of the rectangle.

Then $2\frac{1}{2}x =$ the number of yards in the length of the rectangle.

$$(x + 1\tfrac{1}{2})(2\tfrac{1}{2}x - 10) = x \cdot 2\tfrac{1}{2}x - \tfrac{1260}{9}.$$
$$\frac{5}{2}x^2 - \frac{25x}{4} - 15 = \frac{5}{2}x^2 - 140.$$
$$x = 20,$$

and $2\frac{1}{2}x = 50.$

8. Let $x =$ the number of days required by A and B together.

$$\frac{1}{x} = \frac{1}{10} + \frac{1}{12}.$$
$$60 = 6x + 5x.$$
$$x = 5\tfrac{5}{11}.$$

9. Let $x =$ the number of days A requires to finish.

$$\frac{5}{10} + \frac{5}{15} + \frac{x}{10} = 1.$$
$$x = 1\tfrac{2}{3}.$$

10. Let $x =$ the number of hours to fill the tank.

$$\frac{1}{4} - \frac{1}{6} = \frac{1}{x}.$$
$$3x - 2x = 12.$$
$$x = 12.$$

11. Let $x =$ the number of hours to empty the tank.

$$\frac{1}{6} + \frac{1}{8} - \frac{1}{4} = \frac{1}{x}.$$
$$4x + 3x - 6x = 24.$$
$$x = 24.$$

12. Let $x =$ the number of miles in the diameter of the moon.

Then $3\frac{2}{3}x =$ the number of miles in the diameter of the earth.

$$3\tfrac{2}{3}x - x = 5760.$$
$$x = 2160,$$

and $3\frac{2}{3}x = 7920.$

13. Let $x =$ the number of miles in the diameter of the earth.

Then $10\frac{10}{11}x =$ the number of miles in the diameter of Jupiter.

$$x + 10\tfrac{10}{11}x = 94{,}320.$$
$$x = 7920,$$

and $10\frac{10}{11}x = 86{,}400.$

14. Let $x =$ the number of hours of rowing upstream.

Then $10 - x =$ the number of hours of rowing downstream.

$$(4 + 1\tfrac{1}{2})(10 - x) = x(4 - 1\tfrac{1}{2}).$$
$$55 - 5\tfrac{1}{2}x = 2\tfrac{1}{2}x.$$
$$x = 6\tfrac{7}{8}.$$

15. Let $x =$ the number of miles per hour the freight train travels.

Then $8x =$ the number of miles it travels.

$$8x = 40 \cdot 5\tfrac{1}{4} = 210.$$
$$x = 26\tfrac{1}{4}.$$

16. Let $x =$ the number of dollars invested at 5%.

Then $8000 - x =$ the number of dollars invested at 4%.

$$\tfrac{5}{100}x + \tfrac{4}{100}(8000 - x) = 345.$$
$$5x + 32{,}000 - 4x = 34{,}500.$$
$$x = 2500,$$

and $8000 - x = 5500.$

17. Let $x =$ the number of dollars taxed 4%.

$$\tfrac{2}{100}(10{,}000 - 2000) + \tfrac{4}{100}x = 180.$$
$$16{,}000 + 4x = 18{,}000.$$
$$x = 500.$$
$$\text{Income} = 10{,}000 + 500 = 10{,}500.$$

18. Let $x =$ the number of gallons of water to be added.

$$\frac{\frac{95}{100}\cdot 1}{1+x}=\frac{1}{10}.$$

$$\frac{95}{100(1+x)}=\frac{1}{10}.$$

$$95 = 10 + 10x.$$

$$x = 8.5.$$

19. Let $x =$ the number of pounds of 25-cent coffee.

Then $10 - x =$ the number of pounds of 35-cent coffee.

$$25x + 35(10-x) = 32 \cdot 10.$$

$$25x + 350 - 35x = 320.$$

$$x = 3,$$

and $10 - x = 7.$

Weight of 25-cent coffee used is to weight of 35-cent coffee as 3 : 7.

20. Let $x =$ the number of gallons of water.

$$\tfrac{20}{100}\cdot 25 = \tfrac{18}{100}(25+x).$$

$$500 = 450 + 18x.$$

$$x = 2\tfrac{7}{9}.$$

21. Let $x =$ the number of pounds of tin to be added.

$$\frac{\frac{16}{100}\cdot 410 + x}{410+x}=\frac{18}{100}.$$

$$6560 + 100x = 7380 + 18x.$$

$$x = 10.$$

22. Let $x =$ the number of gallons of 90-per-cent-pure alcohol.

Then $$\frac{\frac{90}{100}x + \frac{96}{100}\cdot 12}{x+12}=\frac{93}{100}.$$

$$90x + 1152 = 93x + 1116.$$

$$x = 12.$$

23. Let $x =$ the number of tons of pure iron.

$$\frac{\frac{93}{100}\cdot 10 + x}{10+x}=\frac{98}{100}.$$

$$930 + 100x = 980 + 98x.$$

$$x = 25.$$

24. Let $x =$ the number of pounds on the shorter arm.

$$6 \cdot 70 = 5 \cdot x.$$

$$x = 84.$$

25. Let $x =$ the number of pounds on the longer arm.

$$7 \cdot 36 = 12 \cdot x.$$
$$x = 21$$

26. Let $x =$ the number of tons stress on one support.

Then $2 - x =$ the number of tons stress on the other support.

$$4x = 8(2 - x).$$
$$x = 1\tfrac{1}{3},$$

and $2 - x = \frac{2}{3}$.

28. Let $x =$ the number of spaces the minute hand travels.

$$x - \frac{x}{12} = 45.$$
$$12x - x = 540.$$

$x = 49\frac{1}{11}$, or $49\frac{1}{11}$ minutes after 9 o'clock.

29. Let $x =$ the number of spaces the minute hand travels.

$$x - \frac{x}{12} = 45.$$

$x = 49\frac{1}{11}$, or $49\frac{1}{11}$ minutes after 3 o'clock.

Also $x - \frac{x}{12} = 15.$

$x = 16\frac{4}{11}$, or $16\frac{4}{11}$ minutes after 3 o'clock.

30. Let $x =$ the number of spaces the minute hand travels.

$$x - \frac{x}{12} = 35.$$
$$12x - x = 420.$$

$x = 38\frac{2}{11}$, or $38\frac{2}{11}$ minutes after 5 o'clock.

Also $x - \frac{x}{12} = 15.$

$x = 16\frac{4}{11}$, or $16\frac{4}{11}$ minutes after 5 o'clock.

31. Let $x =$ the number of spaces the minute hand travels.

$$x - \frac{x}{12} = 35.$$

$x = 38\frac{2}{11}$, or $38\frac{2}{11}$ minutes after 4 o'clock.

Also $x - \frac{x}{12} = 5.$

$x = 5\frac{5}{11}$, or $5\frac{5}{11}$ minutes after 4 o'clock.

32. If $x =$ the number of inches average height, the sum of the heights of n boys is nx inches.

$$nx \text{ inches} = \frac{nx}{36} \text{ yards.}$$

33. One package weighs $\dfrac{p}{3m+5}$ pounds.

Then n packages would weigh $n \cdot \dfrac{p}{3m+5} = \dfrac{np}{3m+5}$ pounds.

34. If a man can do a piece of work in t hours, in one hour he will do $\frac{1}{t}$ part of the work; and n men working at the same rate would do $n \cdot \frac{1}{t}$, or $\frac{n}{t}$, part of the work in one hour. k men in h hours would do $k \cdot h \cdot \frac{1}{t}$, or $\frac{hk}{t}$, part of the work.

35. Let $t =$ the number of hours for x men to do it.

$$t = \frac{nh}{x}.$$

36. Let $g =$ the number of cents gain on n dozen.

$$g = 12n\left(b - \frac{d}{12}\right)$$
$$= n(12b - d).$$

37. Let $l =$ the number of dollars loss.

$$l = \frac{\frac{mc}{100}}{100} - 10$$
$$= \frac{mc}{10000} - 10.$$

38. Let $p =$ the cost in cents of x yards.

$$p = \frac{xc}{n}.$$

39. Let $n =$ the number of yards.

$$n = \frac{100d}{\frac{x}{y}} = \frac{100dy}{x}.$$

40. Let $p =$ the per cent of gain.

$$p = \frac{s-b}{b}.$$

41. Let $x =$ number of days both will take.

$$\frac{1}{d} + \frac{1}{n} = \frac{1}{x}.$$
$$nx + dx = nd.$$
$$x = \frac{nd}{n+d}.$$

42. Let $d =$ the number of 10-hour days.

$$d = \frac{n}{10 \cdot \frac{m}{h}} = \frac{hn}{10m}.$$

43. $$m = \frac{3600\frac{y}{t}}{1760} = \frac{45y}{22t}.$$

44. Let $m =$ the number of men required.

$$m = \frac{nd}{t}.$$

45. Let $x =$ the number of minutes under fire.

$$x = \frac{f}{6080k} \times 60 = \frac{3f}{304k}.$$

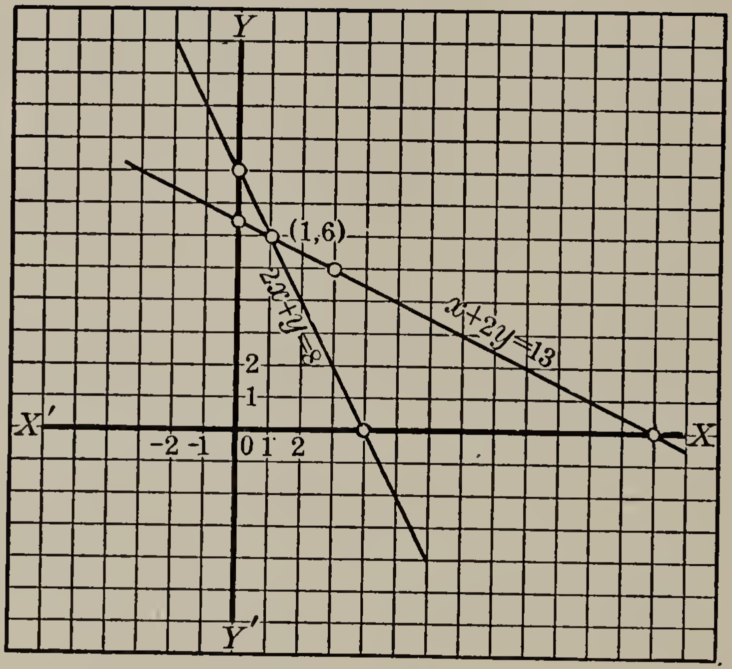
Page 321

1. For $2x + y = 8$.

If $x =$	0	4	1
then $y =$	8	0	6

For $x + 2y = 13$.

If $x =$	0	13	3
then $y =$	$6\frac{1}{2}$	0	5



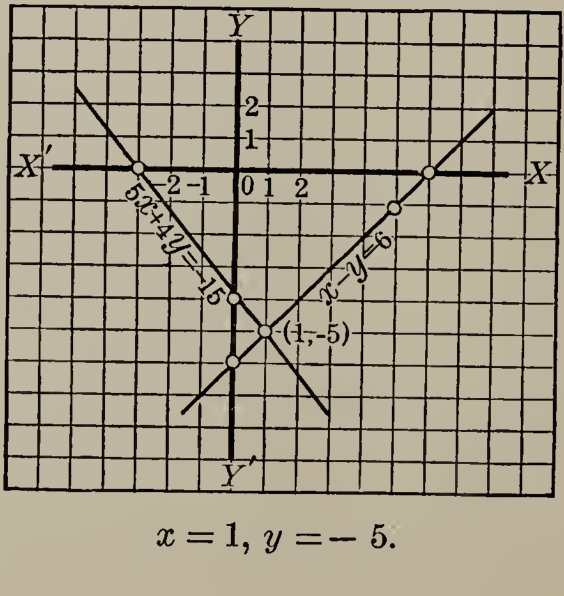
$x = 1, y = 6.$

2. For $x - y = 6$.

If $x =$	0	6	5
then $y =$	-6	0	-1

For $5x + 4y = -15$.

If $x =$	0	-3	1
then $y =$	$-3\frac{3}{4}$	0	-5



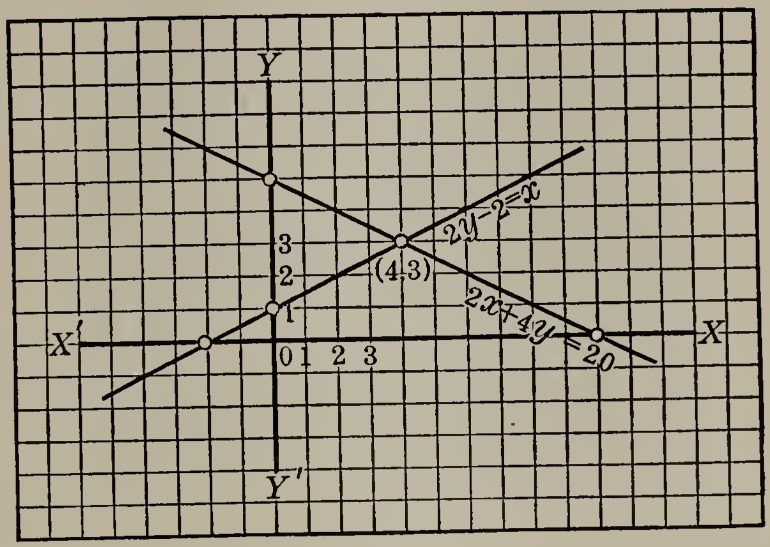
$x = 1, y = -5.$

3. For $2x + 4y = 20$.

If $x =$	0	10	4
then $y =$	5	0	3

For $2y - 2 = x$.

If $x =$	0	-2	4
then $y =$	1	0	3



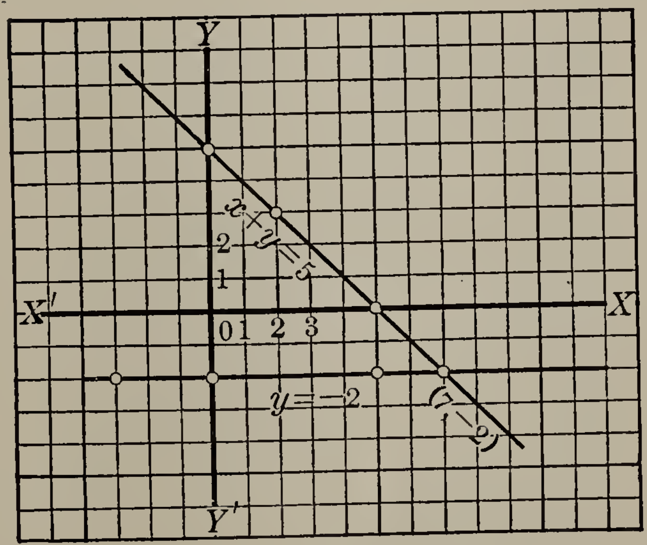
$x = 4$, $y = 3$.

4. For $x + y = 5$.

If $x =$	0	5	2
then $y =$	5	0	3

For $y + 2 = 0$, or $0x + y = -2$.

If $x =$	4	0	-3
then $y =$	-2	-2	-2



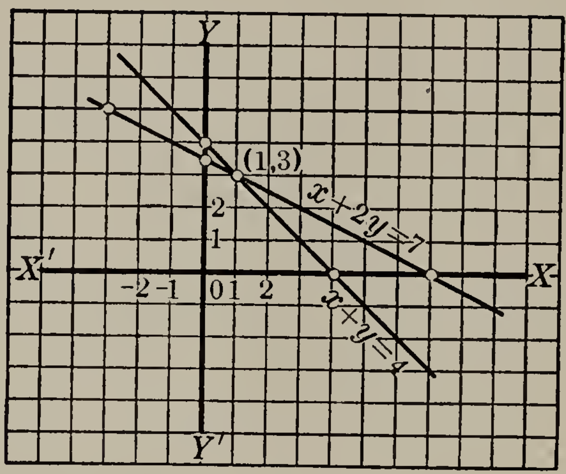
$x = 7$, $y = -2$.

5. For $x + y = 4$.

If $x =$	0	4	1
then $y =$	4	0	3

For $x + 2y = 7$.

If $x =$	0	7	-3
then $y =$	$\frac{7}{2}$	0	5



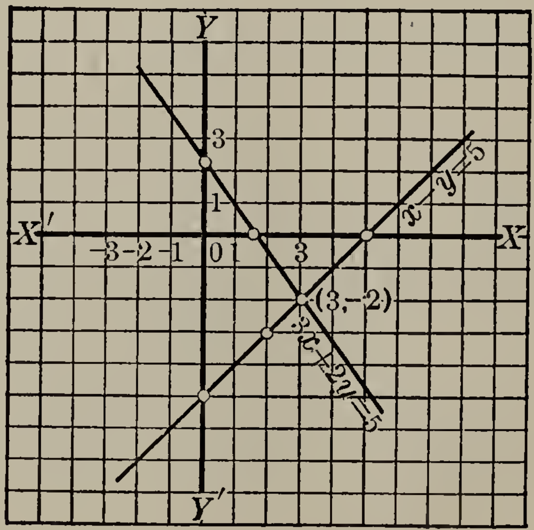
$x = 1, y = 3.$

6. For $x - y = 5$.

If $x =$	0	5	2
then $y =$	-5	0	-3

For $3x + 2y = 5$.

If $x =$	0	$1\frac{2}{3}$	3
then $y =$	$2\frac{1}{2}$	0	-2



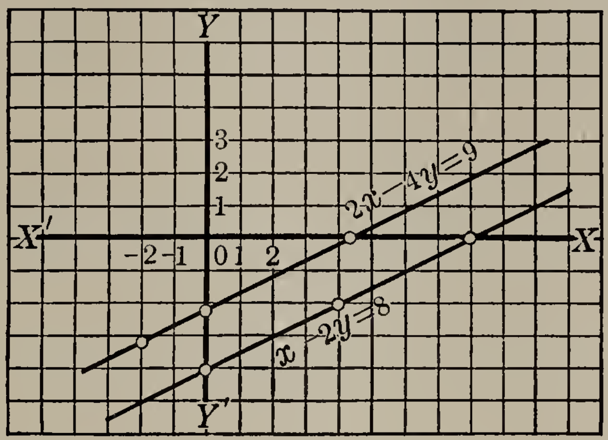
$x = 3, y = -2.$

7. For $2x - 4y = 9$.

If $x =$	0	$\frac{9}{2}$	-2
then $y =$	$-\frac{9}{4}$	0	$-3\frac{1}{4}$

For $x - 2y = 8$.

If $x =$	0	8	4
then $y =$	-4	0	-2

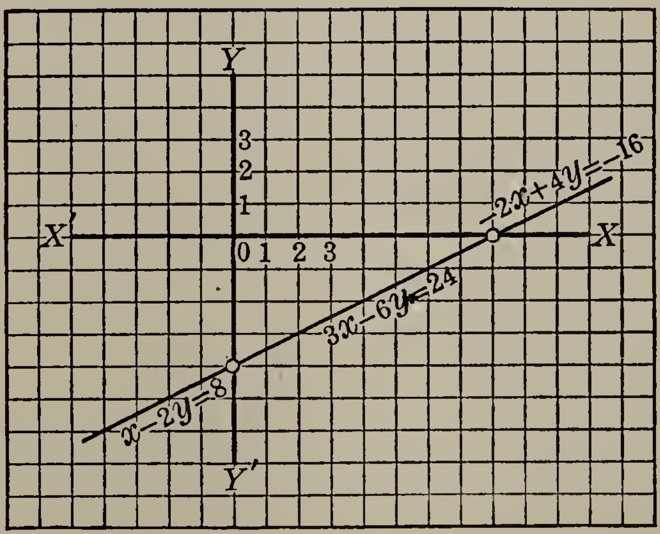


The lines are parallel, and the equations are inconsistent.

8. Yes.

9. For $x - 2y = 8$, for $3x - 6y = 24$, and for $-2x + 4y = -16$.

If $x =$	0	8
then $y =$	-4	0



The three graphs are identical.

An infinite number of sets of roots will satisfy the three equations.

The conclusion that any two dependent equations have the same graph seems warranted.

CR

10. The coördinates of the origin are (0, 0).

11. Yes, if the graphs of the two equations are parallel lines, as in Example 7, above, or in $x + 2y = 5$ and $x + 2y = 7$.

12. The form of the equation of a line parallel to the x-axis is one in which x does not occur, as $y = a$. The form of the equation of a line parallel to the y-axis is one in which y does not occur, as $x = b$.

Page 322

1.

$$3x - 8y = 20, \quad (1)$$
$$x - 6y = 0. \quad (2)$$

From (2), $x = 6y$. (3)

Then in (1), $18y - 8y = 20.$

$y = 2.$

Then in (2), $x - 12 = 0.$

$x = 12.$

2.

$$2x + 5y = 8, \quad (1)$$
$$x - 10y = 9. \quad (2)$$

From (2), $x = 10y + 9.$ (3)

Then in (1),

$2(10y + 9) + 5y = 8.$

$25y = -10.$

$y = -\frac{2}{5}.$

Then in (3), $x = -4 + 9.$

$x = 5.$

3.

$$2(x+y)+3y = 4, \quad (1)$$
$$5 = x + y. \quad (2)$$

Substituting 5 for $x + y$ in (1),

$10 + 3y = 4.$

$y = -2.$

Then in (2), $5 = x - 2.$

$x = 7.$

4.

$$16x + 7 = 15y, \quad (1)$$
$$4x + 5y = 0. \quad (2)$$

From (2), $y = -\frac{4}{5}x.$ (3)

Then in (1),

$16x + 7 = -12x.$

$x = -\frac{1}{4}.$

Then in (3), $y = \frac{1}{5}.$

5.

$$3x - 2y = 18, \quad (1)$$
$$4y + 3x = 0. \quad (2)$$

From (2), $y = -\frac{3}{4}x.$ (3)

Then in (1),

$3x + \frac{3}{2}x = 18.$

$9x = 36.$

$x = 4.$

Then in (3), $y = -3.$

6.

$$\frac{8m - 3n}{2} + 6n = -9, \quad (1)$$
$$4m - 1 = 3n. \quad (2)$$

Clearing (1) of fractions,

$8m + 9n = -18.$ (3)

From (2), $m = \frac{3n + 1}{4}.$ (4)

Then in (3),

$2(3n + 1) + 9n = -18.$

$15n = -20.$

$n = -\frac{4}{3}.$

Then from (4),

$$m = \frac{-4 + 1}{4}$$
$$= -\frac{3}{4}.$$

Page 323

1. $$x + y = 4, \quad (1)$$
$$x - y = 2. \quad (2)$$
From (2), $$x = y + 2. \quad (3)$$
Substituting (3) in (1), $$y + 2 + y = 4.$$
$$2y = 2.$$
$$y = 1.$$
Then from (2), $$x - 1 = 2.$$
$$x = 3.$$

2. $$x + 2y = 3, \quad (1)$$
$$x + y = 2. \quad (2)$$
From (2), $$x = 2 - y. \quad (3)$$
Substituting (3) in (1), $$2 - y + 2y = 3.$$
$$2 + y = 3.$$
$$y = 1.$$
Then from (2), $$x + 1 = 2.$$
$$x = 1.$$

3. $$x - y = 5, \quad (1)$$
$$2x + y = 4. \quad (2)$$
From (1), $$x = 5 + y. \quad (3)$$
Substituting (3) in (2),
$$10 + 3y = 4.$$
$$3y = -6.$$
$$y = -2.$$
Then from (1),
$$x + 2 = 5.$$
$$x = 3.$$

4. $$2x + 3y = 8, \quad (1)$$
$$x - y = 1. \quad (2)$$
From (2), $$x = y + 1. \quad (3)$$
Substituting (3) in (1),
$$5y + 2 = 8.$$
$$5y = 6.$$
$$y = \tfrac{6}{5}.$$
Then from (2),
$$x - \tfrac{6}{5} = 1.$$
$$x = \tfrac{5}{5} + \tfrac{6}{5}.$$
$$x = \tfrac{11}{5}.$$

5. $$x + 3y = 0, \quad (1)$$
$$2x - 3y = 9. \quad (2)$$
From (1), $$x = -3y \quad (3)$$
Substituting (3) in (2),
$$-6y - 3y = 9.$$
$$-9y = 9.$$
$$y = -1.$$
Then from (1),
$$x - 3 = 0.$$
$$x = 3.$$

6. $$5x - 6y = 7, \quad (1)$$
$$4x - 3y = 2. \quad (2)$$
From (2), $$x = \frac{2 + 3y}{4}. \quad (3)$$
Substituting (3) in (1),
$$\frac{10 + 15y}{4} - 6y = 7.$$
$$10 + 15y - 24y = 28.$$
$$-9y = 18.$$
$$y = -2.$$
Then from (2),
$$4x + 6 = 2.$$
$$4x = -4.$$
$$x = -1.$$

Page 324

1. $\dfrac{2x}{3} + y = -7, \quad (1)$

$5x - 3y = 105. \quad (2)$

Clearing (1) of fractions,

$2x + 3y = -21. \quad (3)$

(2) + (3), $7x = 84.$

$x = 12.$

Then from (2), $y = -15.$

2. $\dfrac{3r}{4} - \dfrac{7}{2} = \dfrac{s}{12}, \quad (1)$

$r + 8 = -2s. \quad (2)$

Clearing (1) of fractions,

$9r - 42 = s. \quad (3)$

$2 \cdot (3)$, $18r - 84 = 2s. \quad (4)$

(2) + (4),

$19r - 76 = 0.$

$r = 4.$

Then from (2), $s = -6.$

3. $\dfrac{3y+1}{4} - \dfrac{z+22}{12} = 3, \quad (1)$

$z - 2y = 1. \quad (2)$

From (2), $z = 2y + 1.$

In (1),

$\dfrac{3y+1}{4} - \dfrac{2y+23}{12} = 3.$

$9y + 3 - 2y - 23 = 36.$

$7y = 56.$

$y = 8.$

Then from (2), $z = 17.$

4. $.5x + .7y = \frac{3}{5}, \quad (1)$

$.8x - .2y = 3\frac{3}{5}. \quad (2)$

$(1) \cdot 8$, $4x + 5.6y = 4.8. \quad (3)$

$(2) \cdot 5$, $4x - y = 18. \quad (4)$

(3) − (4), $6.6y = -13.2.$

$y = -2.$

Then in (4), $x = 4.$

5. $\dfrac{m+2n}{5} - \dfrac{2m-n}{10} = \dfrac{5}{2}, \quad (1)$

$\dfrac{m+n}{4} - \dfrac{m-n}{7} = \dfrac{n}{2}. \quad (2)$

Clearing (1) and (2) of fractions,

$2(m + 2n) - (2m - n) = 25. \quad (3)$

$7(m + n) - 4(m - n) = 14n. \quad (4)$

Simplifying (3) and (4),

$5n = 25.$

$n = 5. \quad (5)$

$3m - 3n = 0. \quad (6)$

Substituting from (5), in (6),

$3m = 15.$

$m = 5.$

6. $\dfrac{1}{x} + \dfrac{1}{y} = -\dfrac{1}{6}, \quad (1)$

$\dfrac{2}{x} - \dfrac{3}{y} = \dfrac{4}{3}. \quad (2)$

$3 \cdot (1) + (2)$, $\dfrac{5}{x} = \dfrac{5}{6}.$

$x = 6.$

$2 \cdot (1) - (2)$, $\dfrac{5}{y} = -\dfrac{5}{3}.$

$y = -3.$

7. $\dfrac{2r+4s}{2r-s} = \dfrac{38}{3}, \quad (1)$

$\dfrac{5}{r} - \dfrac{7}{s} = 0. \quad (2)$

Clearing of fractions,

$6r + 12s = 76r - 38s. \quad (3)$

$5s - 7r = 0. \quad (4)$

From (3),

$70r - 50s = 0,$

or $7r - 5s = 0. \quad (5)$

But (5) is equal to $-1 \cdot (4)$; hence the equations are dependent.

8. $$\frac{3}{x-3}-\frac{1}{y}=0, \quad (1)$$
$$\frac{x-6y}{5}=\frac{2}{5}. \quad (2)$$

From (1), $3y-x+3=0. \quad (3)$

From (2),
$$5x-30y-10=0. \quad (4)$$

$5\cdot(3)+(4)$, $-15y+5=0.$
$$y=\tfrac{1}{3}.$$

Then from (3), $x=4.$

9. $$\frac{\frac{3(x+y)}{\frac{3}{5}}}{}+\frac{\frac{x-y}{-11}}{5}=0, \quad (1)$$
$$2x+y=7. \quad (2)$$

From (1),
$$\frac{3(x+y)}{3}+\frac{x-y}{-11}=0.$$
$$-33(x+y)+3(x-y)=0.$$
$$-30x-36y=0.$$
$$y=-\tfrac{5}{6}x. \quad (3)$$

Then in (2), $2x-\frac{5}{6}x=7.$
$$12x-5x=7x=42.$$
$$x=6.$$

Then from (3), $y=-5.$

10. $$\frac{3x+z}{z-x}+2=\frac{12}{x-z}, \quad (1)$$
$$\frac{x+7}{z}+\frac{17+x}{5-z}=0. \quad (2)$$

From (1),
$$3x+z+2(z-x)=-12.$$
$$x+3z=-12. \quad (3)$$

From (2),
$$(x+7)(5-z)+(17+x)z=0.$$
$$5x+10z=-35.$$
$$x+2z=-7. \quad (4)$$

(3) – (4), $z=-5.$

Then in (4), $x=3.$

11. $$5x+4y=10a+4, \quad (1)$$
$$x-2ay=0. \quad (2)$$

From (2), $x=2ay. \quad (3)$

Then in (1),
$$(10a+4)y=10a+4.$$
$$y=1.$$

Then from (3), $x=2a.$

12. $$2ax-3by=7, \quad (1)$$
$$5ax+7by=3. \quad (2)$$

$5\cdot(1)-2\cdot(2)$, $-29by=29.$
$$y=-\frac{1}{b}.$$

Then from (1), $2ax=4.$
$$x=\frac{2}{a}.$$

13. $$\frac{x}{a}-\frac{y}{c}=6, \quad (1)$$
$$\frac{x}{2a}+\frac{y}{3c}=13. \quad (2)$$

Clearing of fractions,
$$cx-ay=6ac. \quad (3)$$
$$3cx+2ay=78ac. \quad (4)$$

$2\cdot(3)+(4)$, $5cx=90ac.$
$$x=18a.$$

Then in (3), $ay=12ac.$
$$y=12c.$$

14. $$\frac{a}{x}+\frac{3a}{y}=1, \quad (1)$$
$$\frac{3a}{x}+\frac{a}{y}=\frac{1}{2}. \quad (2)$$

$3\cdot(1)-(2)$, $\frac{8a}{y}=\frac{5}{2}.$
$$y=\frac{16a}{5}.$$

$3\cdot(2)-(1)$, $\frac{8a}{x}=\frac{1}{2}.$
$$x=16a.$$

15.
$$\frac{x}{a}+\frac{y}{b}=\frac{a+b}{ab}, \qquad (1)$$
$$x-y=\frac{a^2-b^2}{ab}. \qquad (2)$$

From (1), $bx+ay=a+b.$ (3)

From (2), $abx-aby=a^2-b^2.$ (4)

$a\cdot(3)-(4)$, $(a^2+ab)y=ab+b^2.$
$$a(a+b)y=b(a+b).$$
$$y=\frac{b}{a}.$$

$b\cdot(3)+(4)$, $(b^2+ab)x=ab+a^2.$
$$b(b+a)x=a(b+a).$$
$$x=\frac{a}{b}.$$

16.
$$2ax-4by=7c, \qquad (1)$$
$$3ax-6by=5c. \qquad (2)$$

$3\cdot(1)-2\cdot(2)$, $0+0=11c$, and the equations are inconsistent.

Page 325

1. $x+3y-5z=2,$ (1)
$2x-y-z=1,$ (2)
$3x+5y-7z=-10.$ (3)
$2\cdot(1)-(2)$, $7y-9z=3.$ (4)
$3\cdot(1)-(3)$, $4y-8z=16.$ (5)
$4\cdot(4)-7\cdot(5)$, $20z=-100.$
$z=-5.$
Then in (5), $y=-6,$
and in (1), $x=-5.$

2. $x-y+3z=0,$ (1)
$5x+2y+z=14,$ (2)
$2x+3y+4z=-14.$ (3)
$(1)+3\cdot(2)$, $5x+13z=-14.$ (4)
$(3)+2\cdot(2)$, $7x+7z=14.$ (5)
$7\cdot(4)-5\cdot(5)$, $56z=-168.$
$z=-3.$
Then in (4), $x=5,$
and in (2), $y=-4.$

3. $x+2y+z=-1,$ (1)
$2x-y+z=-20,$ (2)
$-x-y-5z=13.$ (3)
$(1)+(3)$, $y-4z=12.$ (4)
$(2)+2\cdot(3)$,
$-3y-9z=6.$ (5)
$3\cdot(4)+(5)$, $-21z=42.$
$z=-2.$
Then in (4), $y=4,$
and in (1), $x=-7.$

4. $x+y+z=1,$ (1)
$x+y-z=2,$ (2)
$x-y+z=3.$ (3)
$(2)+(3)$, $2x=5$ or $x=\frac{5}{2}.$
$(1)-(2)$, $2z=-1$ or $z=-\frac{1}{2}.$
Substituting these values in (1),
$y=-1.$

5. $x + 2y + z = 1,$ (1)

$2x + y - z = 0,$ (2)

$x + 2y + z = 0.$ (3)

Comparison of (1) and (3) shows at once that these equations are inconsistent.

6. $2x - y + 5z = 0,$ (1)

$8x + 7y + z = 38,$ (2)

$x - 5y - z = 7.$ (3)

(1) + 5 · (3), $7x - 26y = 35.$ (4)

(2) + (3), $9x + 2y = 45.$ (5)

(4) + 13 · (5), $124x = 620.$

$x = 5.$

Then in (5), $y = 0,$

and in (2), $z = -2.$

7. $4x - 3y = z,$ (1)

$z = x + y,$ (2)

$2x = 3y + 1.$ (3)

Substituting $x + y$ for z from (2) in (1),

$3x - 4y = 0.$

$x = \frac{4}{3}y.$ (4)

Substituting in (3) the value of x,

$\frac{8}{3}y = 3y + 1.$

$y = -3.$

Then in (4), $x = -4,$

and in (2), $z = -7.$

8. $x - 2y = 10,$ (1)

$3y + 4z = -1,$ (2)

$5x - z = 18.$ (3)

In (3), $z = 5x - 18.$ (4)

Substituting in (2),

$20x + 3y = 71.$ (5)

(5) − 20 · (1), $43y = -129.$

$y = -3.$

Then in (1), $x = 4,$

and in (4), $z = 2.$

9. $x + y = 3a,$ (1)

$x + z = 4a,$ (2)

$y + z = 5a.$ (3)

(2) − (1), $z - y = a.$ (4)

(3) + (4), $2z = 6a.$

$z = 3a.$

Then in (3), $y = 2a,$

and in (1), $x = a.$

10. $.3x + .2y + .4z = 1.9,$ (1)

$.02x = .1 - .01y - .02z,$ (2)

$x + y + z = 6.$ (3)

Clearing (1) and (2) of decimals,

$3x + 2y + 4z = 19,$ (4)

$2x + y + 2z = 10.$ (5)

2 · (3) − (5), $y = 2.$

Substituting in (3) and (4),

$x + z = 4,$ (6)

$3x + 4z = 15.$ (7)

(7) − 3 · (6), $z = 3.$

Then in (6), $x = 1.$

11. $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 3,$ (1)

$\frac{a}{x} - \frac{b}{y} + \frac{2c}{z} = 2,$ (2)

$\frac{5a}{x} - \frac{3b}{y} - \frac{2c}{z} = 0.$ (3)

(1) + (2), $\frac{2a}{x} + \frac{3c}{z} = 5.$ (4)

3 · (1) + (3), $\frac{8a}{x} + \frac{c}{z} = 9.$ (5)

3 · (5) − (4), $\frac{22a}{x} = 22.$

$x = a.$

Then in (5), $z = c,$

and in (1), $y = b.$

12. $$\frac{x+y}{3} - \frac{z}{5} = \frac{2}{3}, \quad (1)$$

$$\frac{y+z}{4} - \frac{x}{2} = \frac{5}{12}, \quad (2)$$

$$3x + 2y = 12 - 3z. \quad (3)$$

Clearing of fractions in (1) and (2),

$$5x + 5y - 3z = 10, \quad (4)$$

$$-6x + 3y + 3z = 5. \quad (5)$$

(4) + (5), $-x + 8y = 15.$ (6)

(4) + (3), $8x + 7y = 22.$ (7)

$8 \cdot (6) + (7)$, $71y = 142.$

$y = 2.$

Then in (6), $x = 1,$

and in (3), $z = \frac{5}{3}.$

13. $$2x + y + z + w = 1, \quad (1)$$

$$x - y - z + 2w = 4, \quad (2)$$

$$x + 2y - z - w = 0, \quad (3)$$

$$x - y + 2z - w = 1. \quad (4)$$

(1) + (2), $3x + 3w = 5.$ (5)

$2 \cdot (1) - (3)$,

$$3x + 3z + 3w = 2. \quad (6)$$

(1) + (4), $3x + 3z = 2.$ (7)

(6) − (5), $3z = -3.$

$z = -1.$

(6) − (7), $3w = 0.$

$w = 0.$

Then in (5), $3x = 5,$

$x = \frac{5}{3},$

and in (1), $y = -\frac{4}{3}.$

14. $$x + y + z = 1, \quad (1)$$

$$x - y - w = -1, \quad (2)$$

$$x - z - w = -5, \quad (3)$$

$$y - z + w = 0. \quad (4)$$

(1) + (2), $2x + z - w = 0.$ (5)

(2) + (4), $x - z = -1.$ (6)

(6) − (3), $w = 4.$

Then (5) becomes $2x + z = 4.$ (7)

(6) + (7), $3x = 3,$

$x = 1,$

and in (7), $z = 2.$

Then in (1), $y = -2.$

Page 326

1. Let x and y respectively represent the numbers.

Then $x + y = 109,$

and $x - y = 49.$

Solving, $x = 79, y = 30.$

2. Let x = tons capacity of a truck,

and y = tons capacity of a dray.

Then $10x + 6y = 39,$

and $8x + 10y = 39.$

Solving, $x = 3, y = 1\frac{1}{2}.$

3. Let x and y = the number of days in which one man and one boy, respectively, can do the work.

Then $$\frac{1}{x}+\frac{1}{y}=\frac{1}{18},$$

and $$\frac{5}{x}+\frac{9}{y}=\frac{1}{3}.$$

Solving, $x=24$, $y=72$.

4. Let x and y = the amounts at 5% and 6% respectively.

Then $\frac{5}{100}x+\frac{6}{100}y=100,$

and $\frac{6}{100}x+\frac{5}{100}y=98.$

Solving, $x=800$, $y=1000$.

5. Substituting the values of x and y which are to satisfy, we get the system in a and b:

$$2a+3b=2,$$
$$6a+5b=2.$$

Solving, $a=-\frac{1}{2}$, $b=1$.

6. Let x and y = the number of ounce and $\frac{3}{8}$-ounce balls respectively.

Then $x+y=23,$

and $x+\frac{3}{8}y=18.$

Solving, $x=15$, $y=8$.

7. Let d and q = the number of dimes and quarters respectively.

Then $d+q=36,$

and $10d+25q=450.$

Solving, $d=30$, $q=6$.

8. Let x = the distance walked, in miles,

and y = the distance by stage, in miles.

Then $x+y=20,$

and $$\frac{x}{4}+\frac{y}{12}=3,$$

since $\frac{\text{Distance}}{\text{Rate}}=\text{Time}.$

Solving, $x=8$, $y=12$.

9. Let x = the faster rate, in miles per hour,

and y = the slower rate, in miles per hour.

Then in the first case the distances covered are x and y miles; in the second, $5x$ and $5y$ miles.

Therefore $x+y=25,$

and $5x-5y=25.$

Solving, $x=15$, $y=10$.

CR

10. Let $d =$ the distance in miles,
and $r =$ the rate in miles per hour.

Then $$\frac{d}{r} = 3,$$

and $$\frac{\frac{3}{2}d}{r + 50} = 3.$$

Solving, $d = 300$, $r = 100$.

11. Let x, y, and $z =$ the respective numbers.

Then $x + y + z = 108$,

and $\frac{1}{3}x + \frac{1}{4}y + \frac{1}{6}z = 25$,

and $3x + 4y + 6z = 504$.

Solving, $x = 24$, $y = 36$, $z = 48$.

12. Let x, y, and $z =$ the respective numbers.

Then $x + y + z = 217$,

and $$\frac{x}{y} = 5,$$

and $$\frac{y}{z} = 5.$$

Solving, $x = 175$, $y = 35$, $z = 7$.

13. Let x, y, and $z =$ the three digits, from left to right.

Then $x + y + z = 14$,

and $100x + 10z + y + 27 = 100x + 10y + z$,

and $100y + 10x + z + 180 = 100x + 10y + z$.

Solving, $x = 7$, $y = 5$, $z = 2$,

and the required number is 752.

14. Let the sides be x, y, z, and u respectively.

Then $x + y = 20$,	(1)		$x + y = 20$,	(5)
$y + z = 27$,	(2)	or	$y + z = 27$,	(6)
$y + u = 23$,	(3)		$y + u = 29$,	(7)
$z + x = 29$,	(4)		$z + x = 23$.	(8)

Then (2) − (1), $z - x = 7$, (9)

and (9) + (4), $2z = 36$,

or $z = 18$.

Then from (4), $x = 11$,

from (1), $y = 9$,

and from (3), $u = 14$.

Hence $x = 11$, $y = 9$, $z = 18$, $u = 14$.

In the second case, (6) – (5),

$$z - x = 7, \tag{10}$$

(10) + (8), $2z = 30$,

or $z = 15$.

Then from (8), $x = 8$,

from (5), $y = 12$,

and from (7), $u = 17$.

Hence $x = 8,\ y = 12,\ z = 15,\ u = 17$.

15. Let x and y = the respective costs of the chairs, in cents.

Then $x + y = 100h$,

and $x - y = m$.

Solving, $x = \dfrac{100h + m}{2}$ (cents),

$y = \dfrac{100h - m}{2}$ (cents).

16. $x + y = a$,

and $x - y = b$.

Solving, $x = \dfrac{a+b}{2},\ y = \dfrac{a-b}{2}$.

17. $x + y = a + b$,

and $x - y = a - b$.

Solving, $x = a,\ y = b$.

18. Let x and y = the respective distances in miles.

Then $x + y = k$,

and $x - y = c$.

Solving, $x = \dfrac{k+c}{2},\ y = \dfrac{k-c}{2}$.

19. Let d and q = the number of dimes and quarters respectively.

Then $d + q = c$,

and $10d + 25q = 100a + b$.

Solving, $q = \dfrac{100a + b - 10c}{15}$,

and $d = \dfrac{25c - b - 100a}{15}$.

20. Let q and n = the number of quarters and nickels respectively.

Then $25q + 5n = 100a$,

and $q - n = b$.

Solving, $q = \dfrac{20a + b}{6}$,

and $n = \dfrac{20a - 5b}{6}$.

21. Let $x =$ rate in miles per hour in still water,
and $y =$ rate of river, in miles per hour.

Then $$\frac{m}{x+y} = t,$$

and $$\frac{m}{x-y} = a.$$

Solving, $$x = \frac{m(a+t)}{2at},$$

and $$y = \frac{m(a-t)}{2at}.$$

22. Solving the equations of the Hint,
$$x = \tfrac{15}{4},\ y = \tfrac{25}{4}.$$

23. Let x and $y =$ the percentage of purity of the respective solutions.

Then $$\frac{\left(\frac{x}{100}\right)(8) + \left(\frac{y}{100}\right)(12)}{20} = \frac{84}{100},$$

and $$\frac{\left(\frac{x}{100}\right)(3) + \left(\frac{y}{100}\right)(2)}{5} = \frac{86}{100}.$$

Solving, $$x = 90\%,\ y = 80\%.$$

Page 333

1. $\left(x + x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)2x^{\frac{3}{2}} = 2x^{\frac{5}{2}} + 2x^2 + 2x.$
2. $\left(a^{\frac{1}{2}}x + ax^{\frac{1}{2}} - a^{\frac{1}{2}}x^{-\frac{1}{2}}\right)ax^{\frac{1}{2}} = a^{\frac{3}{2}}x^{\frac{3}{2}} + a^2x - a^{\frac{3}{2}}.$
3. $(x^2 - 5ax + 6a^2)a^{\frac{2}{3}}x^{-\frac{2}{3}} = a^{\frac{2}{3}}x^{\frac{4}{3}} - 5a^{\frac{5}{3}}x^{\frac{1}{3}} + 6a^{\frac{8}{3}}x^{-\frac{2}{3}}.$
4. $\left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right)\left(x^{\frac{1}{3}} - y^{\frac{1}{3}}\right) = x^{\frac{2}{3}} - y^{\frac{2}{3}}.$
5. $(a^{-2} + 3)(a^{-2} - 5) = a^{-4} - 2a^{-2} - 15.$
6. $(x^{2a} - 3x^a)(x^2 - 2x) = x^{2a+2} - 3x^{a+2} - 2x^{2a+1} + 6x^{a+1}.$
7. $(a^{-1} - a)^2 = a^{-2} - 2 + a^2.$
8. $(a^3 - 2a^{-2})^3 = a^9 - 6a^4 + 12a^{-1} - 8a^{-6}.$
9. $(a^{-1} - 2a + 3a^{-2})^2 = a^{-2} + 4a^2 + 9a^{-4} - 4 + 6a^{-3} - 12a^{-1}.$
10. $(e^x + e^{-x})^2 = e^{2x} + 2 + e^{-2x}.$
11. $(a^{-1} + b^{-2})(a^{-1} - b^{-2}) = a^{-2} - b^{-4}.$
12. $(e^{2x} - 2 + e^{-2x})^2 = e^{4x} + 4 + e^{-4x} - 4e^{2x} + 2 - 4e^{-2x}$
$= e^{4x} - 4e^{2x} + 6 - 4e^{-2x} + e^{-4x}.$
13. $\left(x^{-\frac{1}{2}} + 2x^{\frac{1}{2}} - 3x^{\frac{3}{2}}\right)^2 = x^{-1} + 4x + 9x^3 + 4 - 6x - 12x^2$
$= x^{-1} + 4 - 2x - 12x^2 + 9x^3.$

14. $(e^{2x} - 3\,e^{-x})^4 = (e^{4x} - 6\,e^{x} + 9\,e^{-2x})^2$
$\quad = e^{8x} + 36\,e^{2x} + 81\,e^{-4x} - 12\,e^{5x} + 18\,e^{2x} - 108\,e^{-x}$
$\quad = e^{8x} - 12\,e^{5x} + 54\,e^{2x} - 108\,e^{-x} + 81\,e^{-4x}.$

15.
$$\begin{array}{l}
a^{\frac{4}{5}} - a^{\frac{2}{5}}x + 4x^2 \\
\underline{a^{\frac{2}{5}} + 2x \qquad\qquad\quad} \\
a^{\frac{6}{5}} - \quad a^{\frac{4}{5}}x + 4\,a^{\frac{2}{5}}x^2 \\
\underline{\qquad + 2\,a^{\frac{4}{5}}x - 2\,a^{\frac{2}{5}}x^2 + 8\,x^3} \\
a^{\frac{6}{5}} + \quad a^{\frac{4}{5}}x + 2\,a^{\frac{2}{5}}x^2 + 8\,x^3
\end{array}$$

16. $\left(x^{\frac{1}{3}} + 2\,y^{\frac{1}{3}}\right)\left(x^{\frac{2}{3}} - 2\,x^{\frac{1}{3}}y^{\frac{1}{3}} + 4\,y^{\frac{2}{3}}\right) = x + 8\,y.$

17. $\left(a^{\frac{1}{2}} - a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{2}}\right)\left(a^{\frac{1}{2}} + a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{2}}\right) = a + a^{\frac{1}{2}}b^{\frac{1}{2}} + b.$

18.
$$\begin{array}{llll}
x & + x^{\frac{1}{2}}y^{-\frac{1}{2}} & + y^{-1} & \\
x & - x^{\frac{1}{2}}y^{-\frac{1}{2}} & + y^{-1} & \\
\hline
x^2 & + x^{\frac{3}{2}}y^{-\frac{1}{2}} & + xy^{-1} & \\
 & - x^{\frac{3}{2}}y^{-\frac{1}{2}} & - xy^{-1} & - x^{\frac{1}{2}}y^{-\frac{3}{2}} \\
 & & + xy^{-1} & + x^{\frac{1}{2}}y^{-\frac{3}{2}} + y^{-2} \\
\hline
x^2 & & + xy^{-1} & \qquad\quad + y^{-2}
\end{array}$$

19.
$$\begin{array}{lll}
a^{\frac{5}{3}} & - 5\,a^{\frac{1}{2}} & \\
a^{\frac{5}{3}} & - 5\,a^{\frac{1}{2}} & \\
\hline
a^{\frac{10}{3}} & - 5\,a^{\frac{13}{6}} & \\
 & - 5\,a^{\frac{13}{6}} & + 25\,a \\
\hline
a^{\frac{10}{3}} & - 10\,a^{\frac{13}{6}} & + 25\,a
\end{array}$$

20. $\left(5\,c^{-\frac{5}{2}} + d^{-1} - ac^{-\frac{2}{5}}\right)^2$
$\quad = 25\,c^{-5} + d^{-2} + a^2c^{-\frac{4}{5}} + 10\,c^{-\frac{5}{2}}d^{-1} - 10\,ac^{-\frac{29}{10}} - 2\,ac^{-\frac{2}{5}}d^{-1}.$

21. $\left(a^{\frac{1}{2}} + b^{\frac{1}{2}}c^{-1}\right)\left(a^{\frac{1}{2}} - b^{\frac{1}{2}}c^{-1}\right)^2 = (a - bc^{-2})\left(a^{\frac{1}{2}} - b^{\frac{1}{2}}c^{-1}\right).$

$$\begin{array}{l}
a - bc^{-2} \\
a^{\frac{1}{2}} - b^{\frac{1}{2}}c^{-1} \\
\hline
a^{\frac{3}{2}} - a^{\frac{1}{2}}bc^{-2} - ab^{\frac{1}{2}}c^{-1} + b^{\frac{3}{2}}c^{-3}
\end{array}$$

Page 334

1. $x^4 \div x^6 = x^{-2}.$ 2. $x^3 \div x^{\frac{1}{3}} = x^{\frac{8}{3}}.$ 3. $ax^{\frac{2}{3}} \div a^{\frac{1}{2}}x^{\frac{1}{2}} = a^{\frac{1}{2}}x^{\frac{1}{6}}.$

4. $(ax - a^2x^3) \div a^2x^{\frac{1}{2}} = a^{-1}x^{\frac{1}{2}} - x^{\frac{5}{2}}.$

5. $(x^a - 2\,x^{2a-1} + 3\,x^{3a-2}) \div x^{2a-1} = x^{-a+1} - 2 + 3\,x^{a-1}.$

6. $(6\,a^{3+4n} - 9\,a^{n-2} + 12\,a^{2-n}) \div 3\,a^{n-2} = 2\,a^{3n+5} - 3 + 4\,a^{4-2n}.$

7.
$$\begin{array}{l|l}
x \qquad\qquad\qquad\qquad + y & x^{\frac{1}{3}} + y^{\frac{1}{3}} \\
\cline{2-2}
\underline{x + x^{\frac{2}{3}}y^{\frac{1}{3}}} & x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}} \\
\quad - x^{\frac{2}{3}}y^{\frac{1}{3}} & \\
\quad \underline{- x^{\frac{2}{3}}y^{\frac{1}{3}} - x^{\frac{1}{3}}y^{\frac{2}{3}}} & \\
\qquad\qquad x^{\frac{1}{3}}y^{\frac{2}{3}} & \\
\qquad\qquad \underline{x^{\frac{1}{3}}y^{\frac{2}{3}} + y} &
\end{array}$$

8.
$$\begin{array}{l|l}
x \qquad\qquad\qquad -8y & x^{\frac{1}{3}}-2y^{\frac{1}{3}} \\ \cline{2-2}
\underline{x-2x^{\frac{2}{3}}y^{\frac{1}{3}}} & x^{\frac{2}{3}}+2x^{\frac{1}{3}}y^{\frac{1}{3}}+4y^{\frac{2}{3}} \\ \cline{2-2}
\qquad 2x^{\frac{2}{3}}y^{\frac{1}{3}} & \\
\qquad \underline{2x^{\frac{2}{3}}y^{\frac{1}{3}}-4x^{\frac{1}{3}}y^{\frac{2}{3}}} & \\
\qquad\qquad\quad 4x^{\frac{1}{3}}y^{\frac{2}{3}} & \\
\qquad\qquad\quad \underline{4x^{\frac{1}{3}}y^{\frac{2}{3}}-8y} &
\end{array}$$

9.
$$\begin{array}{l|l}
a^{\frac{3}{2}} \qquad\qquad\qquad +b & a^{\frac{1}{2}}+b^{\frac{1}{3}} \\ \cline{2-2}
\underline{a^{\frac{3}{2}}+ab^{\frac{1}{3}}} & a-a^{\frac{1}{2}}b^{\frac{1}{3}}+b^{\frac{2}{3}} \\ \cline{2-2}
\quad -ab^{\frac{1}{3}} & \\
\quad \underline{-ab^{\frac{1}{3}}-a^{\frac{1}{2}}b^{\frac{2}{3}}} & \\
\qquad\qquad a^{\frac{1}{2}}b^{\frac{2}{3}} & \\
\qquad\qquad \underline{a^{\frac{1}{2}}b^{\frac{2}{3}}+b} &
\end{array}$$

10.
$$\begin{array}{l|l}
a^2 \qquad\qquad +ab^{-1} \qquad\qquad +b^{-2} & a-a^{\frac{1}{2}}b^{-\frac{1}{2}}+b^{-1} \\ \cline{2-2}
\underline{a^2-a^{\frac{3}{2}}b^{-\frac{1}{2}}+ab^{-1}} & a+a^{\frac{1}{2}}b^{-\frac{1}{2}}+b^{-1} \\ \cline{2-2}
\qquad a^{\frac{3}{2}}b^{-\frac{1}{2}} & \\
\qquad \underline{a^{\frac{3}{2}}b^{-\frac{1}{2}}-ab^{-1}+a^{\frac{1}{2}}b^{-\frac{3}{2}}} & \\
\qquad\qquad\qquad ab^{-1}-a^{\frac{1}{2}}b^{-\frac{3}{2}} & \\
\qquad\qquad\qquad \underline{ab^{-1}-a^{\frac{1}{2}}b^{-\frac{3}{2}}+b^{-2}} &
\end{array}$$

11.
$$\begin{array}{l|l}
e^{2x}+2+e^{-2x} & e^{x}+e^{-x} \\ \cline{2-2}
\underline{e^{2x}+1} & e^{x}+e^{-x} \\ \cline{2-2}
\qquad\quad 1 & \\
\qquad\quad \underline{1+e^{-2x}} &
\end{array}$$

12.
$$\begin{array}{l|l}
e^{3x}+3e^{x}+3e^{-x}+e^{-3x} & e^{x}+e^{-x} \\ \cline{2-2}
\underline{e^{3x}+\quad e^{x}} & e^{2x}+2+e^{-2x} \\ \cline{2-2}
\qquad 2e^{x} & \\
\qquad \underline{2e^{x}+2e^{-x}} & \\
\qquad\qquad\quad e^{-x} & \\
\qquad\qquad\quad \underline{e^{-x}+e^{-3x}} &
\end{array}$$

13.
$$\begin{array}{l|l}
a+a^{\frac{3}{5}}b^{\frac{2}{3}}+2a^{\frac{2}{5}}b^{\frac{1}{3}}+2b & a^{\frac{3}{5}}+2b^{\frac{1}{3}} \\ \cline{2-2}
\underline{a \qquad\qquad\quad +2a^{\frac{2}{5}}b^{\frac{1}{3}}} & a^{\frac{2}{5}}+b^{\frac{2}{3}} \\ \cline{2-2}
\qquad a^{\frac{3}{5}}b^{\frac{2}{3}} & \\
\qquad \underline{a^{\frac{3}{5}}b^{\frac{2}{3}} \qquad\qquad\quad +2b} &
\end{array}$$

14.
$$\begin{array}{l|l} m^4 + 7m^2 + 8 - 7m^{-2} + m^{-4} & m^2 + 5 - m^{-2} \\ \underline{m^4 + 5m^2 - 1} & \overline{m^2 + 2 - m^{-2}} \\ \quad 2m^2 + 9 & \\ \quad \underline{2m^2 + 10 - 2m^{-2}} & \\ \qquad -1 - 5m^{-2} & \\ \qquad \underline{-1 - 5m^{-2} + m^{-4}} & \end{array}$$

15.
$$\begin{array}{l|l} 9x^{5n-4} - x^{3n-2} + 2x^{2n-1} & 3x^{2n-2} + 2x^{n-1} \\ \underline{9x^{5n-4} + 6x^{4n-3}} & \overline{3x^{3n-2} - 2x^{2n-1} + x^n} \\ \quad -6x^{4n-3} & \\ \quad \underline{-6x^{4n-3} - 4x^{3n-2}} & \\ \qquad 3x^{3n-2} & \\ \qquad \underline{3x^{3n-2} + 2x^{2n-1}} & \end{array}$$

16.
$$\begin{array}{l|l} -16a^{\frac{19}{5}}b^{\frac{3}{4}} + 40ab - 25a^{-\frac{9}{5}}b^{\frac{5}{4}} & -4a^{\frac{12}{5}}b^{-\frac{5}{8}} + 5a^{-\frac{2}{5}}b^{-\frac{3}{8}} \\ \underline{-16a^{\frac{19}{5}}b^{\frac{3}{4}} + 20ab} & \overline{4a^{\frac{7}{5}}b^{\frac{11}{8}} - 5a^{-\frac{7}{5}}b^{\frac{13}{8}}} \\ \quad 20ab & \\ \quad \underline{20ab - 25a^{-\frac{9}{5}}b^{\frac{5}{4}}} & \end{array}$$

17.
$$\begin{array}{l|l} a^{\frac{3}{2}} - ab^{\frac{1}{2}} + a^{\frac{1}{2}}b - b^{\frac{3}{2}} & a^{\frac{1}{2}} - b^{\frac{1}{2}} \\ \underline{a^{\frac{3}{2}} - ab^{\frac{1}{2}}} & \overline{a + b} \\ \quad a^{\frac{1}{2}}b & \\ \quad \underline{a^{\frac{1}{2}}b - b^{\frac{3}{2}}} & \end{array}$$

18.
$$\begin{array}{l|l} x^6 - 4x^{-6} + 3x^{-10} & x^2 + 2x^{-2} + 3x^{-6} \\ \underline{x^6 + 2x^2 + 3x^{-2}} & \overline{x^4 - 2 + x^{-4}} \\ \quad -2x^2 - 3x^{-2} & \\ \quad \underline{-2x^2 - 4x^{-2} - 6x^{-6}} & \\ \qquad x^{-2} + 2x^{-6} & \\ \qquad \underline{x^{-2} + 2x^{-6} + 3x^{-10}} & \end{array}$$

19. $\left(x^{\frac{3}{5}} - y^{\frac{3}{5}}\right) \div \left(x^{\frac{1}{10}} + y^{\frac{1}{10}}\right) = x^{\frac{1}{10}} - y^{\frac{1}{10}}.$

$$\begin{array}{l|l} x^{\frac{3}{5}} - y^{\frac{3}{5}} & x^{\frac{1}{10}} - y^{\frac{1}{10}} \\ \underline{x^{\frac{3}{5}} - x^{\frac{1}{2}}y^{\frac{1}{10}}} & \overline{x^{\frac{1}{2}} + x^{\frac{2}{5}}y^{\frac{1}{10}} + x^{\frac{3}{10}}y^{\frac{1}{5}} + x^{\frac{1}{5}}y^{\frac{3}{10}} + x^{\frac{1}{10}}y^{\frac{2}{5}} + y^{\frac{1}{2}}} \\ \quad x^{\frac{1}{2}}y^{\frac{1}{10}} & \\ \quad \underline{x^{\frac{1}{2}}y^{\frac{1}{10}} - x^{\frac{2}{5}}y^{\frac{1}{5}}} & \\ \qquad x^{\frac{2}{5}}y^{\frac{1}{5}} & \\ \qquad \underline{x^{\frac{2}{5}}y^{\frac{1}{5}} - x^{\frac{3}{10}}y^{\frac{3}{10}}} & \\ \qquad\quad x^{\frac{3}{10}}y^{\frac{3}{10}} & \\ \qquad\quad \underline{x^{\frac{3}{10}}y^{\frac{3}{10}} - x^{\frac{1}{5}}y^{\frac{2}{5}}} & \\ \qquad\qquad x^{\frac{1}{5}}y^{\frac{2}{5}} & \\ \qquad\qquad \underline{x^{\frac{1}{5}}y^{\frac{2}{5}} - x^{\frac{1}{10}}y^{\frac{1}{2}}} & \\ \qquad\qquad\quad x^{\frac{1}{10}}y^{\frac{1}{2}} - y^{\frac{3}{5}} & \\ \qquad\qquad\quad \underline{x^{\frac{1}{10}}y^{\frac{1}{2}} - y^{\frac{3}{5}}} & \end{array}$$

20.
$$
\begin{array}{llll}
9m & -13 & +4m^{-1} & \big|\ 3m^{\frac{1}{2}}-5+2m^{-\frac{1}{2}} \\
\underline{9m-15m^{\frac{1}{2}}+6} & & & \big|\ \overline{3m^{\frac{1}{2}}+5+2m^{-\frac{1}{2}}} \\
\quad 15m^{\frac{1}{2}}-19 & & & \\
\quad \underline{15m^{\frac{1}{2}}-25+10m^{-\frac{1}{2}}} & & & \\
\qquad 6-10m^{-\frac{1}{2}} & & & \\
\qquad \underline{6-10m^{-\frac{1}{2}}+4m^{-1}} & & &
\end{array}
$$

21.
$$
\begin{array}{llll}
9x^{2a} & -19x^{-a} & +25x^{-4a} & \big|\ 3x^{a}-7x^{-\frac{a}{2}}+5x^{-2a} \\
\underline{9x^{2a}-21x^{\frac{a}{2}}+15x^{-a}} & & & \big|\ \overline{3x^{a}+7x^{-\frac{a}{2}}+5x^{-2a}} \\
\quad 21x^{\frac{a}{2}}-34x^{-a} & & & \\
\quad \underline{21x^{\frac{a}{2}}-49x^{-a}+35x^{-\frac{5a}{2}}} & & & \\
\qquad 15x^{-a}-35x^{-\frac{5a}{2}} & & & \\
\qquad \underline{15x^{-a}-35x^{-\frac{5a}{2}}+25x^{-4a}} & & &
\end{array}
$$

22. $\left(\dfrac{x^{\frac{3}{2}}}{8}+\dfrac{27y^{\frac{3}{2}}}{64}\right)\div\left[\left(\dfrac{3x^{\frac{1}{2}}}{2}+\dfrac{9y^{\frac{1}{2}}}{4}\right)\left(64x-96x^{\frac{1}{2}}y^{\frac{1}{2}}+144y\right)\right]$

$$=\frac{(2x^{\frac{1}{2}}+3y^{\frac{1}{2}})(4x-6x^{\frac{1}{2}}y^{\frac{1}{2}}+9y)}{64}\cdot\frac{4}{3(2x^{\frac{1}{2}}+3y^{\frac{1}{2}})}\cdot\frac{1}{16(4x-6x^{\frac{1}{2}}y^{\frac{1}{2}}+9y)}$$

$$=\frac{1}{768}.$$

Page 335

1. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}.$

2. $4^{-3} = \frac{1}{4^3} = \frac{1}{64}.$

3. $2^{-2} \cdot 3^{-4} = \frac{1}{2^2 \cdot 3^4} = \frac{1}{324}.$

4. $7 \cdot 7^0 \cdot 0 = 0.$

5. $(\frac{1}{2})^{-3} = 2^3 = 8.$

6. $(\frac{2}{3})^{-2} \cdot 4^0 = (\frac{3}{2})^2 = \frac{9}{4}.$

7. $\frac{2}{3^{-2}} = 2 \cdot 3^2 = 18.$

8. $\frac{12}{4^{-1}} = 12 \cdot 4 = 48.$

9. $5 \cdot 2^0 - (5 \cdot 2)^0 = 5 - 1 = 4.$

10. $\frac{4^{-2} \cdot 3^{-2}}{6^{-2}} = \frac{6^2}{4^2 \cdot 3^2} = \frac{1}{4}.$

11. $32^{-\frac{2}{5}} = \frac{1}{\sqrt[5]{(32)^2}} = \frac{1}{4}.$

12. $4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{2}.$

13. $16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}} = \frac{1}{2}.$

14. $8^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{8^2}} = \frac{1}{4}.$

15. $25^{\frac{3}{2}} = 5^3 = 125.$

16. $(-64)^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{(-64)^2}} = \frac{1}{16}.$

17. $(-125)^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{(-125)^2}} = \frac{1}{25}.$

18. $\sqrt[3]{27^{-2}} = \sqrt[3]{\frac{1}{27^2}} = \frac{1}{9}.$

19. $(\sqrt[3]{-8})^2 = 4.$

20. $(\frac{1}{2})^{-4} \cdot (\frac{1}{3})^{-3} \cdot (\frac{1}{2})^0 = 2^4 \cdot 3^3\ 1 = 432.$

21. $(\frac{1}{6})^{-2} = 6^2 = 36.$

22. $(.04)^{\frac{3}{2}} = (.2)^3 = .008.$

23. $(.027)^{-\frac{2}{3}} = \frac{1}{(\sqrt[3]{.027})^2} = \frac{1}{.09} = \frac{100}{9}.$

24. $(.00032)^{\frac{2}{5}} = (.2)^2 = .04.$

25. $\frac{2^{-1}}{2^{-2} - 2^{-3}} = \frac{\frac{1}{2}}{\frac{1}{4} - \frac{1}{8}} = \frac{1}{2} \cdot \frac{8}{1} = 4.$

26. $\frac{3^{-2} - 2^{-2}}{3^{-1} - 2^{-1}} = \frac{\frac{1}{3^2} - \frac{1}{2^2}}{\frac{1}{3} - \frac{1}{2}} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}.$

27. $\frac{3^{-3} - 2^{-3}}{3^{-1} - 2^{-1}} = \frac{\frac{1}{27} - \frac{1}{8}}{\frac{1}{3} - \frac{1}{2}} = -\frac{19}{216} \cdot -\frac{6}{1} = \frac{19}{36}.$

28. $\frac{2}{a^{-2} - b^{-2}} = \frac{2}{\frac{1}{a^2} - \frac{1}{b^2}} = \frac{2a^2b^2}{b^2 - a^2}.$

29. $\frac{3}{a^{-1} + b^{-1}} = \frac{3}{\frac{1}{a} + \frac{1}{b}} = \frac{3}{\frac{b+a}{ab}} = \frac{3ab}{a+b}.$

30. $\dfrac{a}{a^{-2}-b^{-2}}=\dfrac{a}{\dfrac{1}{a^2}-\dfrac{1}{b^2}}=\dfrac{a^3b^2}{b^2-a^2}.$

31. $\dfrac{5\,se^2}{s^{-2}+e^{-2}}=\dfrac{5\,se^2}{\dfrac{1}{s^2}+\dfrac{1}{e^2}}=\dfrac{5\,se^2}{\dfrac{e^2+s^2}{e^2s^2}}=\dfrac{5\,s^3e^4}{e^2+s^2}.$

32. $\dfrac{a^{-2}}{a^{-2}+b^{-2}}=\dfrac{\dfrac{1}{a^2}}{\dfrac{1}{a^2}+\dfrac{1}{b^2}}=\dfrac{1}{a^2}\cdot\dfrac{a^2b^2}{a^2+b^2}=\dfrac{b^2}{a^2+b^2}.$

33. $\dfrac{a^{-3}b^{-3}}{a^{-3}+b^{-3}}=\dfrac{\dfrac{1}{a^3b^3}}{\dfrac{1}{a^3}+\dfrac{1}{b^3}}=\dfrac{1}{a^3+b^3}.$

34. $\dfrac{a^{-2}-b^{-2}}{a^{-1}+b^{-1}}=a^{-1}-b^{-1}=\dfrac{1}{a}-\dfrac{1}{b}=\dfrac{b-a}{ab}.$

35. $\dfrac{a^{-1}+b^{-1}}{a^{-3}+b^{-3}}=\dfrac{\dfrac{1}{a}+\dfrac{1}{b}}{\dfrac{1}{a^3}+\dfrac{1}{b^3}}=\dfrac{a+b}{ab}\cdot\dfrac{a^3b^3}{a^3+b^3}=\dfrac{a^2b^2}{a^2-ab+b^2}.$

36. $\dfrac{a^{-3}-27^{-1}}{a^{-1}-3^{-1}}=\dfrac{\dfrac{1}{a^3}-\dfrac{1}{27}}{\dfrac{1}{a}-\dfrac{1}{3}}=\dfrac{27-a^3}{27\,a^3}\cdot\dfrac{3\,a}{3-a}=\dfrac{9+3\,a+a^2}{9\,a^2}.$

37. $\dfrac{2\,xy}{z^2}=2\,xyz^{-2}.$

38. $\dfrac{3\,x}{a^{-2}b^4}=3\,a^2b^{-4}x.$

39. $\dfrac{4\,sc^{-3}}{2^{-1}s^{-2}}=8\,s^3c^{-3}.$

40. $\dfrac{12\,a^2b^3}{4\,xy^2}=3\,a^2b^3x^{-1}y^{-2}.$

41. $\dfrac{5\,ac^{-2}}{c\,(x-y)^2}=5\,ac^{-3}(x-y)^{-2}.$

42. $\dfrac{1}{5\,a^2\,(c+d)^{-3}}=5^{-1}a^{-2}(c+d)^3.$

43. $\dfrac{42\,m^{-n}n^{2m}}{56\,m^{-2n}n^{-3m}}=3\cdot 4^{-1}m^nn^{5m}.$

44. $\dfrac{r^{-1}s^2}{r^{-2}s^3\,(s-r)^3}=rs^{-1}(s-r)^{-3}.$

45. $(2^3)^3=(8)^3=512.$

46. $(2^3)^{-2}=2^{-6}=\frac{1}{64}.$

47. $(2^{-3})^{-2}=2^6=64.$

48. $[(\frac{3}{2})^{-3}]^2 = (\frac{3}{2})^{-6} = (\frac{2}{3})^6 = \frac{64}{729}$.

49. $(x^{\frac{1}{2}})^4 = x^2$.

50. $(x^{\frac{1}{3}})^{\frac{1}{2}} = x^{\frac{1}{6}}$.

51. $(x^{-\frac{1}{2}})^{-\frac{1}{3}} = x^{\frac{1}{6}}$.

52. $(5\,a^2)^{-3} = \dfrac{1}{125\,a^6}$.

53. $(c^{-2}d)^2 = c^{-4}d^2$.

54. $(5^0 \cdot 2^6 \cdot 3^3)^{\frac{1}{3}} = 2^2 \cdot 3 = 12$.

55. $(2\,x^5)^0 \cdot 8 \cdot 4^{\frac{1}{2}} = 16$.

56. $(a^2b)^3(a+b) = a^7b^3 + a^6b^4$.

57. $(a^3)^{2x} \cdot (a^2)^{3x} = a^{12x}$.

58. $(a^{x+1})^2 \cdot (a^{1-x})^2 = a^4$.

59. $(a^3)^{x+y} \cdot (a^2)^{y-x} = a^{x+5y}$.

60. $(x^2 + xy^{-2})\,x^{-2} = 1 + x^{-1}y^{-2}$.

61. $2^2 \cdot 4^2 = 2^2 \cdot 2^4 = 2^6$.

62. $2^n \cdot 4^n = 2^{3n}$.

63. $\dfrac{2^n \cdot 8^{2n}}{4^{3n}} = \dfrac{2^n \cdot 2^{6n}}{2^{6n}} = 2^n$.

64. $2^n \cdot 4^{n+1} \div 2^n = 2^n \cdot 2^{2n+2} \div 2^n$
$= 2^{2n+2}$.

65.
$$\frac{4^{n+1}}{2^n(4^n-1)^n} \div \frac{8^{n+1}}{(4^n+1)^{n-1}} + 5$$
$$= \frac{2^{2n+2}}{2^n \cdot 2^{2n^2-2n}} \cdot \frac{2^{2n^2-2}}{2^{3n+3}} + 5$$
$$= \frac{2^{2n^2+2n}}{2^{2n^2+2n+3}} + 5$$
$$= \frac{1}{2^3} + 5$$
$$= 5\tfrac{1}{8}.$$

66.
$$3^6 \cdot 9^n = 81^2.$$
$$3^6 \cdot 3^{2n} = 3^8.$$
$$3^{2n+6} = 3^8.$$
$$2\,n + 6 = 8.$$
$$n = 1.$$

67.
$$9^n \cdot 3^3 = 27^n.$$
$$3^{2n} \cdot 3^3 = 3^{3n}.$$
$$3^{2n+3} = 3^{3n}.$$
$$2\,n + 3 = 3\,n.$$
$$n = 3.$$

68.
$$2^{2n+2} \cdot 4^{n+2} = 8^{2n}.$$
$$2^{2n+2} \cdot 2^{2n+4} = 2^{6n}.$$
$$4\,n + 6 = 6\,n.$$
$$n = 3.$$

69.
$$81 \cdot 27^n = (9^n)^{\frac{n}{2}}.$$
$$3^4 \cdot 3^{3n} = 3^{n^2}.$$
$$n^2 = 3\,n + 4.$$
$$(n-4)(n+1) = 0.$$
$$n = 4,\ -1.$$

70.
$$(25^n)^n = \frac{5^{7n}}{(125)^2}.$$
$$5^{2n^2} = 5^{7n-6}.$$
$$2\,n^2 = 7\,n - 6.$$
Whence $n = 2,\ \frac{3}{2}$.

71. $2^{6n+3} \cdot 4^{3n+6} = (8^n)^n$.
$$2^{6n+3} \cdot 2^{6n+12} = 2^{3n^2}.$$
$$3\,n^2 - 12\,n - 15 = 0.$$
Whence $n = 5,\ -1$.

72.
$$x^{\frac{3}{2}} = 8.$$
$$x^3 = 64.$$
$$x = 4.$$

NOTE. In Exercises 72–80, inclusive, only *real* values of x are obtained.

73.
$$x^{-\frac{1}{2}} = 2.$$
$$x^{\frac{1}{2}} = \tfrac{1}{2}.$$
$$x = \tfrac{1}{4}.$$

74.
$$x^{\frac{2}{3}} = 4.$$
$$x^{\frac{1}{3}} = \pm 2.$$
$$x = \pm 8.$$

75.
$$x^{-\frac{2}{3}} = 4.$$
$$x^{-\frac{1}{3}} = \pm 2.$$
$$x = \pm \tfrac{1}{8}.$$

76.
$$x^{-\frac{2}{3}} = 32.$$
$$x^{-\frac{2}{3}} = 2^5.$$
$$x^{\frac{2}{3}} = \frac{1}{2^5}.$$
$$x^2 = \frac{1}{2^{15}}.$$
$$x = \pm\sqrt{\frac{1}{2^{15}}} = \pm\frac{\sqrt{2}}{256}.$$

77.
$$x^{\frac{3}{2}} = 343.$$
$$x^{\frac{1}{2}} = 7.$$
$$x = 49.$$

78.
$$\tfrac{1}{2}x^{-\frac{2}{3}} = 2.$$
$$x^{\frac{2}{3}} = \tfrac{1}{4}.$$
$$x^{\frac{1}{3}} = \pm\tfrac{1}{2}.$$
$$x = \pm\tfrac{1}{8}.$$

79.
$$\left(ax^{\frac{1}{2}}\right)^{-6} = 27.$$
$$a^{-6}x^{-3} = 27.$$
$$a^{-2}x^{-1} = 3.$$
$$x = \frac{1}{3a^2}.$$

80.
$$\frac{\sqrt[3]{x^{\frac{7}{5}}}}{\sqrt[3]{x^{\frac{1}{5}}}} = \frac{\sqrt[5]{25}}{\sqrt[5]{16}}.$$
$$x^{\frac{2}{5}} = \frac{5^{\frac{2}{5}}}{2^{\frac{4}{5}}}.$$
$$x^{\frac{1}{5}} = \pm\frac{5^{\frac{1}{5}}}{2^{\frac{2}{5}}}.$$
$$x = \pm\frac{5}{2^2}$$
$$= \pm\frac{5}{4}.$$

81.
$$\frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x - e^{-x})^2}$$
$$= \frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{e^{2x} - 2 + e^{-2x}} = \frac{4}{e^{2x} - 2 + e^{-2x}}.$$

82.
$$x \cdot \frac{1}{2}(a^2 - x^2)^{-\frac{1}{2}}(-2x) - (a^2 - x^2)^{\frac{1}{2}} = \frac{-x^2 - a^2 + x^2}{(a^2 - x^2)^{\frac{1}{2}}} = \frac{a^2\sqrt{a^2 - x^2}}{x^2 - a^2}.$$

83.
$$\frac{x^3 \cdot a \cdot x^0 - a \cdot 3x^2}{(x^3)^2} \div \frac{a}{x^3} = \frac{ax^3 - 3ax^2}{x^6} \cdot \frac{x^3}{a} = \frac{x - 3}{x}.$$

84.
$$\frac{x^4(3x^2) - (x^3 + 5)4x^3}{(x^4)^2} \div \frac{x^3 + 20}{x^4} = \frac{3x^6 - 4x^6 - 20x^3}{x^8} \cdot \frac{x^4}{x^3 + 20}$$
$$= \frac{-x^6 - 20x^3}{x^4(x^3 + 20)}$$
$$= -\frac{1}{x}.$$

85.
$$\frac{\dfrac{(x^2 - 1)ax^{a-1} - x^a \cdot 2x}{(x^2 - 1)^2}}{\dfrac{x^a}{x^2 - 1}} = \frac{ax^{a+1} - ax^{a-1} - 2x^{a+1}}{(x^2 - 1)^2} \cdot \frac{x^2 - 1}{x^a}$$
$$= \frac{ax - ax^{-1} - 2x}{x^2 - 1}.$$

Page 340

1.
$$\begin{array}{l|l}
x^4 + 4x^3 - 2x^2 - 12x + 9 & \underline{x^2 + 2x - 3} \\
x^4 & \\
\end{array}$$
$$\begin{array}{r|l}
2x^2 + 2x & 4x^3 \\
 & 4x^3 + 4x^2 \\
2x^2 + 4x - 3 & -6x^2 \\
 & -6x^2 - 12x + 9
\end{array}$$

2.
$$\begin{array}{l|l}
 & a^6 - 10a^4 - 4a^3 + 25a^2 + 20a + 4 \;\underline{\big|\, a^3 - 5a - 2} \\
 & a^6 \\
2a^3 - 5a & -10a^4 \\
 & -10a^4 \qquad\quad + 25a^2 \\
2a^3 - 10a - 2 & -4a^3 \\
 & -4a^3 \qquad\quad + 20a + 4
\end{array}$$

3.
$$\begin{array}{l|l}
 & t^6 + 4t^5 + 4t^4 - 6t^3 - 12t^2 + 9 \;\underline{\big|\, t^3 + 2t^2 - 3} \\
 & t^6 \\
2t^3 + 2t^2 & 4t^5 \\
 & 4t^5 + 4t^4 \\
2t^3 + 4t^2 - 3 & -6t^3 \\
 & -6t^3 - 12t^2 + 9
\end{array}$$

4.
$$\begin{array}{l|l}
 & 4a^4 + 12a^3t + 5a^2t^2 - 6at^3 + t^4 \;\underline{\big|\, 2a^2 + 3at - t^2} \\
 & 4a^4 \\
4a^2 + 3at & 12a^3t \\
 & 12a^3t + 9a^2t^2 \\
4a^2 + 6at - t^2 & -4a^2t^2 \\
 & -4a^2t^2 - 6at^3 + t^4
\end{array}$$

5.
$$\begin{array}{l|l}
 & 4a^8 + 12a^4 - 7 - 24a^{-4} + 16a^{-8} \;\underline{\big|\, 2a^4 + 3 - 4a^{-4}} \\
 & 4a^8 \\
4a^4 + 3 & 12a^4 \\
 & 12a^4 + 9 \\
4a^4 + 6 - 4a^{-4} & -16 \\
 & -16 - 24a^{-4} + 16a^{-8}
\end{array}$$

6.
$$\begin{array}{l|l}
 & 49c^{-6} - 28c^{-4} + 74c^{-2} - 20 + 25c^2 \;\underline{\big|\, 7c^{-3} - 2c^{-1} + 5c} \\
 & 49c^{-6} \\
14c^{-3} - 2c^{-1} & -28c^{-4} \\
 & -28c^{-4} + 4c^{-2} \\
14c^{-3} - 4c^{-1} + 5c & 70c^{-2} \\
 & 70c^{-2} - 20 + 25c^2
\end{array}$$

7.
$$\begin{array}{l|l}
 & 9x^4 - 6x^{\frac{7}{2}} + x^3 - 66x^{\frac{5}{2}} + 22x^2 + 121x \;\underline{\big|\, 3x^2 - x^{\frac{3}{2}} - 11x^{\frac{1}{2}}} \\
 & 9x^4 \\
6x^2 - x^{\frac{3}{2}} & -6x^{\frac{7}{2}} \\
 & -6x^{\frac{7}{2}} + x^3 \\
6x^2 - 2x^{\frac{3}{2}} - 11x^{\frac{1}{2}} & -66x^{\frac{5}{2}} \\
 & -66x^{\frac{5}{2}} + 22x^2 + 121x
\end{array}$$

8.

$$\begin{array}{r|l}
 & 25\,x^3 - 10x^2 + 90\,x^{\frac{7}{4}} + x - 18\,x^{\frac{3}{4}} + 81\,x^{\frac{1}{2}} \;\Big|\; \underline{5\,x^{\frac{3}{2}} - x^{\frac{1}{2}} + 9\,x^{\frac{1}{4}}} \\
 & \underline{25\,x^3} \\
10\,x^{\frac{3}{2}} - x^{\frac{1}{2}} & -10\,x^2 \\
 & \underline{-10\,x^2 \qquad\qquad + x} \\
10\,x^{\frac{3}{2}} + 1 + 2\,x^{\frac{1}{2}} + 9\,x^{\frac{1}{4}} & 90\,x^{\frac{7}{4}} \qquad - 18\,x^{\frac{3}{4}} + 81\,x^{\frac{1}{2}} \\
 & \underline{90\,x^{\frac{7}{4}} \qquad - 18\,x^{\frac{3}{4}} + 81\,x^{\frac{1}{2}}}
\end{array}$$

9.

$$\begin{array}{r|l}
 & x^4 + 6\,x^3 + \dfrac{29\,x^2}{3} + 2\,x + \dfrac{1}{9} \;\Big|\; \underline{x^2 + 3\,x + \dfrac{1}{3}} \\
 & \underline{x^4} \\
2\,x^2 + 3\,x & 6\,x^3 \\
 & \underline{6\,x^3 + \;\; 9\,x^2} \\
2\,x^2 + 6\,x + \frac{1}{3} & \dfrac{2\,x^2}{3} \\
 & \underline{\dfrac{2\,x^2}{3} + 2\,x + \dfrac{1}{9}}
\end{array}$$

11.

$$\begin{array}{r|l}
 & 4\,a^4 + \dfrac{4\,a^3}{3} - \dfrac{35\,a^2}{9} - \dfrac{2\,a}{3} + 1 \;\Big|\; \underline{2\,a^2 + \dfrac{a}{3} - 1} \\
 & \underline{4\,a^4} \\
4\,a^2 + \dfrac{a}{3} & \dfrac{4\,a^3}{3} \\
 & \underline{\dfrac{4\,a^3}{3} + \;\; \dfrac{a^2}{9}} \\
4\,a^2 + \dfrac{2\,a}{3} - 1 & -\;4\,a^2 \\
 & \underline{-\;4\,a^2 - \dfrac{2\,a}{3} + 1}
\end{array}$$

12.

$$\begin{array}{r|l}
 & \dfrac{25\,x^4}{4} + \dfrac{10\,x^3}{3} - \dfrac{127\,x^2}{18} - 2\,x + 2\tfrac{1}{4} \;\Big|\; \underline{\dfrac{5\,x^2}{2} + \dfrac{2\,x}{3} - \dfrac{3}{2}} \\
 & \underline{\dfrac{25\,x^4}{4}} \\
5\,x^2 + \dfrac{2\,x}{3} & \dfrac{10\,x^3}{3} \\
 & \underline{\dfrac{10\,x^3}{3} + \;\; \dfrac{4\,x^2}{9}} \\
5\,x^2 + \dfrac{4\,x}{3} - \dfrac{3}{2} & -\;\dfrac{15\,x^2}{2} \\
 & \underline{-\;\dfrac{15\,x^2}{2} \;\; - 2\,x + 2\tfrac{1}{4}}
\end{array}$$

13.

$$\frac{9}{4}a^4 - 2a^3 + \frac{67}{9}a^2 - \frac{28}{9}a + \frac{49}{9} \,\Big|\, \frac{3a^2}{2} - \frac{2a}{3} + \frac{7}{3}$$

$$\frac{9a^4}{4}$$

$$3a^2 - \frac{2a}{3} \,\Big|\, -2a^3$$

$$-2a^3 + \frac{4a^2}{9}$$

$$3a^2 - \frac{4a}{3} + \frac{7}{3} \,\Big|\, 7a^2$$

$$7a^2 - \frac{28a}{9} + \frac{49}{9}$$

14.

$$\frac{100a^2}{9} - 20 + \frac{9}{a^2} \,\Big|\, \frac{10a}{3} - \frac{3}{a}$$

$$\frac{100a^2}{9}$$

$$\frac{20a}{3} - \frac{3}{a} \,\Big|\, -20$$

$$-20 + \frac{9}{a^2}$$

15.

$$\frac{x^2}{a^2} + \frac{5x}{a} + 4\tfrac{1}{4} - \frac{5a}{x} + \frac{a^2}{x^2} \,\Big|\, \frac{x}{a} + \frac{5}{2} - \frac{a}{x}$$

$$\frac{x^2}{a^2}$$

$$\frac{2x}{a} + \frac{5}{2} \,\Big|\, \frac{5x}{a}$$

$$\frac{5x}{a} + \frac{25}{4}$$

$$\frac{2x}{a} + 5 - \frac{a}{x} \,\Big|\, -2$$

$$-2 - \frac{5a}{x} + \frac{a^2}{x^2}$$

16.

$$\frac{a^2}{b^2} + 2a^2 + a^2b^2 + 2 + 2b^2 + \frac{b^2}{a^2} \,\Big|\, \frac{a}{b} + ab + \frac{b}{a}$$

$$\frac{a^2}{b^2}$$

$$\frac{2a}{b} + ab \,\Big|\, 2a^2$$

$$2a^2 + a^2b^2$$

$$\frac{2a}{b} + 2ab + \frac{b}{a} \,\Big|\, 2$$

$$2 + 2b^2 + \frac{b^2}{a^2}$$

17.

$$
\begin{array}{l|l}
 & \dfrac{a^4}{25\,c^4}+\dfrac{2\,a^2}{c^3}+\dfrac{117}{5\,c^2}-\dfrac{40}{a^2c}+\dfrac{16}{a^4}\;\Big|\;\dfrac{a^2}{5\,c^2}+\dfrac{5}{c}-\dfrac{4}{a^2} \\
 & \dfrac{a^4}{25\,c^4} \\
\hline
\dfrac{2\,a^2}{5\,c^2}+\dfrac{5}{c} & \dfrac{2\,a^2}{c^3} \\
 & \dfrac{2\,a^2}{c^3}+\dfrac{25}{c^2} \\
\hline
\dfrac{2\,a^2}{5\,c^2}+\dfrac{10}{c}-\dfrac{4}{a^2} & -\dfrac{8}{5\,c^2} \\
 & -\dfrac{8}{5\,c^2}-\dfrac{40}{a^2c}+\dfrac{16}{a^4} \\
\hline
\end{array}
$$

18.

$$
\begin{array}{l|l}
 & \dfrac{a^2}{4\,c^4}-\dfrac{3\,a}{c^2}+9+\dfrac{2}{5\,ac}-\dfrac{12\,c}{5\,a^2}+\dfrac{4\,c^2}{25\,a^4}\;\Big|\;\dfrac{a}{2\,c^2}-3+\dfrac{2\,c}{5\,a^2} \\
 & \dfrac{a^2}{4\,c^4} \\
\hline
\dfrac{a}{c^2}-3 & -\dfrac{3\,a}{c^2} \\
 & -\dfrac{3\,a}{c^2}+9 \\
\hline
\dfrac{a}{c^2}-6+\dfrac{2\,c}{5\,a^2} & \dfrac{2}{5\,ac} \\
 & \dfrac{2}{5\,ac}-\dfrac{12\,c}{5\,a^2}+\dfrac{4\,c^2}{25\,a^4} \\
\hline
\end{array}
$$

19.

$$
\begin{array}{l|l}
 & \dfrac{a^4}{25\,x^2}-\dfrac{6\,a}{5}+\dfrac{1}{5\,x}+\dfrac{9\,x^2}{a^2}-\dfrac{3\,x}{a^3}+\dfrac{1}{4\,a^4}\;\Big|\;\dfrac{a^2}{5\,x}-\dfrac{3\,x}{a}+\dfrac{1}{2\,a^2} \\
 & \dfrac{a^4}{25\,x^2} \\
\hline
\dfrac{2\,a^2}{5\,x}-\dfrac{3\,x}{a} & -\dfrac{6\,a}{5} \\
 & -\dfrac{6\,a}{5}+\dfrac{9\,x^2}{a^2} \\
\hline
\dfrac{2\,a^2}{5\,x}-\dfrac{6\,x}{a}+\dfrac{1}{2\,a^2} & \dfrac{1}{5\,x} \\
 & \dfrac{1}{5\,x}-\dfrac{3\,x}{a^3}+\dfrac{1}{4\,a^4} \\
\hline
\end{array}
$$

20.

$$\begin{array}{l|l}
 & 1+2x \;\Big|\; 1+x-\frac{x^2}{2}+\frac{x^3}{2} \\
 & 1 \\
2+x & 2x \\
 & 2x+x^2 \\
2+2x-\frac{x^2}{2} & -x^2 \\
 & -x^2-x^3+\frac{x^4}{4} \\
2+2x-x^2+\frac{x^3}{2} & x^3-\frac{x^4}{4}
\end{array}$$

21.

$$\begin{array}{l|l}
 & \frac{25}{9}+a^3 \;\Big|\; \frac{5}{3}+\frac{3a^3}{10}-\frac{27a^6}{1000}+\frac{243a^9}{50000} \\
 & \frac{25}{9} \\
\frac{10}{3}+\frac{3a^3}{10} & a^3 \\
 & a^3+\frac{9a^6}{100} \\
\frac{10}{3}+\frac{3a^3}{5}-\frac{27a^6}{1000} & -\frac{9a^6}{100} \\
 & -\frac{9a^6}{100}-\frac{81a^9}{5000}+\frac{729a^{12}}{1000000} \\
\frac{10}{3}+\frac{3a^3}{5}-\frac{27a^6}{500}+\frac{243a^9}{50000} & \frac{81a^9}{5000}-\frac{729a^{12}}{1000000}
\end{array}$$

Page 341

1.
$$\begin{array}{r|l}
 & 62'41 \;\big|\; 79 \\
 & 49 \\
149 & 13\,41 \\
 & 13\,41
\end{array}$$

2.
$$\begin{array}{r|l}
 & 92'16 \;\big|\; 96 \\
 & 81 \\
186 & 11\,16 \\
 & 11\,16
\end{array}$$

3.
$$\begin{array}{r|l}
 & 1'51'29 \;\big|\; 123 \\
 & 1 \\
22 & 51 \\
 & 44 \\
243 & 7\,29 \\
 & 7\,29
\end{array}$$

4.
$$\begin{array}{r|l}
 & 5'61'69 \;\big|\; 237 \\
 & 4 \\
43 & 1\,61 \\
 & 1\,29 \\
467 & 32\,69 \\
 & 32\,69
\end{array}$$

5.
$$\begin{array}{r|l}
 & 1.46'41 \;\big|\; 1.21 \\
 & 1 \\
22 & 46 \\
 & 44 \\
241 & 2\,41 \\
 & 2\,41
\end{array}$$

6.
$$\begin{array}{r|l}
 & 2'16.09 \;\big|\; 14.7 \\
 & 1 \\
24 & 1\,16 \\
 & 96 \\
287 & 20\,09 \\
 & 20\,09
\end{array}$$

CR

7. 98′80′36 | 994
81
189 | 17 80
| 17 01
1984 | 79 36
| 79 36

8. 7′04′90′25 | 2655
4
46 | 3 04
| 2 76
525 | 28 90
| 26 25
5305 | 2 65 25
| 2 65 25

9. 3.96′01 | 1.99
1
29 | 2 96
| 2 61
389 | 35 01
| 35 01

10. 7 | 2.645+
4
46 | 3 00
| 2 76
524 | .24 00
| 20 96
5285 | 3 04 00
| 2 64 25

11. .63 | .793+
49
149 | 14 00
| 13 41
1583 | 59 00
| 47 49

12. .01′23′50 | .111+
1
21 | 23
| 21
221 | 2 50
| 2 21

13. .96′38′40 | .981+
81
188 | 15 38
| 15 04
1961 | 32 40
| 19 61

14. $\frac{5}{7} = .714285+$.
.71′42′85 | .845+
64
164 | 7 42
| 6 56
1685 | 86 85
| 84 25

15. $4\frac{2}{9} = 4.222222+$.
4.22′22′22 | 2.054+
4
405 | 22 22
| 20 25
4104 | 1 97 22
| 1 64 16

16. $\frac{10}{7} = 1.428571+$.
1.42′85′71 | 1.195+
1
21 | 42
| 21
229 | 21 85
| 20 61
2385 | 1 24 71
| 1 19 25

17. $\frac{4}{5} = .800000.$

```
       .80′00′00 |.894 +
        64
   169 | 16 00
       | 15 21
  1784 | 79 00
       | 71 36
```

18. $23\frac{6}{11} = 23.545454+.$

```
   23.54′54′54 |4.852 +
   16
 88 | 7 54
    | 7 04
 965 | 50 54
     | 48 25
 9702 | 2 29 54
      | 1 94 04
```

19. $89\frac{1}{3} = 89.333333.$

```
   89.33′33′33 |9.451 +
   81
 184 | 8 33
     | 7 36
 1885 | 97 33
      | 94 25
 18901 | 3 08 33
       | 1 89 01
```

20. $\overline{136}^2 + \overline{273}^2$

$= 18{,}496 + 74{,}529$

$= 93{,}025.$

```
   9′30′25 |305
   9
 605 | 30 25
     | 30 25
```

21. $\sqrt{\overline{90}^2 + \overline{90}^2} = 90\sqrt{2}$

$= 90 \cdot 1.4142$

$= 127.28$ feet.

22. $\overline{207}^2 - \overline{83}^2 = 42{,}849 - 6889$

$= 35{,}960.$

```
     3′59′60 |189.63
     1
   28 | 2 59
      | 2 24
  369 | 35 60
      | 33 21
 3786 | 2 39 00
      | 2 27 16
37923 | 11 84 00
      | 11 37 69
```

23. $\overline{5471}^2 - \overline{4059}^2$

$= 29{,}931{,}841 - 16{,}475{,}481$

$= 13{,}456{,}360.$

```
     13′45′63′60 |3668.2 +
     9
   66 | 4 45
      | 3 96
  726 | 49 63
      | 43 56
 7328 | 6 07 60
      | 5 86 24
73362 | 21 36 00
      | 14 67 24
```

24. $\sqrt{(17)^2 - (\frac{17}{2})^2} = \frac{17}{2}\sqrt{3}$

$= 8.5\,(1.7320)$

$= 14.722$

$= 14.7$ inches.

25. $s^2 - \left(\frac{s}{2}\right)^2 = (15)^2.$

$s = 10\sqrt{3}$

$= 17.320$ inches.

26. $\sqrt{37 \cdot 2 \cdot 10 \cdot 25} = 10\sqrt{185}$

$= 10\,(13.60147)$

$= 136.0147$ square inches

$= .945$ square feet.

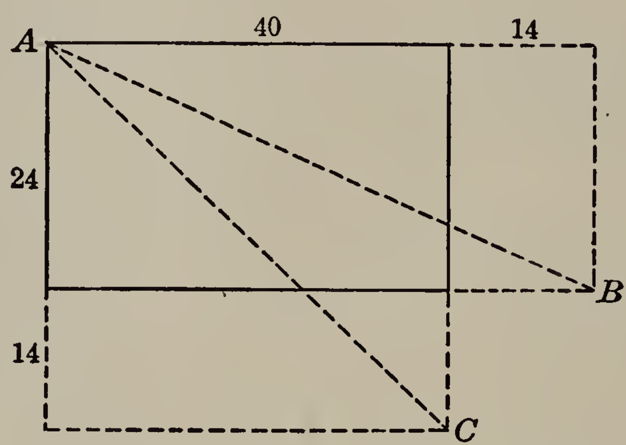
27. $\sqrt{33 \cdot 11 \cdot 11 \cdot 11} = 121\sqrt{3}$
$= 121\,(1.73205)$
$= 209.57$ square inches.

28. $\pi r^2 = 70.$
$r = \sqrt{70 \div 3.1416}$
$= 4.72$ feet.

29. Diagonal $= \sqrt{\overline{15}^2 + \overline{22}^2 + \overline{28}^2}$
$= 38.63$ feet.

30. Diagonal $= \sqrt{8^2 + 8^2 + 8^2}$
$= 8\sqrt{3}$
$= 13.85$ feet.

31. Folding back one end wall and one side wall shows that the shortest line is either AB or AC.



$$AB = \sqrt{(40 + 14)^2 + (24)^2} = \sqrt{3492}.$$
$$AC = \sqrt{(24 + 14)^2 + (40)^2} = \sqrt{3044}.$$

Therefore $\quad AC = 2\sqrt{761} = 2\,(27.586) = 55.172.$ *Ans.*

Page 347

1. $\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}.$
2. $\sqrt{28} = \sqrt{4 \cdot 7} = 2\sqrt{7}.$
3. $\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}.$
4. $\sqrt{52} = \sqrt{4 \cdot 13} = 2\sqrt{13}.$
5. $\sqrt{68} = \sqrt{4 \cdot 17} = 2\sqrt{17}.$
6. $\sqrt{108} = \sqrt{36 \cdot 3} = 6\sqrt{3}.$
7. $\sqrt{192} = \sqrt{64 \cdot 3} = 8\sqrt{3}.$
8. $2\sqrt{45} = 2\sqrt{9 \cdot 5} = 6\sqrt{5}.$
9. $\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = 2\sqrt[3]{2}.$
10. $4\sqrt[3]{54} = 4\sqrt[3]{27 \cdot 2} = 12\sqrt[3]{2}.$
11. $\sqrt[4]{48} = \sqrt[4]{16 \cdot 3} = 2\sqrt[4]{3}.$
12. $3\sqrt[5]{64} = 3\sqrt[5]{32 \cdot 2} = 6\sqrt[5]{2}.$

13. $\sqrt{a^3} = \sqrt{a^2 \cdot a} = a\sqrt{a}.$
14. $\sqrt{ax^3} = \sqrt{x^2\,(ax)} = x\sqrt{ax}.$
15. $\sqrt[3]{a^2x^5} = \sqrt[3]{x^3\,(a^2x^2)} = x\sqrt[3]{a^2x^2}.$
16. $5\sqrt[3]{16a^4} = 5\sqrt[3]{8a^3 \cdot 2a} = 10a\sqrt[3]{2a}.$
18. $\sqrt{\frac{2}{5}} = \sqrt{\frac{10}{25}} = \frac{1}{5}\sqrt{10}.$
19. $\sqrt{\frac{3}{7}} = \sqrt{\frac{21}{49}} = \frac{1}{7}\sqrt{21}.$
20. $\sqrt[3]{\frac{1}{4}} = \sqrt[3]{\frac{2}{8}} = \frac{1}{2}\sqrt[3]{2}.$
21. $\sqrt[3]{\frac{2}{9}} = \sqrt[3]{\frac{6}{27}} = \frac{1}{3}\sqrt[3]{6}.$
23 $\sqrt{\dfrac{1}{2x}} = \sqrt{\dfrac{2x}{4x^2}} = \dfrac{1}{2x}\sqrt{2x}.$
24. $\sqrt{\dfrac{1}{a^3}} = \sqrt{\dfrac{a}{a^4}} = \dfrac{1}{a^2}\sqrt{a}.$

25. $\sqrt[3]{\frac{1}{a^2}} = \sqrt[3]{\frac{a}{a^3}} = \frac{1}{a}\sqrt[3]{a}.$

26. $\sqrt[3]{\frac{1}{ax^3}} = \sqrt[3]{\frac{a^2}{a^3x^3}} = \frac{1}{ax}\sqrt[3]{a^2}.$

27. $\sqrt[3]{-\frac{3}{4}} = \sqrt[3]{-\frac{6}{8}} = -\frac{1}{2}\sqrt[3]{6}.$

28. $\sqrt{1-(\frac{1}{3})^2} = \sqrt{\frac{8}{9}} = \frac{2}{3}\sqrt{2}.$

29. $\sqrt{3^2-(\frac{3}{2})^2} = \sqrt{\frac{27}{4}} = \frac{3}{2}\sqrt{3}.$

30. $\sqrt{s^2-\left(\frac{s}{2}\right)^2} = \sqrt{\frac{3s^2}{4}} = \frac{s}{2}\sqrt{3}.$

31. $\sqrt{s^3+\left(\frac{s}{2}\right)^3} = \sqrt{\frac{9s^3}{8}} = \frac{3s}{4}\sqrt{2s}.$

32. $\sqrt{\left(\frac{a+1}{2}\right)^2 - a} = \sqrt{\frac{a^2-2a+1}{4}} = \frac{a-1}{2}.$

33. $\sqrt{\left(\frac{e^x+e^{-x}}{2}\right)^2 - 1} = \sqrt{\frac{e^{2x}-2+e^{-2x}}{4}} = \frac{e^x-e^{-x}}{2}.$

34. $\sqrt{4-8\sqrt{3}} = \sqrt{4(1-2\sqrt{3})} = 2\sqrt{1-2\sqrt{3}}.$

35. $\sqrt{36+18\sqrt{5}} = 3\sqrt{4+2\sqrt{5}}.$

36. $\sqrt[3]{16-8\sqrt{3}} = 2\sqrt[3]{2-\sqrt{3}}.$

37. $\sqrt[3]{54-9\sqrt{18}} = \sqrt[3]{54-27\sqrt{2}} = 3\sqrt[3]{2-\sqrt{2}}.$

38. $\sqrt{R^2+3R^2\sqrt{5}} = R\sqrt{1+3\sqrt{5}}.$

39. $\sqrt{R^2-\frac{R^2}{2}\sqrt{3}} = \sqrt{\frac{4R^2-2R^2\sqrt{3}}{4}} = \frac{R}{2}\sqrt{4-2\sqrt{3}}.$

41. $\sqrt[4]{9} = 3^{\frac{2}{4}} = 3^{\frac{1}{2}} = \sqrt{3}.$

42. $\sqrt[4]{a^2b^4} = a^{\frac{1}{2}}b = b\sqrt{a}.$

43. $\sqrt[4]{64\,a^2b^4} = 2^{\frac{6}{4}}a^{\frac{2}{4}}b$
$= b\sqrt{2^3a}$
$= 2b\sqrt{2a}.$

44. $\sqrt[6]{9a^2} = 3^{\frac{1}{3}}a^{\frac{1}{3}} = \sqrt[3]{3a}.$

45. $\sqrt[6]{\frac{4a^6}{25}} = a\sqrt[3]{\frac{2}{5}} = \frac{a}{5}\sqrt[3]{50}.$

47. $3\sqrt{5} = \sqrt{45}.$

48. $2\sqrt[3]{8} = \sqrt[3]{64}.$

49. $a\sqrt{a} = \sqrt{a^3}.$

50. $2c\sqrt[3]{c^2} = \sqrt[3]{8c^5}.$

51. $4\sqrt[3]{\frac{1}{4}} = \sqrt[3]{16}.$

52. $\frac{a}{3}\sqrt[3]{\frac{9}{a^2}} = \sqrt[3]{\frac{a}{3}}.$

53. $e^x\sqrt{e^x+e^{-x}} = \sqrt{e^{3x}+e^x}.$

54. $(a+1)\sqrt{\frac{1}{a^2-1}} = \sqrt{\frac{a+1}{a-1}}.$

55. $\frac{x-3a}{5}\sqrt[3]{\frac{125}{(x-3a)^2}} = \sqrt[3]{x-3a}.$

57. $\sqrt{\sqrt{a}} = (a^{\frac{1}{2}})^{\frac{1}{2}} = a^{\frac{1}{4}} = \sqrt[4]{a}.$

58. $\sqrt[3]{\sqrt{a}} = (a^{\frac{1}{2}})^{\frac{1}{3}} = \sqrt[6]{a}.$

59. $\sqrt{\sqrt[3]{a^2}} = (a^{\frac{2}{3}})^{\frac{1}{2}} = \sqrt[3]{a}.$

60. $\sqrt{\sqrt{x^3}} = (x^{\frac{3}{2}})^{\frac{1}{2}} = \sqrt[4]{x^3}.$

61. $\sqrt[3]{\sqrt{x^5}} = (x^{\frac{5}{2}})^{\frac{1}{3}} = \sqrt[6]{x^5}.$

62. $\sqrt[3]{\sqrt{8a^2x}} = ((2^3a^2x)^{\frac{1}{2}})^{\frac{1}{3}}$
$= \sqrt[6]{8a^2x}.$

63. $\sqrt{3\sqrt{3}} = (3\cdot 3^{\frac{1}{2}})^{\frac{1}{2}} = \sqrt[4]{27}.$

64. $\sqrt[4]{\sqrt{8}} = \left((8)^{\frac{1}{2}}\right)^{\frac{1}{4}} = \sqrt[8]{8}$.

65. $2\sqrt[3]{2\sqrt[3]{2}} = 2 \cdot 2^{\frac{1}{3}} \cdot 2^{\frac{1}{9}} = 2\sqrt[9]{16}$.

66. $\sqrt[a]{\sqrt[n]{x^c}} = \left((x^c)^{\frac{1}{n}}\right)^{\frac{1}{a}} = \sqrt[an]{x^c}$.

67. $\sqrt[\frac{m}{n}]{x^{\frac{a}{n}}} = \left(x^{\frac{a}{n}}\right)^{\frac{n}{m}} = \sqrt[m]{x^a}$.

68. $\sqrt{12 \cdot 2 \cdot 4 \cdot 6} = \sqrt{6^2 \cdot 2^2 \cdot 4} = 24$.

69. $\sqrt{28 \cdot 3 \cdot 4 \cdot 21} = \sqrt{7^2 \cdot 3^2 \cdot 4^2} = 84$.

70. $\sqrt{77 \cdot 44 \cdot 21 \cdot 12} = \sqrt{7 \cdot 11^2 \cdot 4 \cdot 3 \cdot 7 \cdot 4 \cdot 3} = 11 \cdot 4 \cdot 3 \cdot 7 = 924$.

71. $\sqrt{221 \cdot 36 \cdot 68 \cdot 117} = \sqrt{13 \cdot 17 \cdot 36 \cdot 4 \cdot 17 \cdot 9 \cdot 13}$
$= 13 \cdot 17 \cdot 6 \cdot 2 \cdot 3$
$= 7956$.

Page 349

2. $\sqrt{20} + 3\sqrt{5} = 2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}$.
3. $\sqrt{50} + \sqrt{98} - \sqrt{32} = 5\sqrt{2} + 7\sqrt{2} - 4\sqrt{2} = 8\sqrt{2}$.
4. $\sqrt{12} + 5\sqrt{75} - 2\sqrt{27} = 2\sqrt{3} + 25\sqrt{3} - 6\sqrt{3} = 21\sqrt{3}$.
5. $\sqrt{75} + 3\sqrt{147} - \sqrt{12} = 5\sqrt{3} + 21\sqrt{3} - 2\sqrt{3} = 24\sqrt{3}$.
6. $2\sqrt{54} + \sqrt{24} - \sqrt{96} = 6\sqrt{6} + 2\sqrt{6} - 4\sqrt{6} = 4\sqrt{6}$.
7. $\sqrt[3]{16} + \sqrt[3]{54} - 3\sqrt[3]{2} = 2\sqrt[3]{2} + 3\sqrt[3]{2} - 3\sqrt[3]{2} = 2\sqrt[3]{2}$.
8. $\sqrt[3]{192} - 4\sqrt[3]{24} + \sqrt[3]{375} = 4\sqrt[3]{3} - 8\sqrt[3]{3} + 5\sqrt[3]{3} = \sqrt[3]{3}$.
9. $\sqrt[3]{625} + \sqrt[3]{40} + \sqrt[3]{135} = 5\sqrt[3]{5} + 2\sqrt[3]{5} + 3\sqrt[3]{5} = 10\sqrt[3]{5}$.
10. $10\sqrt{\frac{6}{5}} - \sqrt{\frac{3}{10}} + \sqrt{\frac{15}{2}} = 2\sqrt{30} - \frac{1}{10}\sqrt{30} + \frac{1}{2}\sqrt{30} = \frac{12}{5}\sqrt{30}$.
11. $a\sqrt{x^3} - \sqrt{a^2x} - 5\sqrt{a^2x} = ax\sqrt{x} - a\sqrt{x} - 5a\sqrt{x} = (ax - 6a)\sqrt{x}$.
12. $\sqrt{x^3} + \sqrt[4]{x^2} - 12\sqrt[6]{x^3} = x\sqrt{x} + \sqrt{x} - 12\sqrt{x} = (x - 11)\sqrt{x}$.
13. $\sqrt{\frac{a}{x^3}} - \sqrt{\frac{a}{x^5}} + \sqrt{\frac{5x^3}{a}} = \frac{1}{x^2}\sqrt{ax} - \frac{1}{x^3}\sqrt{ax} + \frac{x}{a}\sqrt{5ax}$.
14. $\sqrt{\frac{6}{7}} + \sqrt{\frac{3}{14}} - \sqrt{\frac{21}{2}} = \frac{1}{7}\sqrt{42} + \frac{1}{14}\sqrt{42} - \frac{1}{2}\sqrt{42} = -\frac{2}{7}\sqrt{42}$.
15. $\sqrt[4]{32x^5} + \sqrt[4]{1250x} - \sqrt[4]{512x} - \sqrt[4]{2x} = 2x\sqrt[4]{2x} + 5\sqrt[4]{2x} - 4\sqrt[4]{2x} - \sqrt[4]{2x}$
$= 2x\sqrt[4]{2x}$.
16. $\sqrt{(a+c)^3} - c\sqrt[4]{(a+c)^2} + 2c\sqrt[6]{(a+c)^3} = (a+c)\sqrt{a+c} - c\sqrt{a+c} + 2c\sqrt{a+c}$
$= (a + 2c)\sqrt{a+c}$.
17. $\sqrt[3]{(a-c)^4} + c\sqrt[6]{a^2 - 2ac + c^2} + (a+c)\sqrt[3]{a-c}$
$= (a-c)\sqrt[3]{a-c} + c\sqrt[3]{a-c} + (a+c)\sqrt[3]{a-c} = (2a + c)\sqrt[3]{a-c}$.
18. $\sqrt{\frac{a}{c}} - \sqrt{\frac{c}{a}} + \sqrt{\frac{a^2+c^2}{ac} + 2} - \sqrt{\frac{a^2+c^2}{ac} - 2}$
$= \frac{1}{c}\sqrt{ac} - \frac{1}{a}\sqrt{ac} + \frac{a+c}{ac}\sqrt{ac} - \frac{a-c}{ac}\sqrt{ac}$
$= \frac{a+c}{ac}\sqrt{ac}$.

19. $\sqrt[3]{24}+\sqrt[3]{(3a+9)(a+3)^2}-\sqrt[3]{81}+a\sqrt[6]{9}-4\sqrt[3]{3}$

$\quad = 2\sqrt[3]{3}+(a+3)\sqrt[3]{3}-3\sqrt[3]{3}+a\sqrt[3]{3}-4\sqrt[3]{3}=(2a-2)\sqrt[3]{3}.$

20. $2\sqrt{9a^3-9a^2b}-3\sqrt{9ab^2-9b^3}+\sqrt{(a^2-b^2)(a+b)}$

$\quad = 6a\sqrt{a-b}-9b\sqrt{a-b}+(a+b)\sqrt{a-b}$

$\quad = (7a-8b)\sqrt{a-b}.$

21. $(a-b)\sqrt{\dfrac{a+b}{a-b}}+\sqrt{25a^2-25b^2}+\dfrac{a+b}{a-b}\sqrt{\dfrac{36ab^2-36b^3}{a+b}}$

$\quad =\sqrt{a^2-b^2}+5\sqrt{a^2-b^2}+\dfrac{6b}{a-b}\sqrt{a^2-b^2}$

$\quad =\dfrac{6a}{a-b}\sqrt{a^2-b^2}.$

Page 350

1. $\sqrt{12}\sqrt{18}=\sqrt{216}=6\sqrt{6}.$

2. $\sqrt{\frac{2}{3}}\cdot\sqrt{\frac{3}{4}}=\sqrt{\frac{1}{2}}=\frac{1}{2}\sqrt{2}.$

3. $\sqrt{\frac{4}{5}}\cdot\sqrt{\frac{5}{6}}\cdot\sqrt{\frac{3}{4}}=\sqrt{\frac{1}{2}}=\frac{1}{2}\sqrt{2}.$

4. $(\sqrt{2}-3\sqrt{5})\sqrt{5}=\sqrt{10}-15.$

5.
$$\begin{array}{l}
\sqrt{3}-2\sqrt{2}\\
-\sqrt{3}+\sqrt{2}\\ \hline
-3+2\sqrt{6}\\
-4+\sqrt{6}\\ \hline
-7+3\sqrt{6}
\end{array}$$

6.
$$\begin{array}{l}
\sqrt{5}-3\sqrt{2}\\
2\sqrt{5}-\sqrt{3}\\ \hline
10-6\sqrt{10}-\sqrt{15}+3\sqrt{6}
\end{array}$$

7.
$$\begin{array}{l}
\sqrt{a}-\sqrt{ax}\\
\sqrt{a}+2\sqrt{ax}\\ \hline
a-a\sqrt{x}\\
+2a\sqrt{x}-2ax\\ \hline
a+a\sqrt{x}-2ax
\end{array}$$

8.
$$\begin{array}{r}
3\sqrt{5}-\sqrt{2}\\
3\sqrt{5}+\sqrt{2}\\ \hline
45-3\sqrt{10}\\
-2+3\sqrt{10}\\ \hline
43\phantom{+3\sqrt{10}}
\end{array}$$

9.
$$\begin{array}{r}
4\sqrt{5}+2\sqrt{7}\\
4\sqrt{5}-2\sqrt{7}\\ \hline
80+8\sqrt{35}\\
-28-8\sqrt{35}\\ \hline
52\phantom{-8\sqrt{35}}
\end{array}$$

10.
$$\begin{array}{l}
\sqrt{3a}-\sqrt{2a}\\
\sqrt{3a}-\sqrt{2a}\\ \hline
3a-a\sqrt{6}\\
2a-a\sqrt{6}\\ \hline
5a-2a\sqrt{6}
\end{array}$$

11. $(2\sqrt{3x-1})^2=12x-4.$

12.
$$\begin{array}{r}
\sqrt{x}-\sqrt{x-2}\\
\sqrt{x}-\sqrt{x-2}\\ \hline
x-\sqrt{x^2-2x}\\
x-2-\sqrt{x^2-2x}\\ \hline
2x-2-2\sqrt{x^2-2x}
\end{array}$$

13.
$$\begin{array}{r}
\sqrt{5}-\sqrt{3}-\sqrt{2}\\
\sqrt{5}+\sqrt{3}-\sqrt{2}\\ \hline
5-\sqrt{15}-\sqrt{10}\phantom{+\sqrt{6}}\\
-3+\sqrt{15}\phantom{-\sqrt{10}}-\sqrt{6}\\
2\phantom{-\sqrt{15}}-\sqrt{10}+\sqrt{6}\\ \hline
4\phantom{-\sqrt{15}}-2\sqrt{10}\phantom{+\sqrt{6}}
\end{array}$$

14.
$$\begin{array}{l}
3\sqrt{2}+2\sqrt{3}+\sqrt{30}\\
\underline{2\sqrt{2}+2\sqrt{3}-2\sqrt{5}\qquad\qquad}\\
12+\ 4\sqrt{6}+4\sqrt{15}\\
12+\ 6\sqrt{6}\qquad\qquad\qquad +6\sqrt{10}\\
\underline{\quad -10\sqrt{6}-4\sqrt{15}\ -6\sqrt{10}}\\
24
\end{array}$$

15.
$$\begin{array}{l}
R-\frac{R}{2}\sqrt{3}\\
\underline{2R+\frac{3R}{2}\sqrt{3}}\\
2R^2-\quad R^2\sqrt{3}\\
\underline{-\frac{9R^2}{4}+\frac{3R^2}{2}\sqrt{3}}\\
-\frac{R^2}{4}\ +\ \frac{R^2}{2}\sqrt{3}
\end{array}$$

16. $\left(\frac{R}{2}\right)^2+\left(\frac{R}{2}-\frac{R}{2}\sqrt{3}\right)^2=\frac{R^2}{4}+\frac{R^2}{4}-\frac{R^2}{2}\sqrt{3}+\frac{3R^2}{4}=\frac{5R^2}{4}-\frac{R^2}{2}\sqrt{3}.$

17. $R^2-\left(\frac{R}{2}\sqrt{5}-\frac{R}{2}\right)^2=R^2-\frac{5R^2}{4}+\frac{R^2\sqrt{5}}{2}-\frac{R^2}{4}$
$$=\frac{R^2}{2}\sqrt{5}-\frac{R^2}{2}$$
$$=\frac{R^2}{2}(\sqrt{5}-1).$$

18. $(\sqrt{2}-\sqrt{x-3})^2=2-2\sqrt{2x-6}+x-3=x-1-2\sqrt{2x-6}.$

19. $(\sqrt{x-3}+\sqrt{x+5})^2=x-3+2\sqrt{x^2+2x-15}+x+5$
$$=2x+2+2\sqrt{x^2+2x-15}.$$

20. $(2\sqrt{x}-3\sqrt{2x+1})^2=4x-12\sqrt{2x^2+x}+18x+9$
$$=22x+9-12\sqrt{2x^2+x}.$$

21. $(3\sqrt{x-1}+2\sqrt{2-x})^2=9x-9+12\sqrt{-x^2+3x-2}+8-4x$
$$=5x-1+12\sqrt{-x^2+3x-2}.$$

22. $(a+\sqrt{a+b})(a-\sqrt{a+b})=a^2-a-b.$

23. $(\sqrt{2x-3}-\sqrt{3x})(\sqrt{2x-3}+\sqrt{3x})=2x-3-3x=-x-3.$

24. $\sqrt{2}$, $\sqrt[3]{3}=2^{\frac{1}{2}}$, $3^{\frac{1}{3}}=2^{\frac{3}{6}}$, $3^{\frac{2}{6}}=\sqrt[6]{8}$, $\sqrt[6]{9}$ respectively.

25. $\sqrt[4]{ax}$, $\sqrt{a}=\sqrt[4]{ax}$, $\sqrt[4]{a^2}$ respectively.

26. $\sqrt[3]{9}$, $\sqrt{8}=9^{\frac{2}{6}}$, $8^{\frac{3}{6}}=\sqrt[6]{81}$, $2\sqrt[6]{8}$ respectively.

27. $\sqrt{3}\sqrt[3]{3}=\sqrt[6]{3^3\cdot 3^2}=\sqrt[6]{243}.$

28. $\sqrt{a}\sqrt[3]{a}=\sqrt[6]{a^3\cdot a^2}=\sqrt[6]{a^5}.$

29. $\sqrt[4]{a^3}\sqrt{a}=\sqrt[4]{a^3\cdot a^2}=a\sqrt[4]{a}.$

30. $\sqrt[3]{12}\sqrt{\frac{1}{8}}=\sqrt[6]{144\cdot\frac{1}{512}}=\sqrt[6]{\frac{9}{32}}=\frac{1}{2}\sqrt[6]{18}.$

31. $\sqrt[3]{a^2}\sqrt{a^3}=\sqrt[6]{a^4a^9}=a^2\sqrt[6]{a}.$

32. $\sqrt{\frac{x}{a}}\sqrt[3]{\frac{a}{x}}=\sqrt[6]{\frac{x^3}{a^3}}\sqrt[6]{\frac{a^2}{x^2}}=\sqrt[6]{\frac{x}{a}}=\frac{\sqrt[6]{a^5x}}{a}.$

33. $\sqrt[4]{2x^3}\sqrt{3x}=\sqrt[4]{2x^3\cdot 9x^2}=x\sqrt[4]{18x}.$

Page 352

1. $\sqrt{7}$.
2. $\sqrt{5}$.
3. $\sqrt{2}$.
4. $\sqrt[3]{2}$.
5. $\sqrt[3]{25}$.
6. $\sqrt[4]{2}$.
7. $\sqrt{3}+2$.
8. $3\sqrt{6}+2\sqrt{11}$.
9. $\sqrt{3a}+\sqrt{2x}$.
10. $\sqrt{x-3}+2\sqrt{3x}$.
11. $\sqrt{a-b}-\sqrt{a+b}$.
12. $3\sqrt{2}+\sqrt{5}$.
13. $\sqrt{8}\div\sqrt{2}=\sqrt{4}=2$.
14. $6\sqrt{10}\div\sqrt{5}=6\sqrt{2}$.
15. $\sqrt{8}\div\sqrt{24}=\sqrt{\frac{1}{3}}=\frac{1}{3}\sqrt{3}$.

16. $\sqrt{2ax}\div\sqrt{3a^2x}=\sqrt{\dfrac{2}{3a}}=\dfrac{1}{3a}\sqrt{6a}$.

17. $(2\sqrt{10}-3\sqrt{5})\div 2\sqrt{5}=\sqrt{2}-\frac{3}{2}$.

18. $(\sqrt{ax^2}-\sqrt{a^2x})\div\sqrt{ax}=\sqrt{x}-\sqrt{a}$.

19. $8\div 4\sqrt{3}=\dfrac{2}{\sqrt{3}}=\dfrac{2}{3}\sqrt{3}$.

20. $24\div 3\sqrt{3}=\dfrac{8}{\sqrt{3}}=\dfrac{8}{3}\sqrt{3}$.

22. $\sqrt{5}\div\sqrt[3]{3}=\sqrt[6]{\dfrac{5^3}{3^2}}=\dfrac{1}{3}\sqrt[6]{10{,}125}$.

23. $\sqrt{a}\div\sqrt[4]{2}=\sqrt[4]{\dfrac{a^2}{2}}=\dfrac{1}{2}\sqrt[4]{8a^2}$.

24. $\sqrt[4]{8}\div\sqrt{2}=\sqrt[4]{\frac{8}{4}}=\sqrt[4]{2}$.

25. $\sqrt[3]{a}\div\sqrt[6]{a}=\sqrt[6]{a^2\div a}=\sqrt[6]{a}$.

26. $\sqrt[3]{\frac{1}{4}}\div\sqrt{\frac{1}{2}}=\sqrt[6]{\frac{1}{16}\div\frac{1}{8}}=\sqrt[6]{\frac{1}{2}}=\frac{1}{2}\sqrt[6]{32}$.

27. $$\frac{\sqrt{3}}{\sqrt{3}-2}=\frac{\sqrt{3}(\sqrt{3}+2)}{(\sqrt{3}-2)(\sqrt{3}+2)}=\frac{3+2\sqrt{3}}{3-4}=-3-2\sqrt{3}.$$

28. $$\frac{\sqrt{5}}{\sqrt{5}+\sqrt{2}}=\frac{\sqrt{5}(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}=\frac{5-\sqrt{10}}{5-2}=\frac{5-\sqrt{10}}{3}.$$

29. $$\frac{2\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}}=\frac{(2\sqrt{3}+\sqrt{5})(\sqrt{3}+\sqrt{5})}{(\sqrt{3}-\sqrt{5})(\sqrt{3}+\sqrt{5})}=\frac{11+3\sqrt{15}}{3-5}=-\frac{11+3\sqrt{15}}{2}.$$

30. $$\frac{\sqrt{3}}{2\sqrt{6}-\sqrt{3}}=\frac{\sqrt{3}(2\sqrt{6}+\sqrt{3})}{(2\sqrt{6}-\sqrt{3})(2\sqrt{6}+\sqrt{3})}=\frac{6\sqrt{2}+3}{21}=\frac{2\sqrt{2}+1}{7}.$$

31. $$\frac{2\sqrt{5}+3\sqrt{7}}{3\sqrt{5}-2\sqrt{7}}=\frac{(2\sqrt{5}+3\sqrt{7})(3\sqrt{5}+2\sqrt{7})}{(3\sqrt{5}-2\sqrt{7})(3\sqrt{5}+2\sqrt{7})}=\frac{72+13\sqrt{35}}{17}.$$

32. $$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{(\sqrt{a}+\sqrt{b})^2}{a-b}=\frac{a+b+2\sqrt{ab}}{a-b}.$$

33. $$\frac{\sqrt{x-3}+\sqrt{3}}{\sqrt{x-3}-\sqrt{3}}=\frac{(\sqrt{x-3}+\sqrt{3})^2}{x-3-3}=\frac{x+2\sqrt{3x-9}}{x-6}.$$

34. $\dfrac{\sqrt{a+x}}{\sqrt{5}-\sqrt{a+x}}=\dfrac{\sqrt{a+x}(\sqrt{5}+\sqrt{a+x})}{5-a-x}=\dfrac{\sqrt{5a+5x}+a+x}{5-a-x}.$

35. $\dfrac{4}{\sqrt{2-\sqrt{2}}}=\dfrac{4\sqrt{2+\sqrt{2}}}{\sqrt{(2-\sqrt{2})(2+\sqrt{2})}}=\dfrac{4\sqrt{2+\sqrt{2}}}{\sqrt{4-2}}=\dfrac{4\sqrt{4+2\sqrt{2}}}{2}$

$=2\sqrt{4+2\sqrt{2}}.$

36. $\dfrac{\sqrt{10}-\sqrt{5}}{\sqrt{10}+\sqrt{5}}=\dfrac{(\sqrt{10}-\sqrt{5})^2}{10-5}=\dfrac{15-2\sqrt{50}}{5}=3-2\sqrt{2}.$

37. $\dfrac{2-\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\dfrac{(2-\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

$=-5-2\sqrt{2}+2\sqrt{3}+2\sqrt{6}.$

38. $\dfrac{\sqrt{5}+\sqrt{7}-\sqrt{2}}{\sqrt{5}-\sqrt{7}}=\dfrac{(\sqrt{5}+\sqrt{7}-\sqrt{2})(\sqrt{5}+\sqrt{7})}{(\sqrt{5}-\sqrt{7})(\sqrt{5}+\sqrt{7})}$

$=\dfrac{-12-2\sqrt{35}+\sqrt{10}+\sqrt{14}}{2}.$

39. No.

40. $(3-\sqrt{7})^2-6(3-\sqrt{7})+2=16-6\sqrt{7}-18+6\sqrt{7}+2=0.$ Yes.

41. $2\left(\dfrac{7-\sqrt{3}}{2}\right)^2-75\left(\dfrac{7-\sqrt{3}}{2}\right)+161$

$=\dfrac{52-14\sqrt{3}}{2}-\dfrac{525-75\sqrt{3}}{2}+161,$ which does not equal zero. No.

42. $3\left(\dfrac{5\pm\sqrt{109}}{6}\right)^2-5\left(\dfrac{5\pm\sqrt{109}}{6}\right)-7$

$=\dfrac{134\pm10\sqrt{109}}{12}-\dfrac{50\pm10\sqrt{109}}{12}-7=\dfrac{84}{12}-7=0.$ Yes.

Page 354

1. $\sqrt{8-2\sqrt{15}}=\sqrt{3-2\sqrt{15}+5}=\sqrt{5}-\sqrt{3}.$
2. $\sqrt{5-2\sqrt{6}}=\sqrt{3-2\sqrt{6}+2}=\sqrt{3}-\sqrt{2}.$
3. $\sqrt{13+\sqrt{48}}=\sqrt{13+2\sqrt{12}}=\sqrt{12+2\sqrt{12}+1}=\sqrt{12}+1=2\sqrt{3}+1.$
4. $\sqrt{7-\sqrt{40}}=\sqrt{7-2\sqrt{10}}=\sqrt{5-2\sqrt{10}+2}=\sqrt{5}-\sqrt{2}.$
5. $\sqrt{16-6\sqrt{7}}=\sqrt{16-2\sqrt{63}}=\sqrt{7-2\sqrt{63}+9}=3-\sqrt{7}.$
6. $\sqrt{17+12\sqrt{2}}=\sqrt{17+2\sqrt{72}}=\sqrt{9+2\sqrt{72}+8}=2\sqrt{2}+3.$
7. $\sqrt{11-3\sqrt{8}}=\sqrt{11-2\sqrt{18}}=\sqrt{9-2\sqrt{18}+2}=3-\sqrt{2}.$

8. $\sqrt{11-\sqrt{120}}=\sqrt{11-2\sqrt{30}}=\sqrt{6-2\sqrt{30}+5}=\sqrt{6}-\sqrt{5}.$

9. $\sqrt{65x-20\sqrt{3x^2}}=\sqrt{65x-2\sqrt{300x^2}}$
$=\sqrt{5x-2\sqrt{300x^2}+60x}$
$=2\sqrt{15x}-\sqrt{5x}.$

10. $\sqrt{126a-10a\sqrt{5}}=\sqrt{126a-2\sqrt{125a^2}}$
$=\sqrt{125a-2\sqrt{125a^2}+a}$
$=5\sqrt{5a}-\sqrt{a}.$

11. $\sqrt{\frac{13a}{4}-\sqrt{3a^2}}=\sqrt{\frac{13a}{4}-2\sqrt{\frac{3a^2}{4}}}=\sqrt{3a-2\sqrt{\frac{3a^2}{4}}+\frac{a}{4}}$
$=\sqrt{3}a-\frac{1}{2}\sqrt{a}.$

12. $\sqrt{2x+2\sqrt{x^2-49}}=\sqrt{x+7+2\sqrt{x^2-49}+x-7}=\sqrt{x+7}+\sqrt{x-7}.$

13. $\sqrt{a+\sqrt{a^2-1}}=\sqrt{a+2\sqrt{\frac{a^2-1}{4}}}=\sqrt{\frac{a-1}{2}+2\sqrt{\frac{a^2-1}{4}}+\frac{a+1}{2}}$
$=\sqrt{\frac{a-1}{2}}+\sqrt{\frac{a+1}{2}}=\frac{1}{2}\sqrt{2a+2}+\frac{1}{2}\sqrt{2a-2}.$

14. $\sqrt{9+3\sqrt{8}}=\sqrt{9+2\sqrt{18}}=\sqrt{6+2\sqrt{18}+3}=\sqrt{6}+\sqrt{3}.$

15. $\sqrt{15-5\sqrt{8}}=\sqrt{15-2\sqrt{50}}=\sqrt{5-2\sqrt{50}+10}=\sqrt{10}-\sqrt{5}.$

16. $\sqrt{a+\sqrt{a^2-4b^2}}=\sqrt{a+2\sqrt{\frac{a^2-4b^2}{4}}}=\sqrt{\frac{a-2b}{2}+2\sqrt{\frac{a^2-4b^2}{4}}+\frac{a+2b}{2}}$
$=\sqrt{\frac{a-2b}{2}}+\sqrt{\frac{a+2b}{2}}=\frac{1}{2}\sqrt{2a-4b}+\frac{1}{2}\sqrt{2a+4b}.$

17. $\sqrt{m^2+m+2n+2m\sqrt{m+2n}}=\sqrt{m^2+m+2n+2\sqrt{m^2(m+2n)}}$
$=\sqrt{m+2n+2\sqrt{m^2(m+2n)}+m^2}=\sqrt{m+2n}+m.$

Page 355

1. $x^2-11=(x-\sqrt{11})(x+\sqrt{11}).$
2. $3x^2-8=(x\sqrt{3}-2\sqrt{2})(x\sqrt{3}+2\sqrt{2}).$
3. $x^3+3=(x+\sqrt[3]{3})(x^2-x\sqrt[3]{3}+\sqrt[3]{9}).$
4. $x^3-12=(x-\sqrt[3]{12})(x^2+x\sqrt[3]{12}+2\sqrt[3]{18}).$
5. $3x^3-27=3(x-\sqrt[3]{9})(x^2+x\sqrt[3]{9}+3\sqrt[3]{3}).$
6. $5x^3+125=5(x+\sqrt[3]{25})(x^2-x\sqrt[3]{25}+5\sqrt[3]{5}).$

7. $\dfrac{2\sqrt{b}}{a-b}+\dfrac{2}{\sqrt{a}+\sqrt{b}}=\dfrac{2\sqrt{b}}{a-b}+\dfrac{2\sqrt{a}-2\sqrt{b}}{a-b}=\dfrac{2\sqrt{a}}{a-b}.$

8. $$\frac{x+c}{\sqrt{x}-\sqrt{c}}-\frac{x^{\frac{3}{2}}+c^{\frac{3}{2}}}{x-c}=\frac{(x+c)(\sqrt{x}+\sqrt{c})-x^{\frac{3}{2}}-c^{\frac{3}{2}}}{x-c}$$
$$=\frac{x^{\frac{3}{2}}+x\sqrt{c}+c\sqrt{x}+c^{\frac{3}{2}}-x^{\frac{3}{2}}-c^{\frac{3}{2}}}{x-c}$$
$$=\frac{x\sqrt{c}+c\sqrt{x}}{x-c}.$$

9. $$x^2-5=0.$$
$$(x-\sqrt{5})(x+\sqrt{5})=0.$$
$$x=\sqrt{5},\ -\sqrt{5}.$$

10. $$2x^2-3=0.$$
$$(x\sqrt{2}-\sqrt{3})(x\sqrt{2}+\sqrt{3})=0.$$
$$x\sqrt{2}-\sqrt{3}=0.$$
$$x=\frac{\sqrt{3}}{\sqrt{2}}=\frac{1}{2}\sqrt{6}.$$
$$x\sqrt{2}+\sqrt{3}=0.$$
$$x=-\frac{\sqrt{3}}{\sqrt{2}}=-\frac{1}{2}\sqrt{6}.$$

11. $$x^4-26x^2+144=0.$$
$$(x^2-8)(x^2-18)=0.$$
$$(x-\sqrt{8})(x+\sqrt{8})(x-\sqrt{18})(x+\sqrt{18})=0.$$
$$(x-2\sqrt{2})(x+2\sqrt{2})(x-3\sqrt{2})(x+3\sqrt{2})=0.$$
$$x=2\sqrt{2},\ -2\sqrt{2},\ 3\sqrt{2},\ -3\sqrt{2}.$$

12. $$4x^4+c=x^2+4cx^2.$$
$$4x^4-4cx^2-x^2+c=0.$$
$$4x^2(x^2-c)-1(x^2-c)=0.$$
$$(4x^2-1)(x^2-c)=0.$$
$$(2x-1)(2x+1)(x+\sqrt{c})(x-\sqrt{c})=0.$$
$$x=\tfrac{1}{2},\ -\tfrac{1}{2},\ -\sqrt{c},\ \sqrt{c}.$$

Page 356

1. Altitude $= \sqrt{8^2 - 4^2}$
$= 4\sqrt{3}.$

2. Altitude $= \sqrt{s^2 - \left(\frac{s}{2}\right)^2}$
$= \frac{s}{2}\sqrt{3}.$

Area $= \frac{1}{2}\left(s \cdot \frac{s}{2}\sqrt{3}\right)$
$= \frac{s^2}{4}\sqrt{3}.$

3. Let s represent the side.

Then $s^2 - \frac{s^2}{4} = 576.$
$s = 16\sqrt{3}.$

Area $= \frac{24 \cdot 16\sqrt{3}}{2}$
$= 192\sqrt{3}.$

4. $s^2 - \frac{s^2}{4} = a^2.$
$s = \frac{2a}{3}\sqrt{3}.$

5. $x^2 + y^2 = 121.$ (1)
$x^2 + (20 - y)^2 = 169.$ (2)
(2) – (1),
$400 - 40y = 48.$ (3)
From (3), $y = 8.8.$ (4)
From (4) and (1),
$x = 6.6.$ (5)

Area $= \frac{20 \cdot 6.6}{2}$
$= 66.$

6. Referring to the Hint to Exercise 5 of text,

$x^2 + y^2 = 100.$ (1)
$x^2 + (16 - y)^2 = 144.$ (2)
(2)–(1), $256 - 32y = 44.$ (3)
From (3), $y = \frac{53}{8}.$ (4)
From (4) and (1),
$x = \frac{3}{8}\sqrt{399}$
$= 7.49$, or 7.5 nearly.

7. (*a*) Referring to the figure on page 356 of text,

$OK = \sqrt{\overline{18}^2 - 9^2}$
$= 9\sqrt{3}.$

Area of $\triangle ABO = \frac{18 \cdot 9\sqrt{3}}{2}$
$= 81\sqrt{3}.$

Area of the hexagon
$= 6 \cdot 81\sqrt{3}$
$= 486\sqrt{3}.$

(*b*) Referring to the figure on page 356 of text,

$OK = \sqrt{s^2 - \left(\frac{s}{2}\right)^2}$
$= \frac{s}{2}\sqrt{3}.$

Area of $\triangle\ ABO = \frac{s^2}{4}\sqrt{3}.$

Area of the hexagon
$= 6 \cdot \frac{s^2}{4}\sqrt{3}$
$= \frac{3s^2}{2}\sqrt{3}.$

8. (*a*) Referring to the figure on page 356 of text,

$s^2 - \left(\frac{s}{2}\right)^2 = \overline{30}^2.$
$s = 20\sqrt{3}.$

Area of $\triangle ABO = \frac{30 \cdot 20\sqrt{3}}{2}$
$= 300\sqrt{3}.$

Area of the hexagon
$= 6 \cdot 300\sqrt{3}$
$= 1800\sqrt{3}.$

(*b*) Referring to the figure on page 356 of text,

$$s^2 - \left(\frac{s}{2}\right)^2 = h^2.$$

$$s = \frac{2h}{3}\sqrt{3}.$$

$$\text{Area of } \triangle ABO = \frac{1}{2}h \cdot \frac{2h}{3}\sqrt{3}$$

$$= \frac{h^2}{3}\sqrt{3}.$$

Area of the hexagon

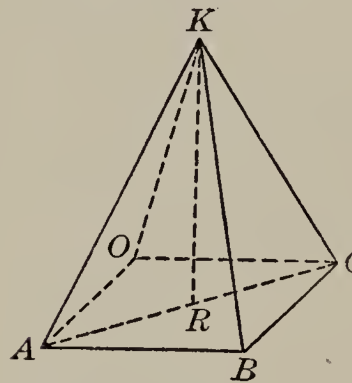
$$= 6 \cdot \frac{h^2}{3}\sqrt{3}$$

$$= 2h^2\sqrt{3}.$$

9. $$AC = \sqrt{\overline{10}^2 + \overline{10}^2}$$

$$= 10\sqrt{2}.$$

$$AR = 5\sqrt{2}.$$



$$\text{Altitude} = KR$$

$$= \sqrt{(20)^2 - (5\sqrt{2})^2}$$

$$= \sqrt{350} = 5\sqrt{14}.$$

$$\text{Volume} = \tfrac{1}{3} \cdot 5\sqrt{14} \cdot \overline{10}^2$$

$$= \tfrac{500}{3}\sqrt{14}.$$

10. $$\text{Altitude} = \sqrt{\overline{36}^2 - \overline{18}^2}$$

$$= 18\sqrt{3}.$$

One part is $\frac{2}{3} \cdot 18\sqrt{3}$.

$$\tfrac{2}{3} \cdot 18\sqrt{3} = 12\sqrt{3}.$$

The other part is $\frac{1}{3} \cdot 18\sqrt{3}$.

$$\tfrac{1}{3} \cdot 18\sqrt{3} = 6\sqrt{3}.$$

11. Referring to the figure on page 357 of text,

$$CR = \sqrt{\overline{24}^2 - \overline{12}^2}$$

$$= 12\sqrt{3}.$$

$$CK = \tfrac{2}{3} \cdot 12\sqrt{3} = 8\sqrt{3}.$$

$$KD = \sqrt{\overline{24}^2 - (8\sqrt{3})^2}$$

$$= 8\sqrt{6}.$$

12. Referring to the figure on page 357 of text,

$$CR = 15\sqrt{3}.$$

$$CK = \tfrac{2}{3} \cdot 15\sqrt{3}$$

$$= 10\sqrt{3}.$$

$$KD = \sqrt{\overline{30}^2 - (10\sqrt{3})^2}$$

$$= 10\sqrt{6}.$$

$$\text{Area of } \triangle ABC = \frac{15\sqrt{3} \cdot 30}{2}$$

$$= 225\sqrt{3}.$$

$$\text{Volume} = \frac{10\sqrt{6} \cdot 225\sqrt{3}}{3}$$

$$= 2250\sqrt{2}.$$

13. Referring to the figure on page 357 of text,

$$CR = \sqrt{e^2 - \left(\frac{e}{2}\right)^2}$$

$$= \frac{e}{2}\sqrt{3}.$$

$$CK = \frac{2}{3} \cdot \frac{e}{2}\sqrt{3}$$

$$= \frac{e}{3}\sqrt{3}.$$

$$KD = \sqrt{e^2 - \left(\frac{e}{3}\sqrt{3}\right)^2}$$

$$= \frac{e}{3}\sqrt{6}.$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot \frac{e}{2}\sqrt{3} \cdot e$$

$$= \frac{e^2}{4}\sqrt{3}.$$

$$\text{Volume} = \frac{1}{3}\left(\frac{e}{3}\sqrt{6} \cdot \frac{e^2}{4}\sqrt{3}\right)$$

$$= \frac{e^3}{12}\sqrt{2}.$$

Page 357

1. $\dfrac{3\sqrt{5}}{\sqrt{3}} = \dfrac{3\sqrt{5}\,\sqrt{3}}{\sqrt{3}\,\sqrt{3}} = \sqrt{15}.$

2. $\dfrac{2}{3\sqrt{7}} = \dfrac{2\sqrt{7}}{3(\sqrt{7})^2} = \dfrac{2}{21}\sqrt{7}.$

3. $\dfrac{2}{\sqrt{3}-\sqrt{2}} = \dfrac{2(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} = 2\sqrt{3}+2\sqrt{2}.$

4. $\dfrac{2\sqrt{3}+3\sqrt{5}}{2\sqrt{3}-3\sqrt{5}} = \dfrac{(2\sqrt{3}+3\sqrt{5})^2}{(2\sqrt{3}-3\sqrt{5})(2\sqrt{3}+3\sqrt{5})} = -\dfrac{57+12\sqrt{15}}{33}$

$= -\dfrac{19+4\sqrt{15}}{11}.$

5. $\dfrac{a\sqrt{m}-b\sqrt{c}}{a\sqrt{m}+b\sqrt{c}} = \dfrac{(a\sqrt{m}-b\sqrt{c})^2}{a^2m-b^2c} = \dfrac{a^2m+b^2c-2\,ab\sqrt{cm}}{a^2m-b^2c}.$

6. $\dfrac{a-\sqrt{a^2-1}}{a+\sqrt{a^2-1}} = \dfrac{(a-\sqrt{a^2-1})^2}{a^2-a^2+1} = \dfrac{a^2-2a\sqrt{a^2-1}+a^2-1}{1}$

$= 2a^2-2a\sqrt{a^2-1}-1.$

7. $\sqrt{28+10\sqrt{3}} = \sqrt{28+2\sqrt{75}} = \sqrt{25+2\sqrt{75}+3} = 5+\sqrt{3}.$

8. $\sqrt{57-12\sqrt{15}} = \sqrt{57-2\sqrt{540}} = \sqrt{45-2\sqrt{540}+12} = \sqrt{45}-\sqrt{12}$

$= 3\sqrt{5}-2\sqrt{3}.$

9. $\sqrt{c^4+c+2c^2\sqrt{c}} = \sqrt{c^4+c+2\sqrt{c^5}} = \sqrt{c^4+2\sqrt{c^5}+c}$

$= \sqrt{c^4}+\sqrt{c} = c^2+\sqrt{c}.$

10. $\left(\dfrac{7+3\sqrt{5}}{2}\right)^2 - 7\left(\dfrac{7+3\sqrt{5}}{2}\right)+1 = \dfrac{49+42\sqrt{5}+45}{4} - \dfrac{49+21\sqrt{5}}{2}+1$

$= \frac{94}{4} - \frac{49}{2} + 1 = 0.$

11. $3\left(\dfrac{-3\pm2\sqrt{3}}{3}\right)^2 + 6\left(\dfrac{-3\pm2\sqrt{3}}{3}\right) = \dfrac{9\mp12\sqrt{3}+12}{3} + \dfrac{-18\pm12\sqrt{3}}{3}$

$= \frac{21}{3} - \frac{18}{3} = 1.$

12. $3a\left(\dfrac{-ab+\sqrt{60ab+a^2b^2}}{6a}\right)^2+ab\left(\dfrac{-ab+\sqrt{60ab+a^2b^2}}{6a}\right)$

$$=\frac{a^2b^2-2ab\sqrt{60ab+a^2b^2}+60ab+a^2b^2}{12a}+\frac{-a^2b^2+ab\sqrt{60ab+a^2b^2}}{6a}$$

$$=\frac{2a^2b^2+60ab}{12a}-\frac{a^2b^2}{6a}=\frac{60ab}{12a}=5b.$$

13. $3\sqrt{\frac{2}{5}}+2\sqrt{\frac{1}{10}}-4\sqrt{\frac{5}{2}}+\sqrt{640}=\frac{3}{5}\sqrt{10}+\frac{1}{5}\sqrt{10}-2\sqrt{10}+8\sqrt{10}$

$$=\tfrac{34}{5}\sqrt{10}.$$

14. $\dfrac{2+\sqrt{3}}{\sqrt{6}}=\dfrac{2\sqrt{6}+3\sqrt{2}}{6}.$

15. $\dfrac{5\sqrt{7}-\sqrt{2}}{\sqrt{2}+\sqrt{7}}=\dfrac{(5\sqrt{7}-\sqrt{2})(\sqrt{2}-\sqrt{7})}{(\sqrt{2}+\sqrt{7})(\sqrt{2}-\sqrt{7})}=-\dfrac{6\sqrt{14}-37}{5}=\dfrac{37-6\sqrt{14}}{5}.$

16. $3\sqrt{\frac{1}{6}}+5\sqrt{24}+\frac{1}{3}\sqrt{96}-\sqrt{66\frac{2}{3}}=\frac{1}{2}\sqrt{6}+10\sqrt{6}+\frac{4}{3}\sqrt{6}-\frac{10}{3}\sqrt{6}$

$$=\tfrac{17}{2}\sqrt{6}.$$

17. $\dfrac{\sqrt{x}}{\sqrt{x}-\sqrt{a}}-\dfrac{\sqrt{a}}{\sqrt{x}+\sqrt{a}}=\dfrac{x+\sqrt{ax}-\sqrt{ax}+a}{x-a}=\dfrac{x+a}{x-a}.$

18. $\sqrt{40}-\dfrac{1}{\sqrt{10}}+\sqrt{\dfrac{2}{5}}+7\sqrt{\dfrac{5}{2}}=2\sqrt{10}-\dfrac{1}{10}\sqrt{10}+\dfrac{1}{5}\sqrt{10}+\dfrac{7}{2}\sqrt{10}$

$$=\tfrac{28}{5}\sqrt{10}.$$

19. $\sqrt{(10)^2-(2\sqrt{2})^2}=\sqrt{100-8}=\sqrt{92}=2\sqrt{23}.$

20. $\sqrt{1-\left(\dfrac{\sqrt{2}-2}{2}\right)^2}=\sqrt{\dfrac{4-(2-4\sqrt{2}+4)}{4}}=\sqrt{\dfrac{-2+4\sqrt{2}}{4}}$

$$=\tfrac{1}{2}\sqrt{4\sqrt{2}-2}.$$

21. $\sqrt{R^2-\left(\dfrac{R}{3}\sqrt{2}\right)^2}=\sqrt{\dfrac{9R^2-2R^2}{9}}=\sqrt{\dfrac{7R^2}{9}}=\dfrac{R}{3}\sqrt{7}.$

22. $\sqrt{(2R)^2-(R-R\sqrt{2})^2}=\sqrt{4R^2-(R^2-2\sqrt{2}R^2+2R^2)}$

$$=\sqrt{R^2+2\sqrt{2}R^2}=R\sqrt{1+2\sqrt{2}}.$$

23. $\sqrt{(R-R\sqrt{3})^2-(2R-R\sqrt{2})^2}$

$$=\sqrt{R^2-2\sqrt{3}R^2+3R^2-(4R^2-4\sqrt{2}R^2+2R^2)}$$

$$=\sqrt{-2R^2-2\sqrt{3}R^2+4\sqrt{2}R^2}$$

$$=R\sqrt{-2(1+\sqrt{3}-2\sqrt{2})}.$$

24. $\sqrt{R^2-\left(\frac{R}{2}\sqrt{5}-\frac{R}{2}\right)^2}=\sqrt{R^2-\left(\frac{5R^2}{4}-\frac{2R^2}{4}\sqrt{5}+\frac{R^2}{4}\right)}$

$$=\sqrt{\frac{-2R^2}{4}+\frac{2R^2}{4}\sqrt{5}}$$

$$=\frac{R}{2}\sqrt{2(\sqrt{5}-1)}.$$

25. $\sqrt{\left(e^x+\frac{2}{e^x}\right)^2-(e^x-2e^{-x})^2+e^{2x}+e^{-2x}-6}$

$$=\sqrt{e^{2x}+4+4e^{-2x}-(e^{2x}-4+4e^{-2x})+e^{2x}+e^{-2x}-6}$$

$$=\sqrt{e^{2x}+2+e^{-2x}}=e^x+e^{-x}.$$

26. $\dfrac{b\sqrt{ax-x^2}-bx(ax-x^2)^{-\frac{1}{2}}(a-2x)}{(\sqrt{ax-x^2})^2}\div\dfrac{bx}{\sqrt{ax-x^2}}$

$$=\frac{b\sqrt{ax-x^2}-\dfrac{bx}{\sqrt{ax-x^2}}(a-2x)}{bx(\sqrt{ax-x^2})}$$

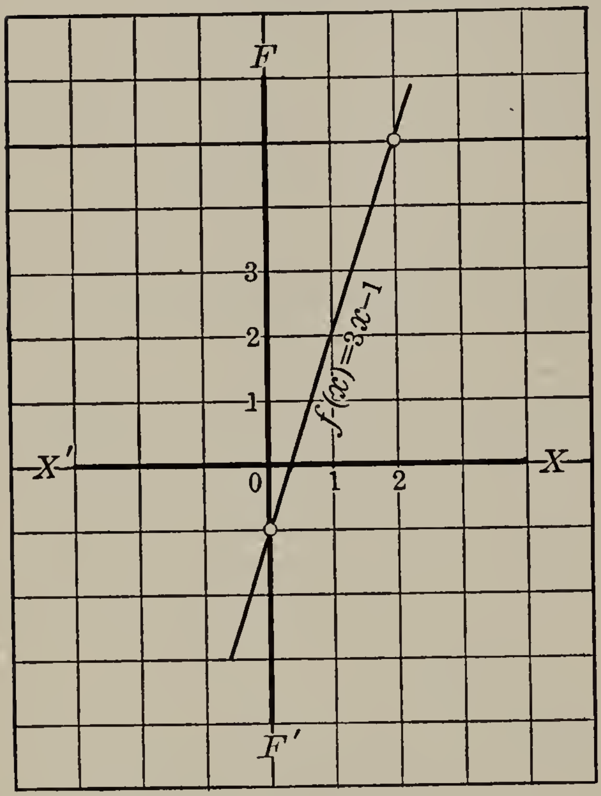
$$=\frac{\sqrt{ax-x^2}-\dfrac{x(a-2x)}{\sqrt{ax-x^2}}}{x(\sqrt{ax-x^2})}=\frac{1}{x}-\frac{a-2x}{ax-x^2}$$

$$=\frac{a-x-(a-2x)}{ax-x^2}=\frac{x}{ax-x^2}=\frac{1}{a-x}.$$

Page 360

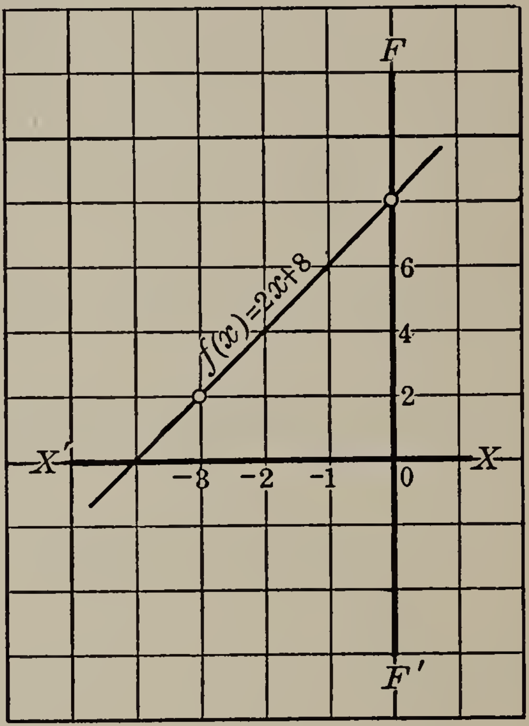
1. $f(x) = 3x - 1$.

If $x =$	0	2
then $f(x) =$	-1	5



2. $f(x) = 2x + 8$.

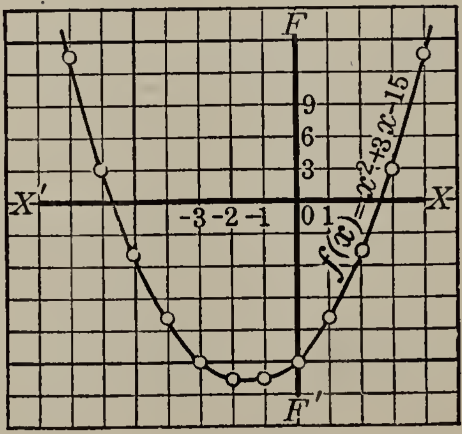
If $x =$	0	-3
then $f(x) =$	8	2



Page 361

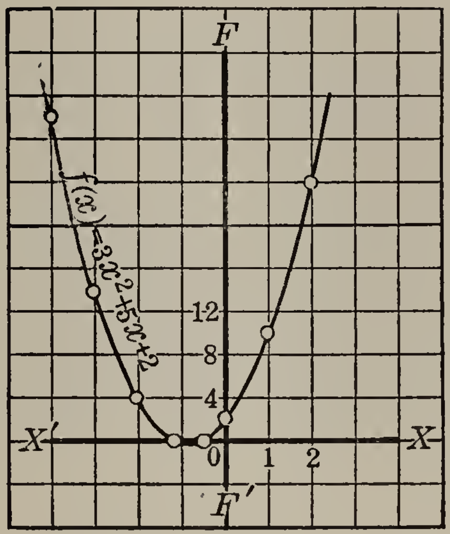
1. $f(x) = x^2 + 3x - 15$.

If $x =$	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
then $f(x) =$	13	3	-5	-11	-15	-17	-17	-15	-11	-5	3	13



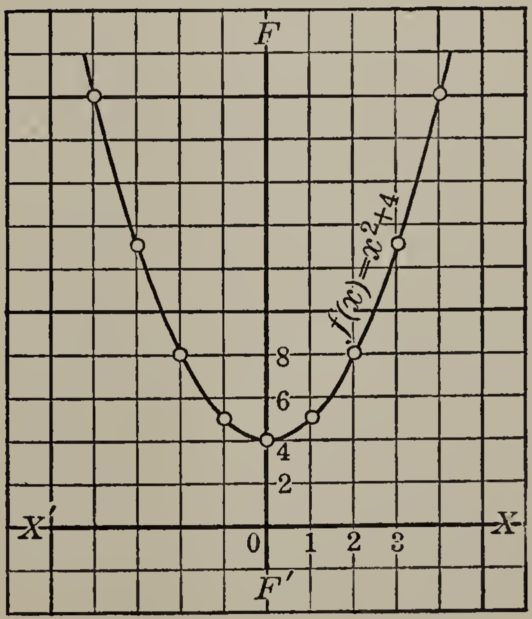
2. $f(x) = 3x^2 + 5x + 2$.

If $x =$	-4	-3	-2	-1	$-\frac{2}{3}$	0	1	2
then $f(x) =$	30	14	4	0	0	2	10	24



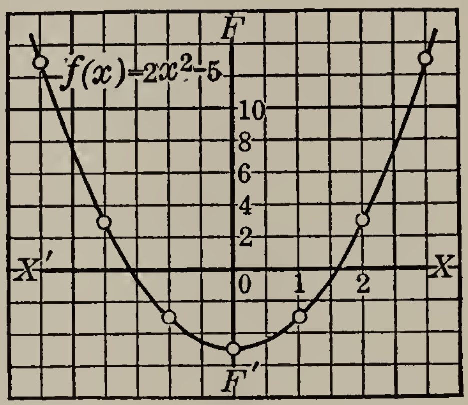
3. $f(x) = x^2 + 4$.

If $x =$	-4	-3	-2	-1	0	1	2	3	4
then $f(x) =$	20	13	8	5	4	5	8	13	20



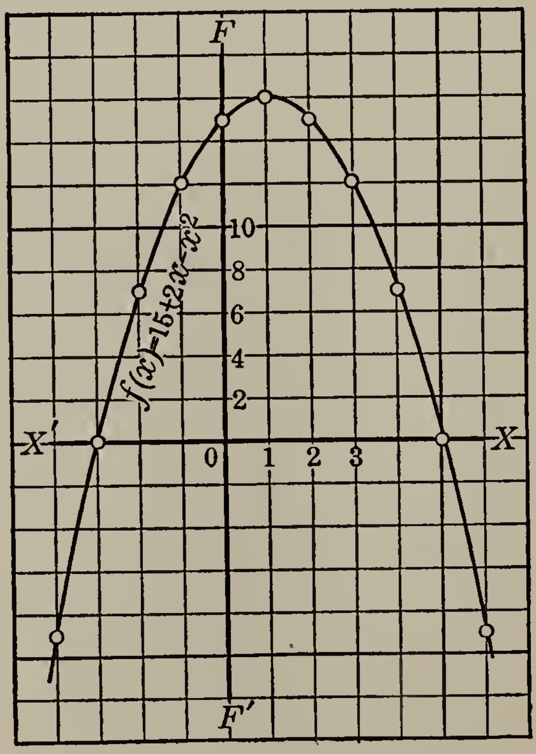
4. $f(x) = 2x^2 - 5$.

If $x =$	-3	-2	-1	0	1	2	3
then $f(x) =$	13	3	-3	-5	-3	3	13



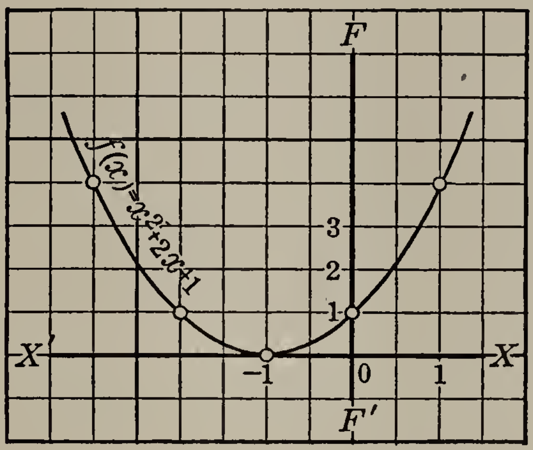
5. $f(x) = 15 + 2x - x^2$.

If $x =$	-4	-3	-2	-1	0	1	2	3	4	5	6
then $f(x) =$	-9	0	7	12	15	16	15	12	7	0	-9



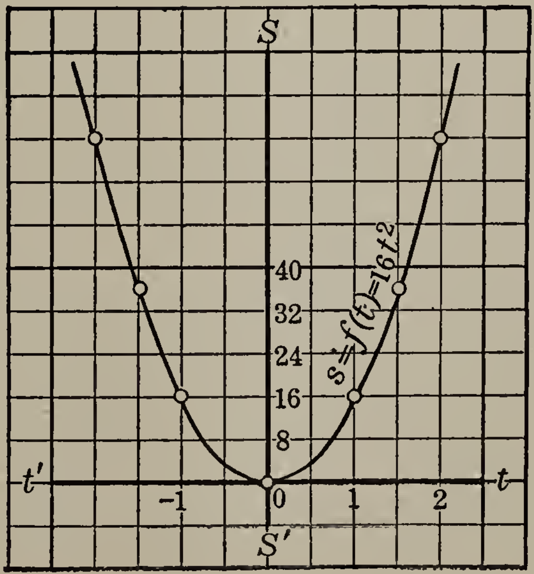
6. $f(x) = x^2 + 2x + 1$.

If $x =$	-3	-2	-1	0	1
then $f(x) =$	4	1	0	1	4



7. $s = f(t) = 16t^2$.

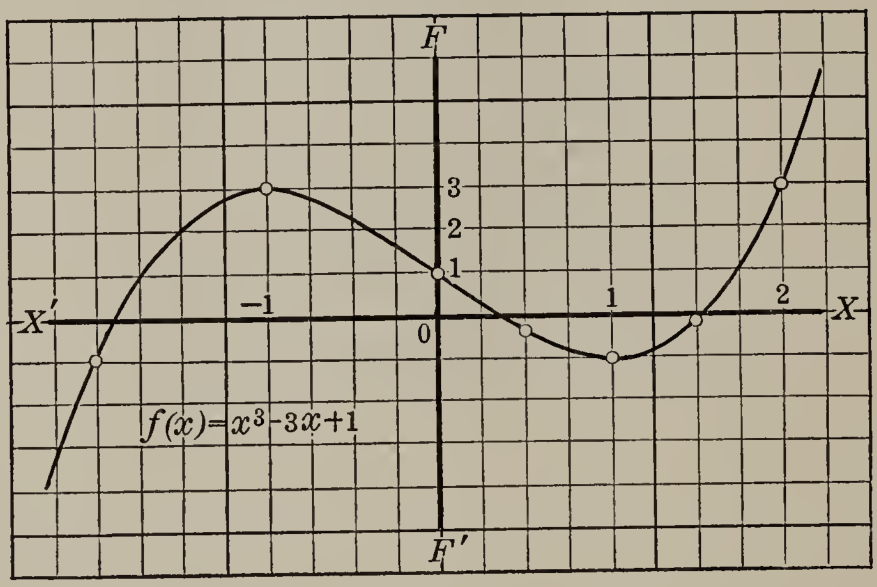
If $t =$	-2	-1.5	-1	0	1	1.5	2
then $f(t) =$	64	36	16	0	16	36	64



Page 363

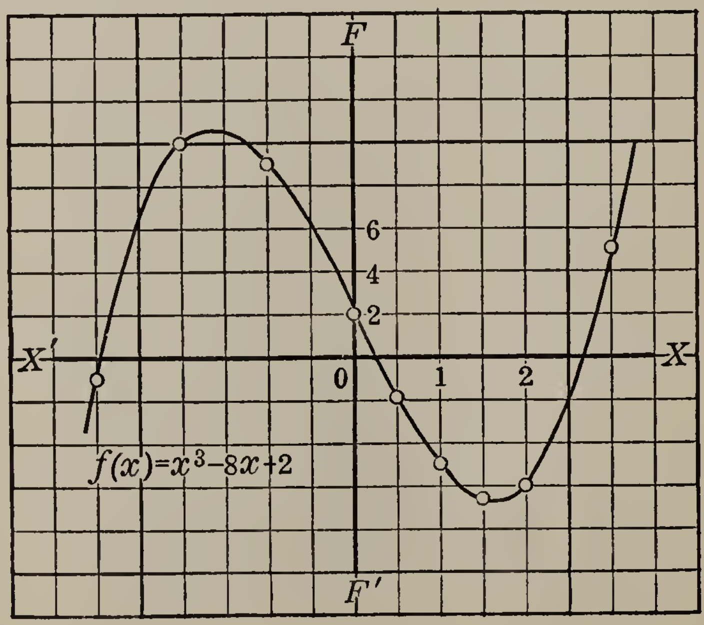
1. $f(x) = x^3 - 3x + 1$.

If $x =$	-3	-2	-1	0	$.5$	1	1.5	2
then $f(x) =$	-17	-1	3	1	$-.375$	-1	$-.125$	3



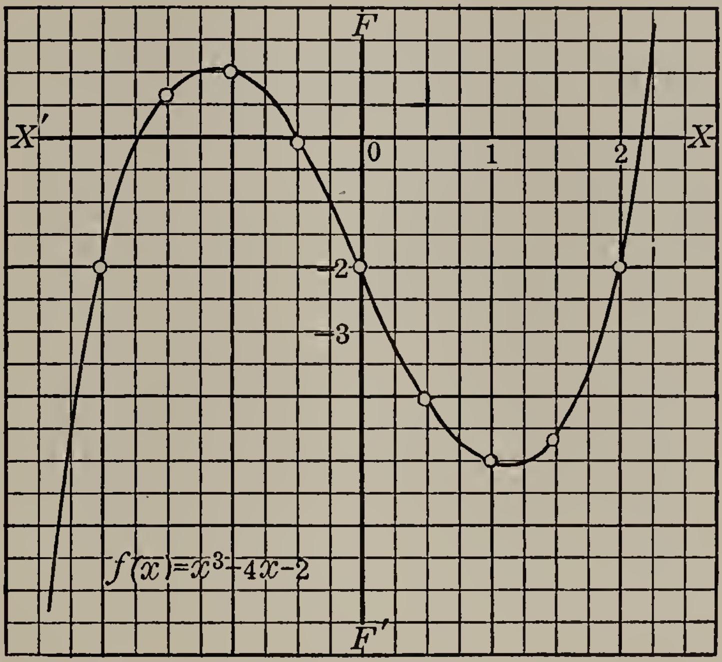
2. $f(x) = x^3 - 8x + 2$.

If $x =$	-4	-3	-2	-1	0	$.5$	1	2	3	4
then $f(x) =$	-30	-1	10	9	2	-1.875	-5	-6	5	34



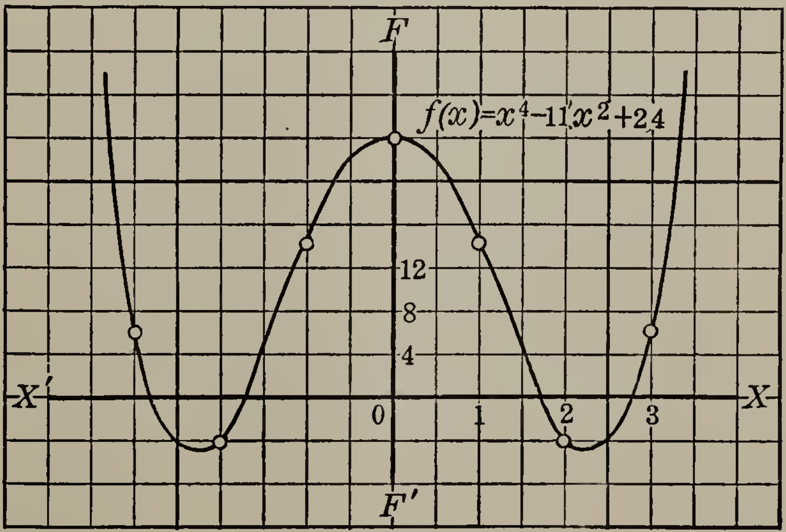
3. $f(x) = x^3 - 4x - 2$.

If $x =$	-2	-1.5	-1	$-.5$	0	$.5$	1	1.5	2
then $f(x) =$	-2	$.6$	1	$-.1$	-2	-3.9	-5	-4.6	-2



4. $f(x) = x^4 - 11x^2 + 24$.

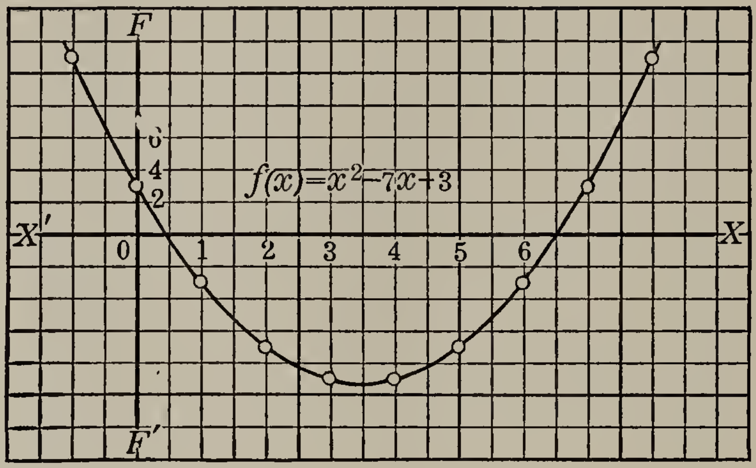
If $x =$	-4	-3	-2	-1	0	1	2	3	4
then $f(x) =$	104	6	-4	14	24	14	-4	6	104



Page 365

1. $f(x) = x^2 - 7x + 3$.

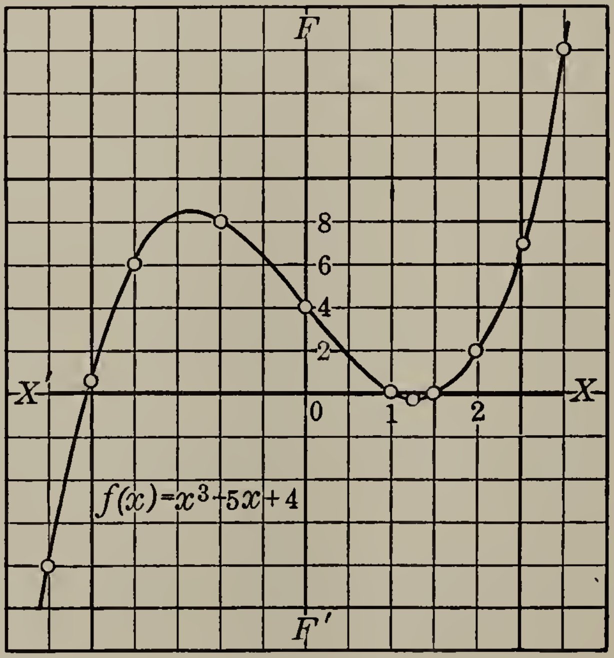
If $x =$	-1	0	1	2	3	4	5	6	7	8
then $f(x) =$	11	3	-3	-7	-9	-9	-7	-3	3	11



From the graph $x = .45, 6.5$.

2. $f(x) = x^3 - 5x + 4$.

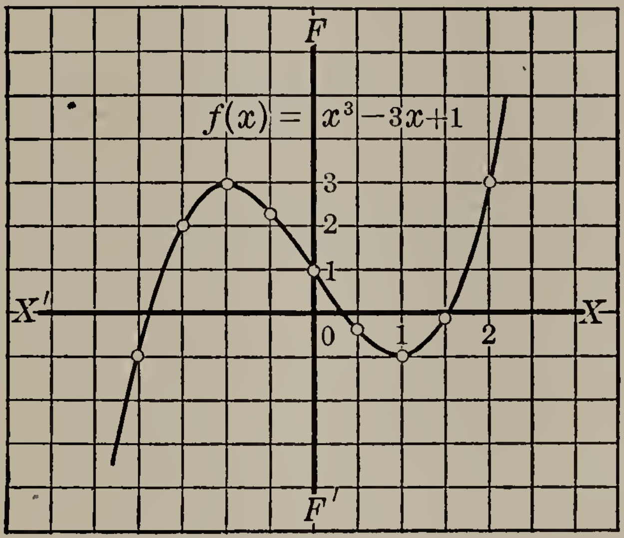
If $x =$	-3	-2.5	-2	-1	0	1	1.25	1.5	2	2.5	3
then $f(x) =$	-8	$.875$	6	8	4	0	$-.296$	$-.125$	2	7.125	16



From the graph $x = 1, 1.6, -2.6$.

3. $f(x) = x^3 - 3x + 1$.

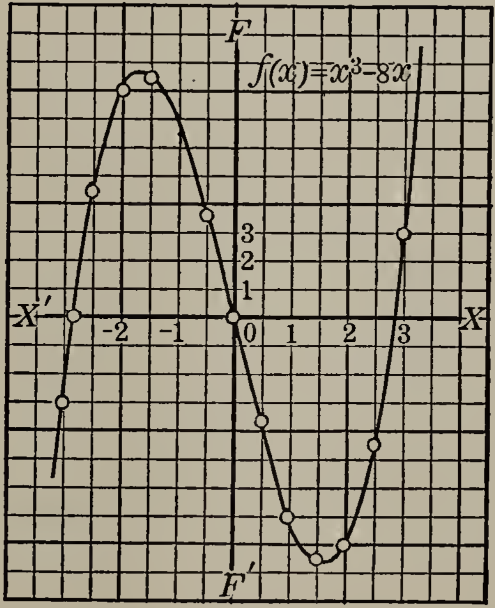
If $x =$	-2	-1.5	-1	$-.5$	0	.5	1	1.5	2	3
then $f(x) =$	-1	2.125	3	2.375	1	$-.375$	-1	$-.125$	3	19



From the graph $x = .3, 1.6, -1.8$.

4. $f(x) = x^3 - 8x$.

If $x =$	-3	-2.5	-2	-1.5	-1	$-.5$	0	.5	1	1.5	2	2.5	3
then $f(x) =$	-3	4.3	8	8.6	7	3.8	0	-3.8	-7	-8.6	-8	-4.3	3

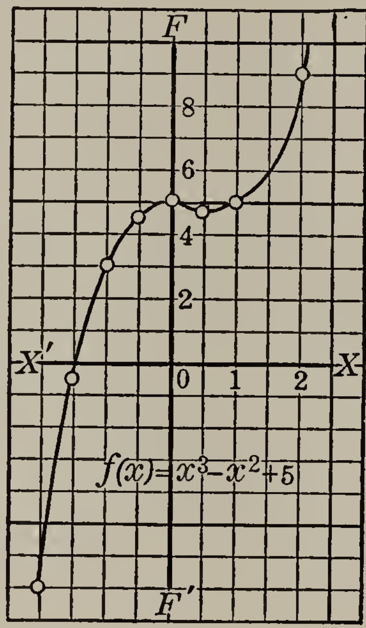


From the graph $x = 2.8$, 0, and -2.8.

CR

5. $f(x) = x^3 - x^2 + 5.$

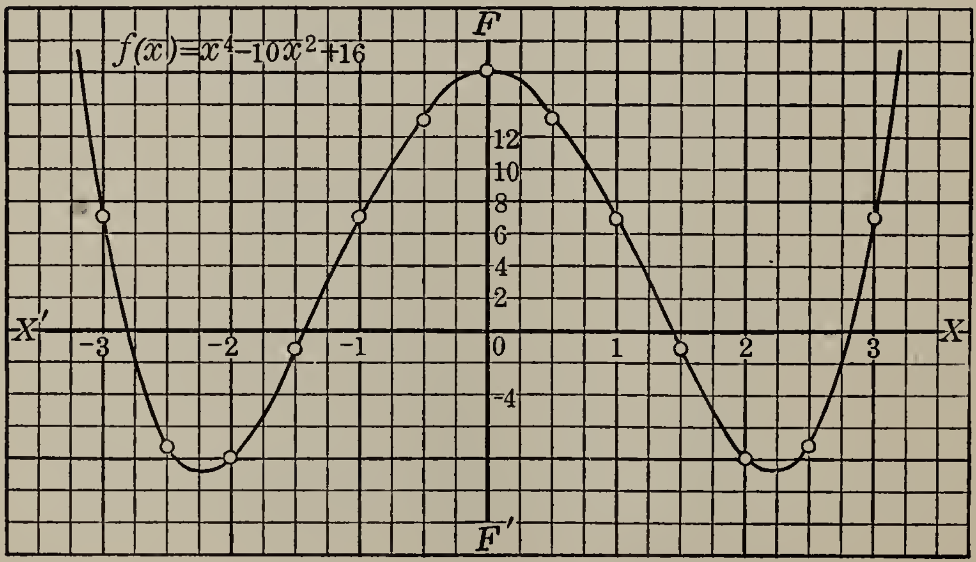
If $x =$	-2	-1.5	-1	$-.5$	0	.5	1	2
then $f(x) =$	-7	$-.625$	3	4.625	5	4.875	5	9



From the graph $x = -1.4$.

6. $f(x) = x^4 - 10x^2 + 16.$

If $x =$	-3	-2.5	-2	-1.5	-1	$-.5$	0	.5	1	1.5	2	2.5	3
then $f(x) =$	7	-7.4	-8	-1.4	7	13.5	16	13.5	7	-1.4	-8	-7.4	7

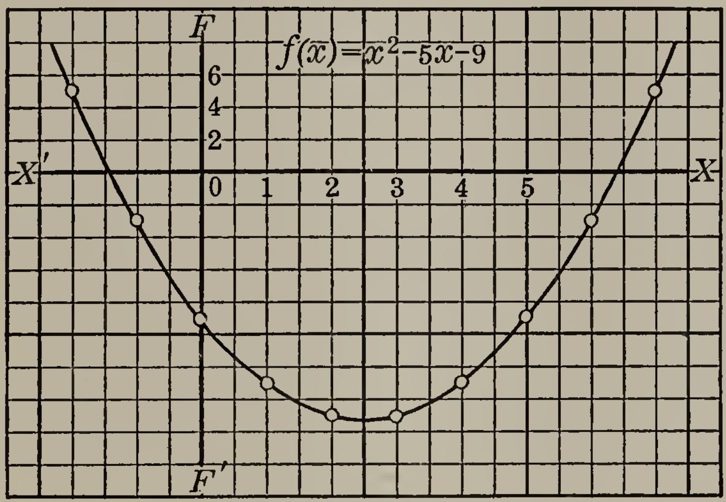


From the graph $x = 2.8,\ 1.4,\ -1.4$, and -2.8.

Page 368

1. $f(x) = x^2 - 5x - 9$.

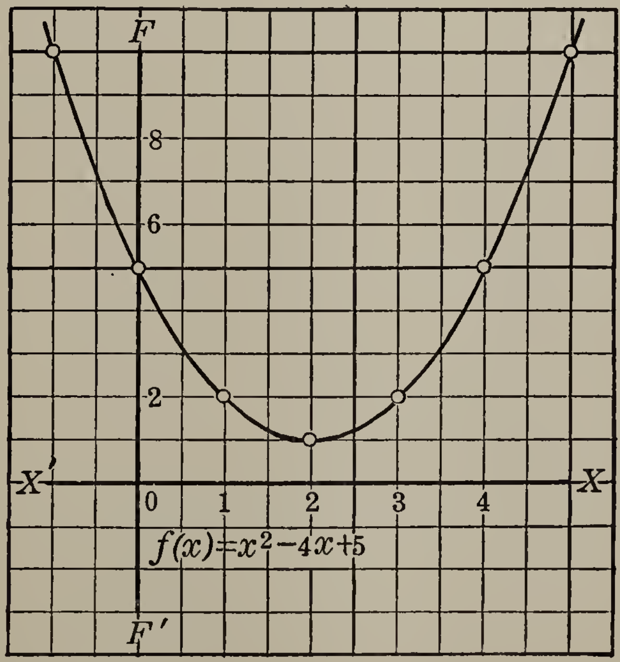
If $x =$	-3	-2	-1	0	1	2	3	4	5	6	7	8
then $f(x) =$	15	5	-3	-9	-13	-15	-15	-13	-9	-3	5	15



From the graph $x = 6.4, -1.4$.

2. $f(x) = x^2 - 4x + 5$.

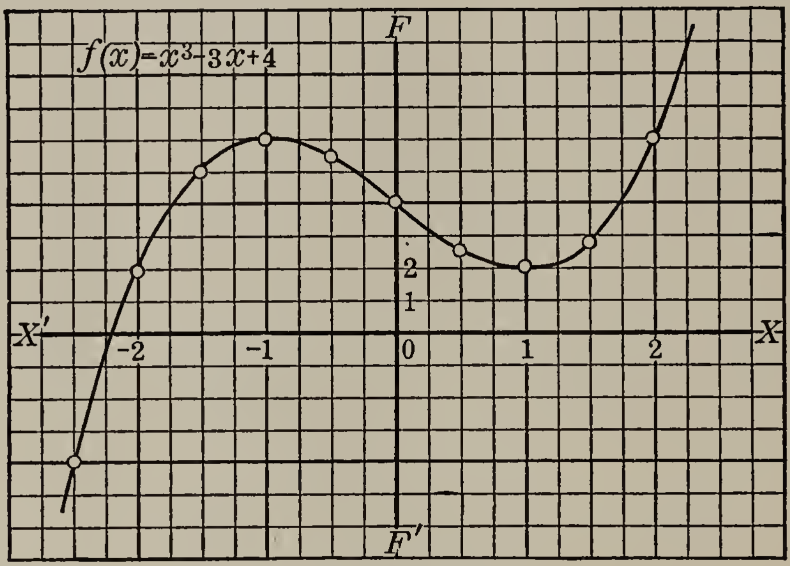
If $x =$	-1	0	1	2	3	4	5
then $f(x) =$	10	5	2	1	2	5	10



There are no real roots.

3. $f(x) = x^3 - 3x + 4.$

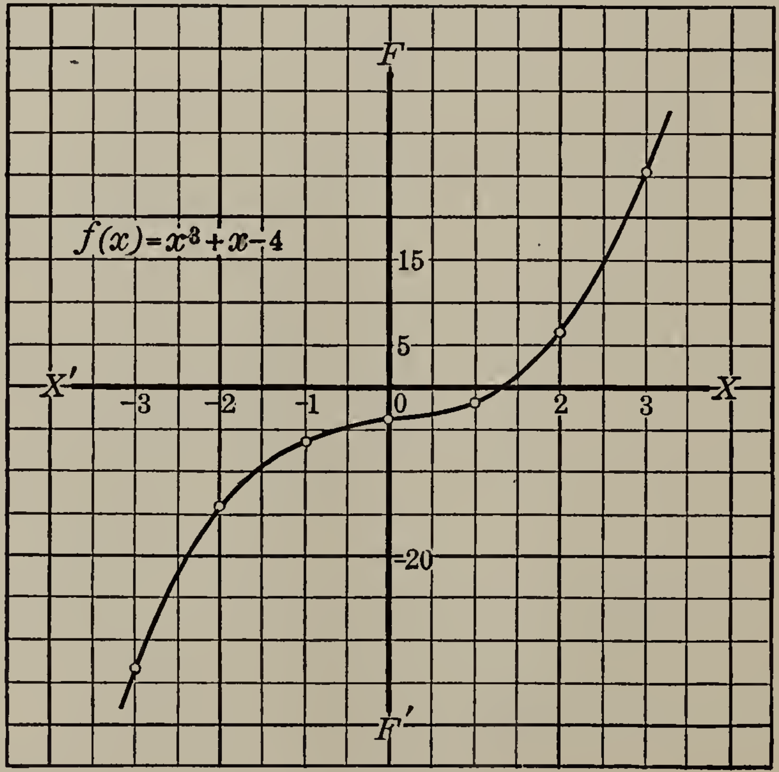
If $x =$	-2.5	-2	-1.5	-1	$-.5$	0	$.5$	1	1.5	2
then $f(x) =$	-4.1	2	5.1	6	5.3	4	2.6	2	2.8	6



From the graph $x = -2.2$, approximately.

4. $f(x) = x^3 + x - 4.$

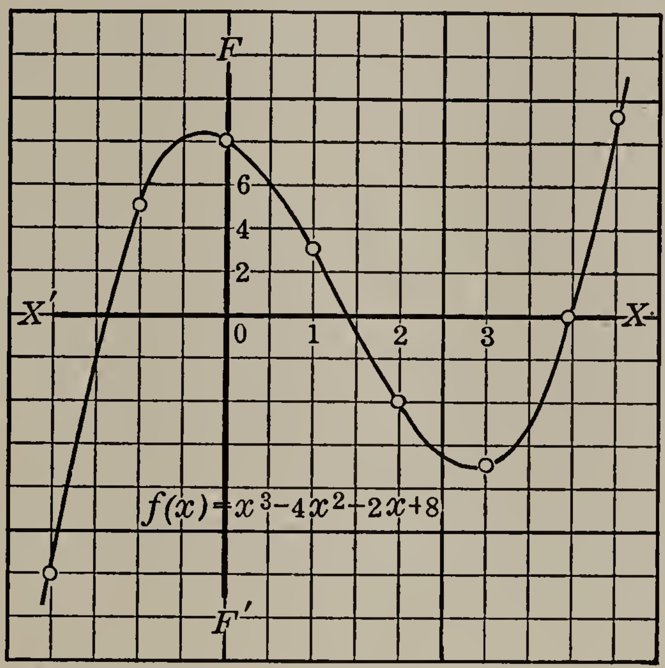
If $x =$	-3	-2	-1	0	1	2	3
then $f(x) =$	-34	-14	-6	-4	-2	6	26



From the graph $x = 1.3$, approximately.

5. $f(x) = x^3 - 4x^2 - 2x + 8$.

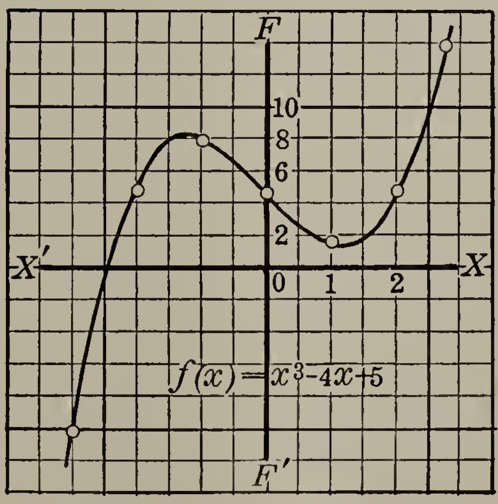
If $x =$	-2	-1	0	1	2	3	4	4.5
then $f(x) =$	-12	5	8	3	-4	-7	0	9.125



From the graph $x = 4$, 1.4, and -1.4.

6. $f(x) = x^3 - 4x + 5$.

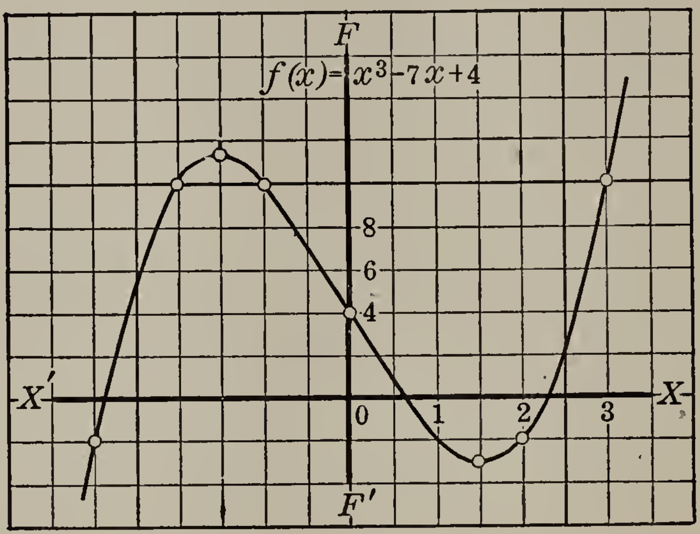
If $x =$	-3	-2	-1	0	1	2	3
then $f(x) =$	-10	5	8	5	2	5	20



From the graph $x = -2.4$.

7. $f(x) = x^3 - 7x + 4$.

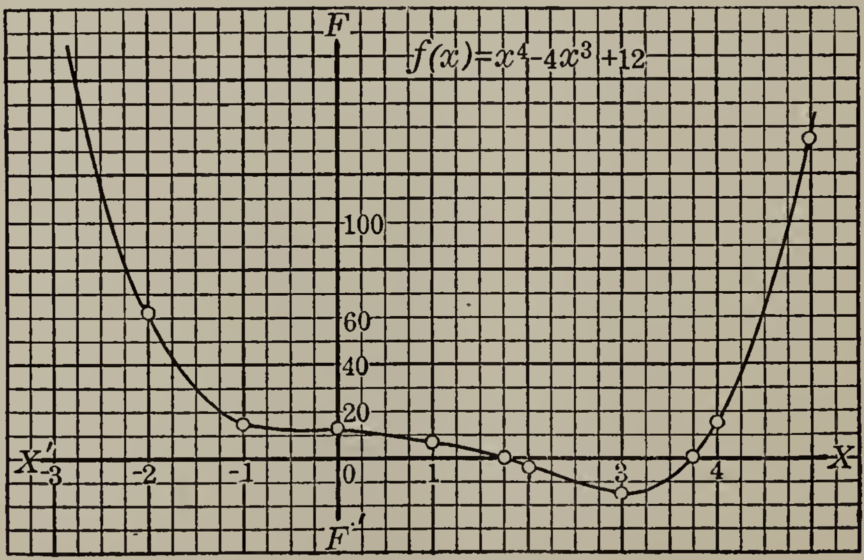
If $x =$	-4	-3	-2	-1.5	-1	0	1	1.5	2	3
then $f(x) =$	-32	-2	10	11.125	10	4	-2	-3.125	-2	10



From the graph $x = .7$, 2.3, and -2.8.

8. $f(x) = x^4 - 4x^3 + 12$.

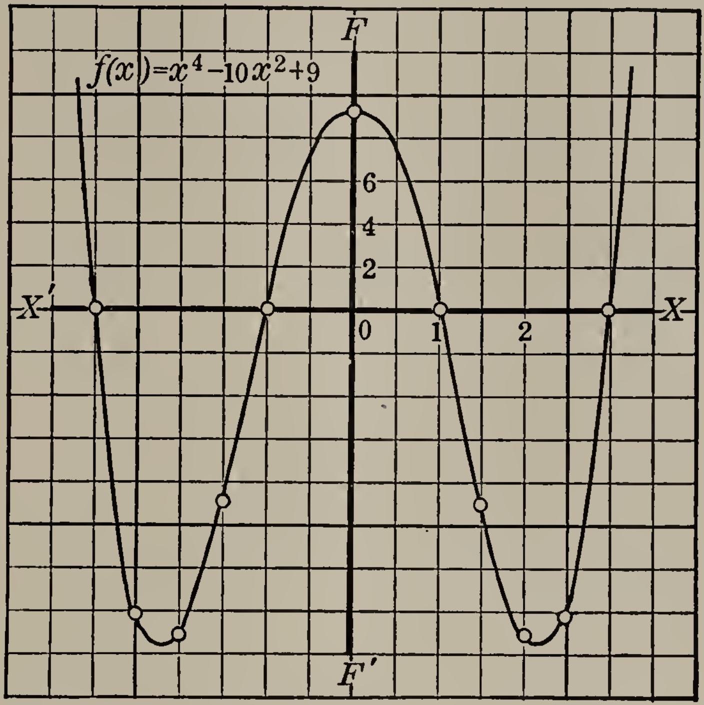
If $x =$	-2	-1	0	1	2	3	4	5
then $f(x) =$	60	17	12	9	-4	-15	12	137



From the graph $x = 1.7$ and 3.8.

9. $f(x) = x^4 - 10x^2 + 9.$

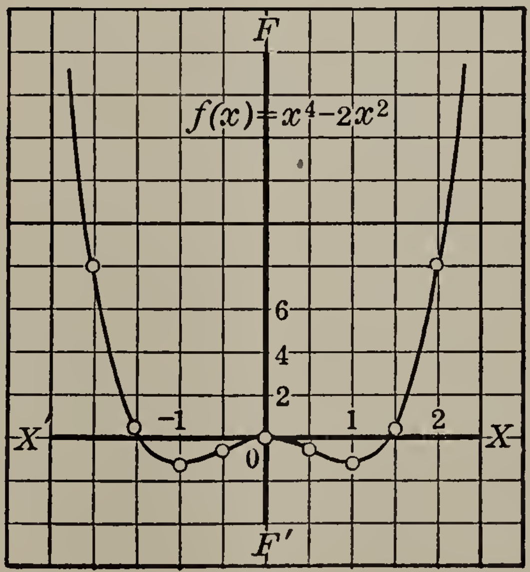
If $x =$	-3.5	-3	-2.5	-2	-1.5	-1	0	1	1.5	2	2.5	3	3.5
then $f(x) =$	36.5	0	-14.4	-15	-8.4	0	9	0	-8.4	-15	-14.4	0	36.5



From the graph $x = 1, -1, 3, -3$.

10. $f(x) = x^4 - 2x^2.$

If $x =$	-2.5	-2	-1.5	-1	$-.5$	0	.5	1	1.5	2	2.5
then $f(x) =$	26.5	8	.5	-1	$-.4$	0	$-.4$	-1	.5	8	26.5



From the graph $x = 0, 0, 1.4, -1.4$.

Page 369

1. $x^2 + 4x + 3 = 0.$
$x^2 + 4x = -3.$
$x^2 + 4x + 4 = 1.$
$(x + 2)^2 = 1.$
$x + 2 = \pm 1.$
$x = -1, -3.$

2. $s^2 - s - 2 = 0.$
$s^2 - s = 2.$
$s^2 - s + \frac{1}{4} = 2\frac{1}{4}.$
$(s - \frac{1}{2})^2 = \frac{9}{4}.$
$s - \frac{1}{2} = \pm \frac{3}{2}.$
$s = 2, -1.$

3. $x^2 + 2 = -3x.$
$x^2 + 3x = -2.$
$x^2 + 3x + \frac{9}{4} = \frac{1}{4}.$
$(x + \frac{3}{2})^2 = \frac{1}{4}.$
$x + \frac{3}{2} = \pm \frac{1}{2}.$
$x = -1, -2.$

4. $2x^2 + 5x + 3 = 0.$
$2x^2 + 5x = -3.$
$x^2 + \frac{5}{2}x = -\frac{3}{2}.$
$x^2 + \frac{5}{2}x + \frac{25}{16} = \frac{1}{16}.$
$(x + \frac{5}{4})^2 = \frac{1}{16}.$
$x + \frac{5}{4} = \pm \frac{1}{4}.$
$x = -1, -\frac{3}{2}.$

5. $3x^2 + 7x - 6 = 0.$
$x^2 + \frac{7}{3}x = 2.$
$x^2 + \frac{7}{3}x + \frac{49}{36} = 2 + \frac{49}{36} = \frac{121}{36}.$
$(x + \frac{7}{6})^2 = \frac{121}{36}.$
$x + \frac{7}{6} = \pm \frac{11}{6}.$
$x = \frac{2}{3}, -3.$

6. $2x^2 - 3x - 5 = 0.$
$x^2 - \frac{3}{2}x = \frac{5}{2}.$
$(x - \frac{3}{4})^2 = \frac{5}{2} + \frac{9}{16} = \frac{49}{16}.$
$x - \frac{3}{4} = \pm \frac{7}{4}.$
$x = \frac{5}{2}, -1.$

7. $5x^2 - 7x - 6 = 0.$
$x^2 - \frac{7}{5}x = \frac{6}{5}.$
$(x - \frac{7}{10})^2 = \frac{6}{5} + \frac{49}{100} = \frac{169}{100}.$
$x - \frac{7}{10} = \pm \frac{13}{10}.$
$x = 2, -\frac{3}{5}.$

8. $x - 4 + x^2 = 6 - 2x^2 + 8x.$
$3x^2 - 7x = 10.$
$x^2 - \frac{7}{3}x = \frac{10}{3}.$
$(x - \frac{7}{6})^2 = \frac{10}{3} + \frac{49}{36} = \frac{169}{36}.$
$x - \frac{7}{6} = \pm \frac{13}{6}.$
$x = \frac{10}{3}, -1.$

10. $n^2 + 2 = 5n.$
$n^2 - 5n = -2.$
$(n - \frac{5}{2})^2 = -2 + \frac{25}{4} = \frac{17}{4}.$
$n - \frac{5}{2} = \pm \frac{1}{2}\sqrt{17}$
$= \pm 2.062.$
$n = 2.5 \pm 2.062$
$= 4.562, .438.$

11. $3x^2 - 12x + 9 = 2.$
$x^2 - 4x = -\frac{7}{3}.$
$(x - 2)^2 = -\frac{7}{3} + 4 = \frac{5}{3}.$
$x - 2 = \pm \sqrt{\frac{5}{3}}$
$= \pm \frac{1}{3}\sqrt{15}$
$= \pm 1.291.$
$x = 2 \pm 1.291$
$= 3.291, .709.$

12. $5x^2 + 8x + 2 = 0.$
$x^2 + \frac{8}{5}x = -\frac{2}{5}.$
$(x + \frac{4}{5})^2 = -\frac{2}{5} + \frac{16}{25} = \frac{6}{25}.$
$x + \frac{4}{5} = \pm \frac{1}{5}\sqrt{6}$
$= \pm .489.$
$x = -.8 \pm .489$
$= -.311, -1.289.$

13. $(y+1)(y+2) = 3y(y-4).$
$$y^2 + 3y + 2 = 3y^2 - 12y.$$
$$2y^2 - 15y = 2.$$
$$y^2 - \tfrac{15}{2}y = 1.$$
$$(y - \tfrac{15}{4})^2 = 1 + \tfrac{225}{16} = \tfrac{241}{16}.$$
$$y - \tfrac{15}{4} = \pm\tfrac{1}{4}\sqrt{241}$$
$$= \pm 3.881.$$
$$y = 3.750 \pm 3.881$$
$$= 7.631,\ -.131.$$

15. $x^4 - 5x^2 + 4 = 0.$
$$x^4 - 5x^2 = -4.$$
$$(x^2 - \tfrac{5}{2})^2 = -4 + \tfrac{25}{4} = \tfrac{9}{4}.$$
$$x^2 - \tfrac{5}{2} = \pm\tfrac{3}{2}.$$
$$x^2 = 4,\ 1.$$
$$x = \pm 2,\ \pm 1.$$

16. $x^4 - 13x^2 + 36 = 0.$
$$(x^2 - \tfrac{13}{2})^2 = -36 + \tfrac{169}{4}$$
$$= \tfrac{25}{4}.$$
$$x^2 - \tfrac{13}{2} = \pm\tfrac{5}{2}.$$
$$x^2 = 9,\ 4.$$
$$x = \pm 3,\ \pm 2.$$

17.
$$x^4 - 4x^2 + 3 = 0.$$
$$x^4 - 4x^2 = -3.$$
$$(x^2 - 2)^2 = -3 + 4 = 1.$$
$$x^2 - 2 = \pm 1.$$
$$x^2 = 3,\ 1.$$
$$x = \pm\sqrt{3},\ \pm 1.$$

18.
$$6x^4 - 11x^2 + 3 = 0.$$
$$x^4 - \tfrac{11}{6}x^2 = -\tfrac{1}{2}.$$
$$(x^2 - \tfrac{11}{12})^2 = -\tfrac{1}{2} + \tfrac{121}{144} = \tfrac{49}{144}.$$
$$x^2 - \tfrac{11}{12} = \pm\tfrac{7}{12}.$$
$$x^2 = \tfrac{3}{2},\ \tfrac{1}{3}.$$
$$x = \pm\sqrt{\tfrac{3}{2}},\ \pm\sqrt{\tfrac{1}{3}}$$
$$= \pm\tfrac{1}{2}\sqrt{6},\ \pm\tfrac{1}{3}\sqrt{3}.$$

19.
$$x^{-2} + 16x^{-1} - 17 = 0.$$
$$(x^{-1} + 17)(x^{-1} - 1) = 0.$$
$$x = -\tfrac{1}{17},\ 1.$$

20.
$$y^{-4} - 10y^{-2} + 9 = 0.$$
$$(y^{-2} - 9)(y^{-2} - 1) = 0.$$
$$(y^{-1} + 3)(y^{-1} - 3)(y^{-1} + 1)(y^{-1} - 1) = 0.$$
$$y = -\tfrac{1}{3},\ \tfrac{1}{3},\ -1,\ 1.$$

21.
$$x^{-1} - 13x^{-\frac{1}{2}} + 36 = 0.$$
$$(x^{-\frac{1}{2}} - 9)(x^{-\frac{1}{2}} - 4) = 0.$$
$$(x^{-\frac{1}{4}} + 3)(x^{-\frac{1}{4}} - 3)(x^{-\frac{1}{4}} + 2)(x^{-\frac{1}{4}} - 2) = 0.$$
$$x = \tfrac{1}{81},\ \tfrac{1}{16}.$$

CR

23. If $$y = x - 1,$$
$$(x-1)^2 + 4(x-1) = 5 \text{ becomes}$$
$$y^2 + 4y = 5.$$
$$(y+2)^2 = 5 + 4 = 9.$$
$$y + 2 = \pm 3.$$
$$y = x - 1 = 1, -5.$$
$$x = 2, -4.$$

24. If $$y = x^2 - 4x,$$
$$(x^2 - 4x)^2 - 5(x^2 - 4x) - 24 = 0 \text{ becomes}$$
$$y^2 - 5y = 24.$$
$$(y - \tfrac{5}{2})^2 = 24 + \tfrac{25}{4} = \tfrac{121}{4}.$$
$$y - \tfrac{5}{2} = \pm \tfrac{11}{2}.$$
$$y = 8, -3.$$

Then
$$\left.\begin{aligned} x^2 - 4x &= 8, \\ (x-2)^2 &= 8 + 4 = 12, \\ x - 2 &= \pm\sqrt{12} = \pm 2\sqrt{3}, \\ x &= 2 \pm 2\sqrt{3}, \end{aligned}\right\} \text{ and } \left\{\begin{aligned} x^2 - 4x &= -3, \\ (x-2)^2 &= -3 + 4 = 1, \\ x - 2 &= \pm 1, \\ x &= 3, 1. \end{aligned}\right.$$

25. If $$y = x - \frac{6}{x},$$
$$\left(x - \frac{6}{x}\right)^2 + 4\left(x - \frac{6}{x}\right) - 5 = 0 \text{ becomes}$$
$$y^2 + 4y = 5.$$
$$(y+2)^2 = 5 + 4 = 9.$$
$$y + 2 = \pm 3.$$
$$y = 1, -5.$$

Then
$$\left.\begin{aligned} x - \frac{6}{x} &= 1, \\ x^2 - 6 &= x, \\ x^2 - x &= 6, \\ (x - \tfrac{1}{2})^2 &= 6 + \tfrac{1}{4} = \tfrac{25}{4}, \\ x - \tfrac{1}{2} &= \pm \tfrac{5}{2}, \\ x &= 3, -2, \end{aligned}\right\} \text{ and } \left\{\begin{aligned} x - \frac{6}{x} &= -5, \\ x^2 - 6 &= -5x, \\ x^2 + 5x &= 6, \\ (x + \tfrac{5}{2})^2 &= 6 + \tfrac{25}{4} = \tfrac{49}{4}, \\ x + \tfrac{5}{2} &= \pm \tfrac{7}{2}, \\ x &= 1, -6. \end{aligned}\right.$$

27.
$$x^2 + 3ax + 2a^2 = 0.$$
$$\left(x + \frac{3a}{2}\right)^2 = -2a^2 + \frac{9a^2}{4} = \frac{a^2}{4}.$$
$$x + \frac{3a}{2} = \pm \frac{a}{2}.$$
$$x = -a, -2a.$$

28.
$$2y^2 + by = 6b^2.$$
$$y^2 + \frac{b}{2}y = 3b^2.$$
$$\left(y + \frac{b}{4}\right)^2 = 3b^2 + \frac{b^2}{16} = \frac{49b^2}{16}.$$
$$y + \frac{b}{4} = \pm\frac{7b}{4}.$$
$$y = \tfrac{3}{2}b, \ -2b.$$

29.
$$bx^2 + x = 1 + bx.$$
$$bx^2 + (1-b)x = 1.$$
$$x^2 + \left(\frac{1-b}{b}\right)x = \frac{1}{b}.$$
$$x^2 + \left(\frac{1-b}{b}\right)x + \left(\frac{1-b}{2b}\right)^2 = \frac{1}{b} + \frac{1-2b+b^2}{4b^2} = \frac{1+2b+b^2}{4b^2}.$$
$$\left(x + \frac{1-b}{2b}\right)^2 = \left(\frac{1+b}{2b}\right)^2.$$
$$x + \frac{1-b}{2b} = \pm\frac{1+b}{2b}.$$
$$x = \frac{b-1}{2b} \pm \frac{1+b}{2b}$$
$$= 1, \ -\frac{1}{b}.$$

30.
$$\frac{x-c}{c} - \frac{c}{x-c} = \frac{3}{2}.$$
$$2x^2 - 4cx + 2c^2 - 2c^2 = 3cx - 3c^2.$$
$$2x^2 - 7cx + 3c^2 = 0.$$
$$x = 3c, \ \frac{c}{2}.$$

31.
$$bx^2 - a(b+1)x + a^2 = 0.$$
$$x^2 - \frac{a}{b}(b+1)x = -\frac{a^2}{b}.$$
$$x^2 - \frac{a}{b}(b+1)x + \frac{a^2}{4b^2}(b+1)^2 = -\frac{a^2}{b} + \frac{a^2(b+1)^2}{4b^2}$$
$$\left[x - \frac{a}{2b}(b+1)\right]^2 = \frac{a^2(b-1)^2}{4b^2}.$$
$$x - \frac{a}{2b}(b+1) = \pm\frac{a}{2b}(b-1).$$
$$x = \frac{a}{2b}(b+1) \pm \frac{a}{2b}(b-1)$$
$$= a, \ \frac{a}{b}.$$

32.
$$\frac{b}{2a}x^2 = \frac{b}{2a} + 1 - x.$$
$$\frac{b}{2a}x^2 + x = \frac{b}{2a} + 1 \cdot$$
$$x^2 + \frac{2a}{b}x = \frac{b+2a}{2a} \cdot \frac{2a}{b} = \frac{b+2a}{b}.$$
$$\left(x + \frac{a}{b}\right)^2 = \frac{b+2a}{b} + \frac{a^2}{b^2} = \frac{b^2 + 2ab + a^2}{b^2}.$$
$$x + \frac{a}{b} = \pm\frac{a+b}{b}.$$
$$x = -\frac{a}{b} \pm \frac{a+b}{b}$$
$$= 1, -\frac{2a+b}{b}.$$

33.
$$cx^2 - (c^2 + d)x + cd = 0.$$
$$x^2 - \frac{c^2+d}{c}x = -d.$$
$$\left(x - \frac{c^2+d}{2c}\right)^2 = -d + \frac{c^4 + 2c^2d + d^2}{4c^2}$$
$$= \frac{c^4 - 2c^2d + d^2}{4c^2}.$$
$$x - \frac{c^2+d}{2c} = \pm\frac{c^2-d}{2c}.$$
$$x = \frac{c^2 + d \pm (c^2 - d)}{2c}$$
$$= c, \frac{d}{c}.$$

34.
$$abx^2 - x(a+b)\sqrt{ab} + ab = 0.$$
$$x^2 - \frac{a+b}{\sqrt{ab}}x = -1.$$
$$\left(x - \frac{a+b}{2\sqrt{ab}}\right)^2 = -1 + \frac{a^2 + 2ab + b^2}{4ab}$$
$$= \frac{a^2 - 2ab + b^2}{4ab}.$$
$$x - \frac{a+b}{2\sqrt{ab}} = \pm\frac{a-b}{2\sqrt{ab}}.$$
$$x = \frac{a + b \pm (a-b)}{2\sqrt{ab}}$$
$$= \frac{a}{\sqrt{ab}}, \frac{b}{\sqrt{ab}}$$
$$= \frac{1}{b}\sqrt{ab}, \frac{1}{a}\sqrt{ab}.$$

35. $$6a^2y^2 - 7ary + 2r^2 = 0.$$

$$y^2 - \frac{7r}{6a}y = -\frac{r^2}{3a^2}.$$

$$\left(y - \frac{7r}{12a}\right)^2 = -\frac{r^2}{3a^2} + \frac{49r^2}{144a^2} = \frac{r^2}{144a^2}.$$

$$y - \frac{7r}{12a} = \pm\frac{r}{12a}.$$

$$y = \frac{2r}{3a}, \frac{r}{2a}.$$

36. If $$y = ax + b,$$

$(ax + b)^2 + 3(ax + b) + 2 = 0$ becomes

$$y^2 + 3y + 2 = 0.$$

$$(y + \tfrac{3}{2})^2 = -2 + \tfrac{9}{4} = \tfrac{1}{4}.$$

$$y + \tfrac{3}{2} = \pm\tfrac{1}{2}.$$

$$y = ax + b = -2, -1.$$

Then $$x = -\frac{2+b}{a}, -\frac{1+b}{a}.$$

Page 372

2. $$x^2 - 6x - 16 = 0.$$

$$x = \frac{6 \pm \sqrt{36 + 64}}{2} = \frac{6 \pm 10}{2} = 8, -2.$$

3. $$x^2 - x - 1 = 0.$$

$$x = \frac{1 \pm \sqrt{1 + 4}}{2} = \frac{1 \pm \sqrt{5}}{2}.$$

4. $$3x^2 + 7x - 3 = 0.$$

$$x = \frac{-7 \pm \sqrt{49 + 36}}{6} = \frac{-7 \pm \sqrt{85}}{6}.$$

5. $$2x + 4 = x^2.$$

$$x^2 - 2x - 4 = 0.$$

$$x = \frac{2 \pm \sqrt{4 + 16}}{2} = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}.$$

6. $$x = 1 - x^2.$$

$$x^2 + x - 1 = 0.$$

$$x = \frac{-1 \pm \sqrt{1 + 4}}{2} = \frac{-1 \pm \sqrt{5}}{2}.$$

7. $$3x + 4 = x^2.$$

$$x^2 - 3x - 4 = 0.$$

$$x = \frac{3 \pm \sqrt{9 + 16}}{2} = \frac{3 \pm 5}{2} = 4, -1.$$

8.
$$11x^2 - 15x = 26.$$
$$11x^2 - 15x - 26 = 0.$$
$$x = \frac{15 \pm \sqrt{225 + 1144}}{22}$$
$$= \frac{15 \pm \sqrt{1369}}{22}$$
$$= \frac{15 \pm 37}{22}$$
$$= \frac{26}{11}, -1.$$

9.
$$\tfrac{3}{2}x - \tfrac{1}{2} = x^2.$$
$$x^2 - \tfrac{3}{2}x + \tfrac{1}{2} = 0.$$

The equation could be solved in the above form, but clearing of fractions makes the solution easier.

$$2x^2 - 3x + 1 = 0.$$
$$x = \frac{3 \pm \sqrt{9 - 8}}{4} = \frac{3 \pm 1}{4} = 1, \frac{1}{2}.$$

11.
$$x^2 - 2kx - 3k^2 = 0.$$
$$x = \frac{2k \pm \sqrt{4k^2 + 12k^2}}{2}$$
$$= \frac{2k \pm 4k}{2}$$
$$= 3k, -k.$$

12.
$$3a^2 = 7ax + 6x^2.$$
$$6x^2 + 7ax - 3a^2 = 0.$$
$$x = \frac{-7a \pm \sqrt{49a^2 + 72a^2}}{12}$$
$$= \frac{-7a \pm 11a}{12}$$
$$= \frac{a}{3}, -\frac{3a}{2}.$$

13.
$$4x^2 + kx - 14k^2 = 0.$$
$$x = \frac{-k \pm \sqrt{k^2 + 224k^2}}{8}$$
$$= \frac{-k \pm 15k}{8}$$
$$= \frac{7k}{4}, -2k.$$

14.
$$bx = 12x^2 - b^2.$$
$$12x^2 - bx - b^2 = 0.$$
$$x = \frac{b \pm \sqrt{b^2 + 48b^2}}{24} = \frac{b \pm 7b}{24}$$
$$= \frac{b}{3}, -\frac{b}{4}.$$

15.
$$a^2x^2 + 4abx + 3b^2 = 0.$$
$$x = \frac{-4ab \pm \sqrt{16a^2b^2 - 12a^2b^2}}{2a^2}$$
$$= \frac{-4ab \pm 2ab}{2a^2}$$
$$= -\frac{3b}{a}, -\frac{b}{a}.$$

16.
$$12p^2x^2 - 4prx - r^2 = 0.$$
$$x = \frac{4pr \pm \sqrt{16p^2r^2 + 48p^2r^2}}{24p^2}$$
$$= \frac{4pr \pm 8pr}{24p^2}$$
$$= \frac{r}{2p}, -\frac{r}{6p}.$$

17.
$$3a^2x^2 + 8abx + 4b^2 = 0.$$
$$x = \frac{-8ab \pm \sqrt{64a^2b^2 - 48a^2b^2}}{6a^2}$$
$$= \frac{-8ab \pm 4ab}{6a^2}$$
$$= -\frac{2b}{a}, -\frac{2b}{3a}.$$

18.
$$x - 3\sqrt{c} - \frac{4c}{x} = 0.$$
$$x^2 - 3\sqrt{c}\cdot x - 4c = 0.$$
$$x = \frac{3\sqrt{c} \pm \sqrt{9c + 16c}}{2}$$
$$= \frac{3\sqrt{c} \pm 5\sqrt{c}}{2}$$
$$= 4\sqrt{c}, -\sqrt{c}.$$

19.

$$6h^2 + \frac{13hn}{x} = \frac{8n^2}{x^2}.$$

$$6h^2x^2 + 13hnx - 8n^2 = 0.$$

$$x = \frac{-13hn \pm \sqrt{169h^2n^2 + 192h^2n^2}}{12h^2}$$

$$= \frac{-13hn \pm \sqrt{361h^2n^2}}{12h^2}$$

$$= \frac{-13hn \pm 19hn}{12h^2}$$

$$= \frac{n}{2h}, \frac{-8n}{3h}.$$

20.

$$13a^2b^2x^2 = 9b^4 + 4a^4x^4.$$

$$4a^4x^4 - 13a^2b^2x^2 + 9b^4 = 0.$$

This is a quadratic in x^2 (cf. text, p. 370, Ex. 14).

$$x^2 = \frac{13a^2b^2 \pm \sqrt{169a^4b^4 - 144a^4b^4}}{8a^4}$$

$$= \frac{13a^2b^2 \pm 5a^2b^2}{8a^4}.$$

$$x^2 = \frac{9b^2}{4a^2}, \frac{b^2}{a^2}.$$

$$x = \pm\frac{3b}{2a}, \pm\frac{b}{a}.$$

22. $2x^2 + (a + 2b)x - a^2 - ab = 0.$

$$x = \frac{-a - 2b \pm \sqrt{a^2 + 4ab + 4b^2 + 8a^2 + 8ab}}{4}$$

$$= \frac{-a - 2b \pm (3a + 2b)}{4}$$

$$= \frac{a}{2} \text{ and } -a - b.$$

23.

$$ax^2 - 2ax - 3a = bx - \frac{b}{3}x^2.$$

$$3ax^2 - 6ax - 9a = 3bx - bx^2.$$

$$(3a + b)x^2 - 3(2a + b)x - 9a = 0.$$

$$x = \frac{+3(2a + b) \pm \sqrt{36a^2 + 36ab + 9b^2 + 108a^2 + 36ab}}{6a + 2b}$$

$$= \frac{6a + 3b \pm (12a + 3b)}{6a + 2b}$$

$$= 3 \text{ or } \frac{-3a}{3a + b}.$$

24. $$p^2 + x^2 = 2px + 2x - 2p.$$
$$x^2 - (2p + 2)x + (p^2 + 2p) = 0.$$
$$x = \frac{2p + 2 \pm \sqrt{4p^2 + 8p + 4 - 4p^2 - 8p}}{2}$$
$$= \frac{2p + 2 \pm 2}{2}$$
$$= p + 2,\ p.$$

25. $$x^2 - cx = \tfrac{1}{2}(ax - ac).$$
$$2x^2 - 2cx = ax - ac.$$
$$2x^2 - (a + 2c)x + ac = 0.$$
$$x = \frac{a + 2c \pm \sqrt{a^2 + 4ac + 4c^2 - 8ac}}{4}$$
$$= \frac{a + 2c \pm (a - 2c)}{4}$$
$$= \frac{a}{2},\ c.$$

26. $$4a^2x^4 + 4c^2 = 16a^2x^2 + c^2x^2.$$
$$4a^2x^4 - (16a^2 + c^2)x^2 + 4c^2 = 0.$$
$$x^2 = \frac{16a^2 + c^2 \pm \sqrt{256a^4 + 32a^2c^2 + c^4 - 64a^2c^2}}{8a^2}$$
$$= \frac{16a^2 + c^2 \pm (16a^2 - c^2)}{8a^2}$$
$$= 4,\ \frac{c^2}{4a^2}.$$
$$x = \pm 2,\ \pm \frac{c}{2a}.$$

27. $$x^6 - a^3x^3 = 8b^3x^3 - 8a^3b^3.$$
$$x^6 - (a^3 + 8b^3)x^3 + 8a^3b^3 = 0.$$
$$x^3 = \frac{a^3 + 8b^3 \pm \sqrt{a^6 + 16a^3b^3 + 64b^6 - 32a^3b^3}}{2}$$
$$= \frac{a^3 + 8b^3 \pm (a^3 - 8b^3)}{2}$$
$$= a^3,\ 8b^3.$$
$$x = a,\ 2b.$$

28. $$a^2 + x^2 = 2ax + b^2.$$
$$x^2 - 2ax + (a^2 - b^2) = 0.$$
$$x = \frac{2a \pm \sqrt{4a^2 - 4a^2 + 4b^2}}{2} = \frac{2a \pm 2b}{2} = a \pm b.$$

29. $12\,pq = x^2 + 4\,qx - 3\,px.$

$$x^2 + (4\,q - 3\,p)\,x - 12\,pq = 0.$$

$$x = \frac{3\,p - 4\,q \pm \sqrt{16\,q^2 - 24\,qp + 9\,p^2 + 48\,qp}}{2}$$

$$= \frac{3\,p - 4\,q \pm (4\,q + 3\,p)}{2}$$

$$= 3\,p,\ -4\,q.$$

30. $x^4 - a^2x^2 + a^2b^4 = b^4x^2.$

$$x^4 - (a^2 + b^4)\,x^2 + a^2b^4 = 0.$$

$$x^2 = \frac{a^2 + b^4 \pm \sqrt{a^4 + 2\,a^2b^4 + b^8 - 4\,a^2b^4}}{2}$$

$$= \frac{a^2 + b^4 \pm (a^2 - b^4)}{2}$$

$$= a^2,\ b^4.$$

$$x = \pm\, a,\ \pm\, b^2.$$

Page 375

1. $3\,x^2 - 7\,x - 10 = 0.$

$$x = \frac{7 \pm \sqrt{49 + 120}}{6} = \frac{7 \pm 13}{6}$$

$$= \tfrac{10}{3},\ -1.$$

2. $5\,x^2 + 14\,x + 8 = 0.$

$$x = \frac{-14 \pm \sqrt{196 - 160}}{10}$$

$$= \frac{-14 \pm 6}{10}$$

$$= -\tfrac{4}{5},\ -2.$$

(Note that even when an expression of the type in Exs. 1 and 2 can be factored, it is often easier to solve by formula than by factoring.)

3. $4\,x^2 + 3\,x - 7 = 0.$

$$x = \frac{-3 \pm \sqrt{9 + 112}}{8} = \frac{-3 \pm 11}{8}.$$

$$= 1,\ -\tfrac{7}{4}.$$

4. $x^2 - 1.7\,x - .84 = 0.$

$$(x - 2.1)\,(x + .4) = 0.$$

$$x = 2.1,\ -.4.$$

5.
$$8x^2 + 2\sqrt{5}x - 15 = 0.$$
$$x = \frac{-2\sqrt{5} \pm \sqrt{20 + 480}}{16}$$
$$= \frac{-2\sqrt{5} \pm 10\sqrt{5}}{16}$$
$$= \tfrac{1}{2}\sqrt{5},\ -\tfrac{3}{4}\sqrt{5}.$$

6.
$$x^4 - 7x^2 + 12 = 0.$$
$$(x^2 - 4)(x^2 - 3) = 0.$$
$$x^2 = 4,\ 3.$$
$$x = \pm 2,\ \pm\sqrt{3}.$$

7.
$$x^3 - x = 0.$$
$$x(x-1)(x+1) = 0.$$
$$x = 0,\ \pm 1.$$

8.
$$x^4 = x^2.$$
$$x^2(x^2 - 1) = 0.$$
$$x^2(x-1)(x+1) = 0.$$
$$x = 0,\ 0,\ \pm 1.$$

9.
$$x^4 - 64x^2 = 0.$$
$$x^2(x^2 - 64) = 0.$$
$$x = 0,\ 0,\ \pm 8.$$

10. If
$$y = x^2 - 3x,$$
$$(x^2 - 3x)^2 - 2(x^2 - 3x) = 8 \text{ becomes}$$
$$y^2 - 2y - 8 = 0.$$
$$(y - 4)(y + 2) = 0.$$

Then
$$\left.\begin{aligned} x^2 - 3x - 4 &= 0,\\ (x-4)(x+1) &= 0,\\ x &= 4,\ -1, \end{aligned}\right\} \text{ and } \left\{\begin{aligned} x^2 - 3x + 2 &= 0,\\ (x-2)(x-1) &= 0,\\ x &= 1,\ 2. \end{aligned}\right.$$

11.
$$5x^2 - 9x + 3 = 0.$$
$$x = \frac{9 \pm \sqrt{81 - 60}}{10} = \frac{9 \pm \sqrt{21}}{10}.$$

12.
$$.03x^2 + .01x = .1.$$
$$3x^2 + x - 10 = 0.$$
$$x = \frac{-1 \pm \sqrt{1 + 120}}{6}$$
$$= \frac{-1 \pm 11}{6}$$
$$= \tfrac{5}{3},\ -2.$$

13.
$$x^2 + x\sqrt{2} - \sqrt{6} = x\sqrt{3}.$$
$$x^2 + x(\sqrt{2} - \sqrt{3}) - \sqrt{6} = 0.$$
$$(x - \sqrt{3})(x + \sqrt{2}) = 0.$$
$$x = \sqrt{3},\ -\sqrt{2}.$$

14.
$$\frac{x-1}{x} = \frac{x+1}{6}.$$
$$6x - 6 = x^2 + x.$$
$$x^2 - 5x + 6 = 0.$$
$$(x-2)(x-3) = 0.$$
$$x = 2,\ 3.$$

15.
$$\frac{x-3}{x+4} = \frac{x-4}{2x+6}.$$
$$2(x^2 - 9) = x^2 - 16.$$
$$x^2 = 2.$$
$$x = \pm\sqrt{2}.$$

16.
$$\frac{x+1}{x+2} + \frac{x+3}{x+4} = \frac{22}{15}.$$
$$15(x+1)(x+4) + 15(x+2)(x+3) = 22(x+2)(x+4).$$
$$8x^2 + 18x - 26 = 0.$$
$$4x^2 + 9x - 13 = 0.$$
$$x = \frac{-9 \pm \sqrt{81 + 208}}{8}$$
$$= \frac{-9 \pm 17}{8}$$
$$= 1,\ -\tfrac{13}{4}.$$

17.
$$\frac{2x-1}{x-1} + \frac{1-x}{x-2} = \frac{x+1}{x+2}.$$
$$(2x-1)(x^2-4) - (x-1)^2(x+2) = (x^2-1)(x-2).$$
$$x^2 - 4x = 0.$$
$$x = 0,\ 4.$$

18.
$$pqx^2 - rqx + psx = rs.$$
$$pqx^2 - rqx + psx - rs = 0.$$
$$(qx + s)(px - r) = 0.$$
$$x = \frac{r}{p},\ -\frac{s}{q}.$$

19.
$$x^2 - 2ax + a^2 - b^2 = 0.$$
$$(x - a - b)(x - a + b) = 0.$$
$$x = a + b,\ a - b.$$

20.
$$\frac{m}{m-x} - \frac{m-x}{m} = 2.$$
$$m^2 - m^2 + 2\,mx - x^2 = 2\,m^2 - 2\,mx.$$
$$x^2 - 4\,mx + 2\,m^2 = 0.$$
$$x = \frac{4\,m \pm \sqrt{16\,m^2 - 8\,m^2}}{2}$$
$$= 2\,m \pm m\sqrt{2}.$$

21.
$$3\,x^2 + 2\,a = x + 6\,ax.$$
$$3\,x^2 - (1 + 6\,a)\,x + 2\,a = 0.$$
$$x = \frac{1 + 6\,a \pm \sqrt{(1 + 6\,a)^2 - 24\,a}}{6}$$
$$= \frac{1 + 6\,a \pm (1 - 6\,a)}{6}$$
$$x = \tfrac{1}{3},\ 2\,a.$$

22.
$$5\,x^2 + 3\,m = 5\,mx + 3\,x.$$
$$5\,x^2 - (5\,m + 3)\,x + 3\,m = 0.$$
$$x = \frac{5\,m + 3 \pm \sqrt{(5\,m + 3)^2 - 60\,m}}{10}$$
$$= \frac{5\,m + 3 \pm (5\,m - 3)}{10}$$
$$x = m,\ \tfrac{3}{5}.$$

23.
$$ax^2 + hm = mx - ahx.$$
$$ax^2 - (m - ah)\,x + hm = 0.$$
$$x = \frac{m - ah \pm \sqrt{(m - ah)^2 + 4\,ahm}}{2\,a}$$
$$= \frac{m - ah \pm (m + ah)}{2\,a}$$
$$= \frac{m}{a},\ -h.$$

24.
$$cx^2 + kx = 2\,mk + 2\,cmx.$$
$$cx^2 + (k - 2\,cm)\,x - 2\,mk = 0.$$
$$x = \frac{-k + 2\,cm \pm \sqrt{(k - 2\,cm)^2 + 8\,cmk}}{2\,c}$$
$$= \frac{-k + 2\,cm \pm (k + 2\,cm)}{2\,c}.$$
$$x = 2\,m,\ -\frac{k}{c}.$$

25.

$$x^2 + 2s + s^2 = 2sx + 2x.$$
$$x^2 - (2s + 2)x + 2s + s^2 = 0.$$
$$x = \frac{2s + 2 \pm \sqrt{4(s+1)^2 - 8s - 4s^2}}{2}$$
$$= \frac{2s + 2 \pm 2}{2}.$$
$$x = s,\ s + 2.$$

26.

$$c^2x^2 + a^2 = m^2 + 2acx.$$
$$c^2x^2 - 2acx + a^2 - m^2 = 0.$$
$$x = \frac{2ac \pm \sqrt{4a^2c^2 - 4c^2(a^2 - m^2)}}{2c^2}$$
$$= \frac{2ac \pm 2cm}{2c^2}.$$
$$x = \frac{a - m}{c},\ \frac{a + m}{c}.$$

27.

$$cmx^2 + m^2x + a^2 = am + acx + amx.$$
$$cmx^2 + (m^2 - ac - am)x + a^2 - am = 0.$$
$$x = \frac{-(m^2 - ac - am) \pm \sqrt{(m^2 - ac - am)^2 - 4a^2cm + 4acm^2}}{2cm}$$
$$= \frac{-(m^2 - ac - am) \pm (m^2 + ac - am)}{2cm}$$
$$= \frac{a}{m},\ \frac{a - m}{c}.$$

28.

$$ax^2 - 4a = bx^2 + 2bx.$$
$$(a - b)x^2 - 2bx - 4a = 0.$$
$$x = \frac{2b \pm \sqrt{4b^2 + 16a(a - b)}}{2(a - b)}$$
$$= \frac{2b \pm (4a - 2b)}{2(a - b)}$$
$$= \frac{2a}{a - b},\ -2.$$

29.

$$2mx^2 + 3ax = 18m + ax^2.$$
$$(2m - a)x^2 + 3ax - 18m = 0.$$
$$x = \frac{-3a \pm \sqrt{9a^2 + 72m(2m - a)}}{2(2m - a)}$$
$$= \frac{-3a \pm (3a - 12m)}{2(2m - a)}$$
$$x = -\frac{6m}{2m - a},\ 3.$$

30.
$$(s-x)^2+(t-x)^2=(t-s)^2.$$
$$2x^2-2(s+t)x+2ts=0.$$
$$(x-s)(x-t)=0.$$
$$x=s,\ t.$$

31.
$$3cx^2+50a^2=15acx+2x^2.$$
$$(3c-2)x^2-15acx+50a^2=0.$$
$$x=\frac{15ac\pm\sqrt{215a^2c^2-200a^2(3c-2)}}{2(3c-2)}$$
$$=\frac{15ac\pm(15ac-20a)}{2(3c-2)}.$$
$$x=5a,\ \frac{10a}{3c-2}.$$

32.
$$2x^2+5x=cx^2+3cx+3.$$
$$(2-c)x^2+(5-3c)x-3=0.$$
$$x=\frac{-(5-3c)\pm\sqrt{(5-3c)^2+12(2-c)}}{2(2-c)}$$
$$=\frac{-5+3c\pm(7-3c)}{2(2-c)}.$$
$$x=\frac{1}{2-c},\ -3.$$

33.
$$x^2+2x+1=mx^2+mx.$$
$$(x+1)^2=mx(x+1).$$
$$(x+1)^2-mx(x+1)=0.$$
$$(x+1)(x+1-mx)=0.$$
$$x=-1,\ \frac{1}{m-1}.$$

34.
$$ax^2+3x=2x^2+2ax-2.$$
$$(a-2)x^2+(3-2a)x+2=0.$$
$$x=\frac{-(3-2a)\pm\sqrt{(3-2a)^2-8(a-2)}}{2(a-2)}$$
$$=\frac{-3+2a\pm(2a-5)}{2(a-2)}.$$
$$x=2,\ \frac{1}{a-2}.$$

35. $$2x^2 + 5x = hx^2 + 3hx + 3.$$
$$(2-h)x^2 + (5-3h)x - 3 = 0.$$
$$x = \frac{-(5-3h) \pm \sqrt{(5-3h)^2 + 12(2-h)}}{2(2-h)}$$
$$= \frac{-5+3h \pm (7-3h)}{2(2-h)}.$$
$$x = \frac{1}{2-h}, \quad -3.$$

36. $$2x^2 + (a+2b)x = a^2 + ab.$$
$$2x^2 + (a+2b)x - a^2 - ab = 0.$$
$$x = \frac{-(a+2b) \pm \sqrt{(a+2b)^2 + 8(a^2+ab)}}{4}$$
$$= \frac{-a-2b \pm (3a+2b)}{4}.$$
$$x = \tfrac{1}{2}a, \quad -a-b.$$

37. $$(3a+b)x^2 - (2a+b)x = a.$$
$$(3a+b)x^2 - (2a+b)x - a = 0.$$
$$x = \frac{2a+b \pm \sqrt{(2a+b)^2 + 4a(3a+b)}}{2(3a+b)}$$
$$= \frac{2a+b \pm (4a+b)}{2(3a+b)}.$$
$$x = 1, \quad -\frac{a}{3a+b}.$$

38. $$\frac{t^2}{x^2} = \frac{t+1}{x+1}.$$
$$t^2x + t^2 = tx^2 + x^2.$$
$$(t+1)x^2 - t^2x - t^2 = 0.$$
$$x = \frac{t^2 \pm \sqrt{t^4 + 4t^2(t+1)}}{2(t+1)}$$
$$= \frac{t^2 \pm t(t+2)}{2(t+1)}.$$
$$x = t, \quad -\frac{t}{t+1}.$$

39. $$\frac{x-4a+c}{c-a}=\frac{9c+x}{x}.$$

$$x^2-4ax+cx=9c^2+cx-9ac-ax.$$

$$x^2-3ax-9c^2+9ac=0.$$

$$x=\frac{3a\pm\sqrt{9a^2+36c^2-36ac}}{2}$$

$$=\frac{3a\pm(3a-6c)}{2}.$$

$$x=3a-3c,\ 3c.$$

40. $$\frac{1}{mx+4}+\frac{mx-4}{16}=1.$$

$$16+m^2x^2-16=16mx+64.$$

$$m^2x^2-16mx=64.$$

$$m^2x^2-16mx+64=128.$$

$$mx-8=\pm 8\sqrt{2}.$$

$$x=\frac{8}{m}(1\pm\sqrt{2}).$$

41. $$\frac{2a+x}{2a-x}+\frac{a-2x}{a+2x}=\frac{8}{3}.$$

$$6a^2+15ax+6x^2+6a^2-15ax+6x^2=16a^2+24ax-16x^2.$$

$$28x^2-24ax-4a^2=0.$$

$$7x^2-6ax-a^2=0.$$

$$(7x+a)(x-a)=0.$$

$$x=-\frac{a}{7},\ a.$$

42. $$\frac{x^2+1}{x}=\frac{a+m}{t}+\frac{t}{a+m}.$$

Let $$\frac{a+m}{t}=b.$$

Then $$\frac{x^2+1}{x}=b+\frac{1}{b}.$$

$$bx^2-(b^2+1)x+b=0.$$

$$x=\frac{b^2+1\pm\sqrt{(b^2+1)^2-4b^2}}{2b}$$

$$=\frac{b^2+1\pm(b^2-1)}{2b}.$$

$$x=b,\ \frac{1}{b}.$$

Replacing $\frac{a+m}{t}$ for b, $$x=\frac{a+m}{t},\ \frac{t}{a+m}.$$

43. $$\frac{2x-r}{s}-\frac{4r}{2x-s}+3=0.$$

$$4x^2-2sx-2rx+rs-4rs+3s(2x-s)=0.$$

$$4x^2+(4s-2r)x-3rs-3s^2=0.$$

$$x=\frac{-(4s-2r)\pm\sqrt{(4s-2r)^2+16(3rs+3s^2)}}{8}$$

$$=\frac{-4s+2r\pm(8s+2r)}{8}$$

$$x=\frac{r+s}{2},\ -\frac{3s}{2}.$$

44. $$\frac{1}{m+c+x}=\frac{1}{m}+\frac{1}{c}+\frac{1}{x}.$$

$$mcx=cx(m+c+x)+mx(m+c+x)+mc(m+c+x).$$

$$(c+m)x^2+(c^2+2cm+m^2)x+mc(c+m)=0.$$

$$x^2+(c+m)x+mc=0.$$

$$(x+m)(x+c)=0.$$

$$x=-m,\ -c.$$

45. $$\frac{x+a}{x-t}+\frac{x-a}{x+t}=1+\frac{a^2+2t^2}{x^2-t^2}.$$

$$x^2+ax+tx+at+x^2-ax-tx+at=x^2-t^2+a^2+2t^2.$$

$$x^2+2at-a^2-t^2=0.$$

$$x-(a-t)^2=0.$$

$$x=a-t,\ t-a.$$

46. $$\frac{x^2+2ax}{a-b}-\frac{x}{2b}=\frac{a}{b}.$$

$$2bx^2+4abx-(a-b)x=2a(a-b).$$

$$2bx^2+(4ab-(a-b))x-2a(a-b)=0.$$

$$(x+2a)(2bx-(a-b))=0.$$

$$x=-2a,\ \frac{a-b}{2b}.$$

47. If $$y=x^2+5x+2,$$

$(x^2+5x+2)^2-6(x^2+5x+2)=16$ becomes

$$y^2-6y-16=0.$$

$$(y-8)(y+2)=0.$$

Then
$$\left.\begin{aligned}x^2+5x+2-8&=0,\\x^2+5x-6&=0,\\(x+6)(x-1)&=0,\\x&=-6,\ 1,\end{aligned}\right\}\text{ and }\left\{\begin{aligned}x^2+5x+2+2&=0,\\x^2+5x+4&=0,\\(x+4)(x+1)&=0,\\x&=-4,\ -1.\end{aligned}\right.$$

48.
$$\frac{3x-1}{5x+1}-\frac{x+1}{x-1}=\frac{48}{(5x+1)(1-x)}.$$
$$(3x-1)(x-1)-(x+1)(5x+1)+48=0.$$
$$x^2+5x-24=0.$$
$$(x-3)(x+8)=0.$$
$$x=3,\ -8.$$

49.
$$x-3=\frac{x^3-10x^2+1}{(x-3)^2}.$$
$$(x-3)^3=x^3-10x^2+1.$$
$$x^2+27x-28=0.$$
$$(x-1)(x+28)=0.$$
$$x=1,\ -28.$$

50.
$$15x^2-1.95x+.054=0.$$
$$x^2-.13x+.0036=0.$$
$$(x-.04)(x-.09)=0.$$
$$x=.04,\ .09.$$

51.
$$10-9x=7x^2.$$
$$7x^2+9x-10=0.$$
$$x=\frac{-9\pm\sqrt{81+280}}{14}$$
$$=\frac{-9\pm 19}{14}$$
$$=\tfrac{5}{7},\ -2.$$

52.
$$x^2+961a^2=62ax.$$
$$x^2-62ax+961a^2=0.$$
$$(x-31a)^2=0.$$
$$x=31a,\ 31a.$$

53.
$$(2x+3)(x-4)=(3x-8)(4x-1).$$
$$2x^2-5x-12=12x^2-35x+8.$$
$$10x^2-30x+20=0.$$
$$x^2-3x+2=0.$$
$$(x-2)(x-1)=0.$$
$$x=2,\ 1.$$

54.
$$(3x-2)(x-5)=(4x-3)(x+1).$$
$$3x^2-17x+10=4x^2+x-3.$$
$$x^2+18x-13=0.$$
$$x=\frac{-18\pm\sqrt{324+52}}{2}$$
$$=\frac{-18\pm\sqrt{376}}{2}$$
$$=-9\pm\sqrt{94}.$$

55.

$$\frac{1}{x+7}=\frac{2}{5}-\frac{1}{3-x}.$$

$$5(3-x)=2(x+7)(3-x)-5(x+7).$$

$$15-5x=42-8x-2x^2-5x-35.$$

$$2x^2+8x+8=0.$$

$$x^2+4x+4=0.$$

$$(x+2)^2=0.$$

$$x=-2,\ -2.$$

56.

$$\frac{x}{x^2+3x-4}=\frac{x+2}{x+4}.$$

$$\frac{x}{(x+4)(x-1)}=\frac{x+2}{x+4}.$$

$$x=x^2+x-2.$$

$$x^2-2=0.$$

$$x=\pm\sqrt{2}.$$

57.

$$\frac{x+3}{x^2-3x+2}=\frac{x-1}{x-2}-\frac{x-2}{x-1}.$$

$$\frac{x+3}{(x-1)(x-2)}=\frac{(x-1)^2-(x-2)^2}{(x-1)(x-2)}.$$

$$x+3=2x-3.$$

$$x=6.$$

58.

$$\frac{5x-11}{x-1}+\frac{1-3x}{x+2}=\frac{2x-1}{x+1}.$$

$$(5x-11)(x^2+3x+2)+(1-3x)(x^2-1)=(2x-1)(x^2+x-2).$$

Expanding and collecting,

$$4x^2-15x-25=0.$$

$$x=\frac{15\pm\sqrt{225+400}}{8}$$

$$=\frac{15\pm 25}{8}$$

$$=5,\ -\tfrac{5}{4}.$$

59.

$$17x=6x^2-10.$$

$$6x^2-17x-10=0.$$

$$x=\frac{17\pm\sqrt{289+240}}{12}$$

$$=\frac{17\pm 23}{12}$$

$$=\tfrac{10}{3},\ -\tfrac{1}{2}.$$

60.

$$2k^2x^2 + 8kx = 5(3kx - 1).$$
$$2k^2x^2 - 7kx + 5 = 0.$$
$$(2kx - 5)(kx - 1) = 0.$$
$$x = \frac{5}{2k}, \frac{1}{k}.$$

61.

$$37pqx = 210q^2 - p^2x^2.$$
$$p^2x^2 + 37pqx - 210q^2 = 0.$$
$$(px + 42q)(px - 5q) = 0.$$
$$x = \frac{5q}{p}, -\frac{42q}{p}.$$

62.

$$\frac{3x-2}{x-1} + \frac{2}{3} = 9\tfrac{1}{6} - \frac{5(x+1)}{4x-1}.$$
$$\frac{3x-2}{x-1} = \frac{51}{6} - \frac{5(x+1)}{4x-1}.$$
$$6(12x^2 - 11x + 2) = 51(4x^2 - 5x + 1) - 30(x^2 - 1).$$
$$102x^2 - 189x + 69 = 0.$$
$$34x^2 - 63x + 23 = 0.$$
$$x = \frac{63 \pm \sqrt{3969 - 3128}}{68} = \frac{63 \pm \sqrt{841}}{68} = \frac{63 \pm 29}{68}$$
$$= \tfrac{23}{17}, \tfrac{1}{2}.$$

63.

$$3 + \frac{4k}{c - 2x} = \frac{k - 2x}{c}.$$
$$3c^2 - 6cx + 4ck = ck - 2cx - 2kx + 4x^2.$$
$$4x^2 + (4c - 2k)x - (3c^2 + 3ck) = 0.$$
$$x = \frac{2k - 4c \pm \sqrt{16c^2 - 16ck + 4k^2 + 48c^2 + 48ck}}{8}$$
$$= \frac{2k - 4c \pm \sqrt{64c^2 + 32ck + 4k^2}}{8}$$
$$= \frac{2k - 4c \pm (8c + 2k)}{8}$$
$$= \frac{k + c}{2}, -\frac{3c}{2}.$$

64.

$$3x^2 + .7x = .2.$$
$$30x^2 + 7x - 2 = 0.$$
$$x = \frac{-7 \pm \sqrt{49 + 240}}{60}$$
$$= \frac{-7 \pm 17}{60}$$
$$= \tfrac{1}{6}, -\tfrac{2}{5}.$$

65. $$\frac{1}{x^2-3x+2}+\frac{1}{x^2+x-6}=\frac{1}{3-2x-x^2}.$$

$$\frac{1}{(x-2)(x-1)}+\frac{1}{(x-2)(x+3)}+\frac{1}{(x+3)(x-1)}=0.$$

$$(x+3)+(x-1)+(x-2)=0.$$

$$3x=0.$$

$$x=0.$$

66. If $$y=x^2-2,$$

$$\frac{2(x^2-2)-1}{x^2-1}=\frac{x^2+2}{(x^2-2)-4} \text{ becomes}$$

$$\frac{2y-1}{y+1}=\frac{y+4}{y-4}.$$

$$2y^2-9y+4=y^2+5y+4.$$

$$y^2-14y=0.$$

Then $$\left.\begin{array}{r} x^2-2=0,\\ x^2=2,\\ x=\pm\sqrt{2}, \end{array}\right\} \text{ and } \left\{\begin{array}{r} x^2-2=14,\\ x^2=16,\\ x=\pm 4. \end{array}\right.$$

67. $$(x-2)^2-(1-2x)(3x+5)=5-(1-2x)(3x+2).$$

Combining the terms in $1-2x$,

$$(x-2)^2-3(1-2x)=5.$$

$$x^2+2x-4=0.$$

$$x=\frac{-2\pm\sqrt{4+16}}{2}$$

$$=-1\pm\sqrt{5}.$$

68. If $$y=x^2-2x-2,$$

$$(x^2-2x-2)^2+2(x^2-2x-2)-3=0 \text{ becomes}$$

$$y^2+2y-3=0.$$

$$(y+3)(y-1)=0.$$

Then $$\left.\begin{array}{r} x^2-2x-2+3=0,\\ x^2-2x+1=0,\\ (x-1)^2=0,\\ x=1,1, \end{array}\right\} \text{ and } \left\{\begin{array}{r} x^2-2x-2-1=0,\\ x^2-2x-3=0,\\ (x-3)(x+1)=0,\\ x=3,-1 \end{array}\right.$$

69. If $$y=x^2-4x+1,$$

$$(x^2-4x+1)^2-4(x^2-4x)-16=0 \text{ becomes}$$

$$y^2-4y-12=0.$$

$$(y-6)(y+2)=0.$$

Then $$\left.\begin{array}{r} x^2-4x+1-6=0,\\ (x-5)(x+1)=0,\\ x=5,-1, \end{array}\right\} \text{ and } \left\{\begin{array}{r} x^2-4x+1+2=0,\\ (x-3)(x-1)=0,\\ x=3,1 \end{array}\right.$$

70. If $y = 2x - b,$

$$(2x - b)^2 = a(2x - b) + 2a^2 \text{ becomes}$$

$$y^2 - ay - 2a^2 = 0.$$

$$(y - 2a)(y + a) = 0.$$

Then $\left.\begin{array}{l} 2x - b - 2a = 0, \\ x = \dfrac{b + 2a}{2}, \end{array}\right\}$ and $\left\{\begin{array}{l} 2x - b + a = 0, \\ x = \dfrac{b - a}{2}. \end{array}\right.$

Page 377

1. Let $x =$ the second part.
Then $42 - x =$ the first part,
and $42 - x = x^2.$
Whence $x = 6$ or $-7,$
and $42 - x = 36$ or $49.$

2. Let $x =$ the first number.
Then $x + 2 =$ the second,
and $x(x + 2) = 143.$
Whence $x = 11$ or $-13,$
and $x + 2 = 13$ or $-11.$

3. Let x and $x + 1 =$ the numbers, respectively.

Then $$\frac{1}{x} + \frac{1}{x + 1} = \frac{13}{42}.$$

$$42(x + 1) + 42x = 13x(x + 1).$$

Whence $x = 6$ or $-\frac{7}{13},$

and $x + 1 = 7$ or $\frac{6}{13}.$

But the term *consecutive numbers* implies integers; therefore we must reject the fractional solutions.

4. Let $x =$ the rate in miles per hour.
Then $x + 6 =$ the time in hours,
and $x(x + 6) = 216,$ or the distance in miles.
Whence $x = 12,$ (Root -18 rejected.)
and $x + 6 = 18.$

5. Let $x =$ the base in feet.
Then $x - 6 =$ the altitude in feet,

and $$\frac{x(x - 6)}{2} = 56.$$

Whence $x = 14,$ (Root -8 rejected.)
and $x - 6 = 8.$

6. Let x and $x - 9 =$ the legs, respectively, in feet,
and $\sqrt{x^2 + (x - 9)^2} =$ the hypotenuse.

Then $$\frac{x(x - 9)}{2} = 45.$$

Whence $x = 15,$ (Root -6 rejected.)
and $x - 9 = 6.$

$$\sqrt{x^2 + (x - 9)^2} = \sqrt{15^2 + 6^2} = \sqrt{261} = 3\sqrt{29}.$$

7. Let $x =$ the altitude.
Then $x + 4$ and $3x =$ the bases, respectively,
and $\left(\frac{x + 4 + 3x}{2}\right)x = 180$, or the area.
Whence $x = 9$, (Root -10 rejected.)
and $x + 4 = 13$,
and $3x = 27$.

8. Let $x =$ the shorter side in inches.
Then $\frac{21}{2} - x =$ the longer side in inches,
and $x(\frac{21}{2} - x) = 27$, or the area.
Whence $x = 4\frac{1}{2}, 6$,
or $\frac{21}{2} - x = 6, 4\frac{1}{2}$, which duplicates the first answer and may therefore be ignored.

9. Let $n =$ the number of sides in a polygon with 119 diagonals.
Then $\frac{1}{2}n(n-3) = 119$.
Whence $n = 17$. (Root -14 rejected.)

10. Let $BC = x$.
Then $6^2 = x(x + 10)$.
Whence $x = -5 + \sqrt{61}$. (Root $-5 - \sqrt{61}$ rejected.)
$= 2.8$.

11. Let $x =$ the number of days for B.
Then $x + 4 =$ the number of days for A,
and $\frac{1}{x} + \frac{1}{x+4} = 1 \div 2\frac{2}{3} = \frac{3}{8}$.
Whence $x = 4$. (Root $-\frac{8}{3}$ rejected.)
$x + 4 = 8$.

12. Let $x =$ the original cost in dollars.
Then $20 - x =$ the profit in dollars,
and $\frac{20-x}{x} =$ per cent of profit *expressed as a decimal.*
Therefore $\frac{20-x}{x} = \frac{x+9}{100}$.
Whence $x = 16$, (Root -125 rejected.)
and $\frac{20-x}{x} = \frac{1}{4} = 25\%$.

13. $(x-6)(x-6)(3) = 12$, or the volume of the box.
Whence $x = 4$ and 8.
But if $x = 4$, $x - 6 = -2$, which is absurd.
Therefore $x = 8$ inches is the only solution.

14. Let $x =$ the number of letters.

Then $x - 20 =$ the number of lines,

and $(x - 16)(x - 20 + 15) = x(x - 20).$

Whence $x = 80,$

and $x - 20 = 60.$

15. $(x - 5)(x + 5) - 1 = x + 5 - (x - 5).$

$x^2 - 26 = 10.$

$x = 6.$ (Root -6 rejected.)

Or $(x - 5)(x + 5) - 1 = x - 5 \div (x + 5) = -10.$

Solving, $x = 4.$ (Root -4 rejected.)

16. $(4x - 5)(x - 2) = x + 4.$

$4x^2 - 13x + 10 = x + 4.$

Whence $x = 3, \frac{1}{2}.$

17. Let $x =$ rate per cent of interest.

Then $4000 \cdot \frac{x}{100}$, or $40x =$ interest in dollars for first year,

and $4000 + 40x + 400$, or $4400 + 40x =$ principal for second year. (1)

Therefore

$\frac{x}{100}(4400 + 40x)$, or $\frac{x}{5}(220 + 2x) =$ interest for second year. (2)

Now $400 + (1) + (2) = 5230$, principal for third year.

Then

$$(4400 + 40x) + \frac{x}{5}(220 + 2x) + 400 = 5230.$$

Whence $x = 5.$ (Root -215 rejected.)

Therefore the rate is 5%.

18. Let $x =$ the number in the party.

Then $\frac{60}{x} =$ the share of each,

and $$\frac{60}{x} - 1 = \frac{60}{x + 2}.$$

Whence $x = 10.$ (Root -12 rejected.)

19. Let x, $x + 1$, and $x + 2 =$ the numbers, respectively.

Then $2x(x + 1) + 2x(x + 2) + 2(x + 1)(x + 2) = 214.$

$6x^2 + 12x - 210 = 0.$

Whence $x = 5,$ (Root -7 rejected.)

and $x + 1 = 6,\ x + 2 = 7.$

20. Let $x =$ the radius of the reduced circle in inches,

and $28 - x =$ the amount in inches the radius of the given circle must be shortened.

Then $\frac{22}{7}x^2 = \frac{22}{7} \cdot 28^2 - 1078 = 2464 - 1078 = 1386.$

Whence $x = 21,$ (Root -21 rejected.)

and $28 - x = 28 - 21 = 7.$

21. Let A be the point from which the body A starts, and let B be the point from which the body B starts.

Let $x =$ the number of seconds after which the distance $AB = 75$.

In x seconds A travels $10x$ feet away from the vertex, while B travels $5x$ feet toward it.

After x seconds A will be $5 + 10x$ feet from the vertex, and B will be $35 - 5x$ feet from the vertex.

Then $(5 + 10x)^2 + (35 - 5x)^2 = \overline{75}^2 = 5625.$

Whence $x^2 - 2x - 35 = 0,$

and $x = 7, -5.$

7 is positive and gives a point of time in the future at which A and B will be 75 feet apart.

-5 is negative and gives a point of time in the past at which A and B were 75 feet apart.

Therefore the times are 5 seconds ago and 7 seconds hence.

22. C represents the base and B the top of the mountain. A is the point 40 miles distant (from B, *not* from C). AB is the line of sight, tangent to the earth's surface. DB here passes through O, the earth's center, therefore $DC = 8000$ miles. Let $BC = x$, and we have

$$\overline{AB}^2 = BC \cdot BD.$$

$$1600 = x(x + 8000).$$

$$x^2 + 8000x - 1600 = 0.$$

$$x = \frac{-8000 \pm \sqrt{64,006,400}}{2}$$

$$= -4000 \pm \sqrt{16,001,600}$$

$$= -4000 \pm 4000.2 \text{ (approximately).}$$

Rejecting the negative root, $x = .2$ miles.

23. $s = \frac{1}{2} \cdot 32 \cdot 8^2 = 16 \cdot 64 = 1024$ feet.

24. $100t - 16t^2 = 136.$

Whence $t = 2, 4\frac{1}{4}.$

The stone goes up beyond a height of 136 feet and passes that height twice — once going up (after 2 seconds) and once going down (after $4\frac{1}{4}$ seconds).

25. $100t - 16t^2 = 0.$

$$t = 0, 6\tfrac{1}{4}.$$

The stone was at the starting point to begin with (that is, after 0 seconds) and returns there after $6\frac{1}{4}$ seconds.

Page 383

1. $$\sqrt{x+1}=5.$$
$$x+1=25.$$
$$x=24.$$

2. $$\sqrt{x+1}=\sqrt{3x-5}.$$
$$x+1=3x-5.$$
$$x=3.$$

3. $$3\sqrt{2x-8}-7=17.$$
$$3\sqrt{2x-8}=24.$$
$$\sqrt{2x-8}=8.$$
$$2x-8=64.$$
$$x=\tfrac{72}{2}=36.$$

4. $$3\sqrt{2x+6}=\sqrt{6x^2-6}.$$
$$9(2x+6)=6x^2-6.$$
$$6x^2-18x-60=0.$$
$$x^2-3x-10=0.$$
$$(x-5)(x+2)=0.$$
$$x=5,\ -2.$$

5. $$\sqrt[3]{2x+3}=3.$$
$$2x+3=27.$$
$$x=\tfrac{24}{2}=12.$$

6. $$7+2\sqrt[3]{2x-1}=13.$$
$$2\sqrt[3]{2x-1}=6.$$
$$\sqrt[3]{2x-1}=3.$$
$$2x-1=27.$$
$$x=\tfrac{28}{2}=14.$$

7. $$\sqrt[3]{4x+3}-\sqrt[3]{4-3x}=0.$$
$$\sqrt[3]{4x+3}=\sqrt[3]{4-3x}.$$
$$4x+3=4-3x.$$
$$7x=1.$$
$$x=\tfrac{1}{7}.$$

8. $$\sqrt[3]{2x+2}=\sqrt{2x-2}.$$
$$(\sqrt[3]{2x+2})^6=(\sqrt{2x-2})^6.$$
$$(2x+2)^2=(2x-2)^3.$$
$$4x^2+8x+4=8x^3-24x^2+24x-8.$$
$$2x^3-7x^2+4x-3=0.$$
$$(x-3)(2x^2-x+1)=0.$$
$$x=3,\ \frac{1\pm\sqrt{-7}}{4}.$$

9. $$\sqrt[4]{x+1}=\sqrt{x-1}.$$
$$x+1=(x-1)^2.$$
$$x^2-3x=x(x-3)=0.$$
$$x=0,\ 3. \qquad \text{(Root 0 rejected.)}$$

11. $$\sqrt{9x}-4=\sqrt{x}.$$
$$\sqrt{9x}=\sqrt{x}+4.$$
$$9x=x+16+8\sqrt{x}.$$
$$8x-16=8\sqrt{x}.$$
$$x-2=\sqrt{x}.$$
$$x^2-4x+4=x.$$
$$x^2-5x+4=0.$$
$$(x-1)(x-4)=0.$$
$$x=1,\ 4. \qquad \text{(Root 1 rejected.)}$$

12.
$$\sqrt{3x-2}+5=\sqrt{x+35}.$$
$$3x-2+25+10\sqrt{3x-2}=x+35.$$
$$2x-12=-10\sqrt{3x-2}.$$
$$x-6=-5\sqrt{3x-2}.$$
$$x^2-12x+36=25(3x-2).$$
$$x^2-87x+86=0.$$
$$(x-86)(x-1)=0.$$
$$x=86,\ 1. \qquad \text{(Root 86 rejected.)}$$

13.
$$\sqrt{3x+4}-\sqrt{x+5}=\sqrt{5-x}.$$
$$3x+4+x+5-2\sqrt{3x^2+19x+20}=5-x.$$
$$5x+4=2\sqrt{3x^2+19x+20}.$$
$$25x^2+40x+16=12x^2+76x+80.$$
$$13x^2-36x-64=0.$$
$$(13x+16)(x-4)=0.$$
$$x=-\tfrac{16}{13},\ 4. \qquad \text{(Root } -\tfrac{16}{13} \text{ rejected.)}$$

14.
$$\sqrt{x+2}=\sqrt{x}+\sqrt{2}.$$
$$x+2=x+2+2\sqrt{2x}.$$
$$2\sqrt{2x}=0.$$
$$8x=0.$$
$$x=0.$$

15.
$$x+1=\sqrt{2x^2+3x-1}.$$
$$x^2+2x+1=2x^2+3x-1.$$
$$x^2+x-2=0.$$
$$(x+2)(x-1)=0.$$
$$x=-2,\ 1. \qquad \text{(Root } -2 \text{ rejected.)}$$

16.
$$\sqrt{x+2}+\sqrt{x-1}-\sqrt{3x+3}=0.$$
$$\sqrt{x+2}+\sqrt{x-1}=\sqrt{3x+3}.$$
$$2x+1+2\sqrt{x^2+x-2}=3x+3.$$
$$2\sqrt{x^2+x-2}=x+2.$$
$$4x^2+4x-8=x^2+4x+4.$$
$$3x^2-12=3(x^2-4)=0.$$
$$x=\pm 2.$$

18.
$$x^3+5x^{\frac{3}{2}}-14=0.$$
$$x^3+5x^{\frac{3}{2}}+(\tfrac{5}{2})^2=14+\tfrac{25}{4}=\tfrac{81}{4}.$$
$$x^{\frac{3}{2}}+\tfrac{5}{2}=\pm\tfrac{9}{2}.$$
$$x^{\frac{3}{2}}=2,\ -7.$$
$$x=\sqrt[3]{4},\ \sqrt[3]{49}. \qquad \text{(Root } \sqrt[3]{49} \text{ rejected.)}$$

19.
$$6 = x^{\frac{1}{2}} + x.$$
$$x + x^{\frac{1}{2}} + \tfrac{1}{4} = \tfrac{25}{4}.$$
$$x^{\frac{1}{2}} + \tfrac{1}{2} = \pm \tfrac{5}{2}.$$
$$x^{\frac{1}{2}} = 2, -3.$$
$$x = 4, 9. \qquad \text{(Root 9 rejected.)}$$

20.
$$x^{-4} - 17x^{-2} + 52 = 0.$$
$$(x^{-2} - 13)(x^{-2} - 4) = 0.$$
$$x^{-2} = 4, 13.$$
$$x = \pm \tfrac{1}{2}, \pm \tfrac{1}{13}\sqrt{13}.$$

21.
$$2x^{\frac{2}{3}} - 7\sqrt[3]{x} + 6 = 0.$$
$$2x^{\frac{2}{3}} - 7x^{\frac{1}{3}} + 6 = 0.$$
$$\left(2x^{\frac{1}{3}} - 3\right)\left(x^{\frac{1}{3}} - 2\right) = 0.$$
$$x^{\frac{1}{3}} = \tfrac{3}{2}, 2.$$
$$x = \tfrac{27}{8}, 8.$$

22.
$$x^{-\frac{3}{2}} = 4x^{-\frac{3}{4}} + 32.$$
$$x^{-\frac{3}{2}} - 4x^{-\frac{3}{4}} - 32 = 0.$$
$$\left(x^{-\frac{3}{4}} - 8\right)\left(x^{-\frac{3}{4}} + 4\right) = 0.$$
$$x^{-\frac{3}{4}} = 8, -4.$$

Whence
$$x^{\frac{3}{4}} = \tfrac{1}{8}, -\tfrac{1}{4},$$

and
$$x = \tfrac{1}{16}, \sqrt[3]{\tfrac{1}{256}}. \quad \left(\text{Root } \sqrt[3]{\tfrac{1}{256}} \text{ rejected.}\right)$$

23.
$$x^{\frac{1}{3}} - x^{\frac{1}{6}} - 6 = 0.$$
$$\left(x^{\frac{1}{6}} - 3\right)\left(x^{\frac{1}{6}} + 2\right) = 0.$$
$$x^{\frac{1}{6}} = 3, -2.$$
$$x = 3^6, (-2)^6$$
$$= 729, 64. \qquad \text{(Root 64 rejected.)}$$

24.
$$x^{-1} - \frac{25}{36x^{\frac{1}{2}}} + \frac{1}{9} = 0.$$
$$36x^{-1} - 25x^{-\frac{1}{2}} + 4 = 0.$$
$$\left(4x^{-\frac{1}{2}} - 1\right)\left(9x^{-\frac{1}{2}} - 4\right) = 0.$$
$$x^{-\frac{1}{2}} = \tfrac{1}{4}, \tfrac{4}{9}.$$
$$x = 16, \tfrac{81}{16}.$$

25. If $$y = x^2 + 5x,$$

$$x^2 + 5x + 3\sqrt{x^2 + 5x} - 54 = 0 \text{ becomes}$$

$$y + 3y^{\frac{1}{2}} - 54 = 0.$$

$$\left(y^{\frac{1}{2}} + 9\right)\left(y^{\frac{1}{2}} - 6\right) = 0.$$

$$y^{\frac{1}{2}} = 6, -9.$$

$$y = 36, 81.$$

But $y = 81$ does not satisfy $y + 3y^{\frac{1}{2}} - 54 = 0.$

Therefore $$x^2 + 5x = 36 \text{ only}.$$

$$x^2 + 5x - 36 = (x + 9)(x - 4) = 0.$$

$$x = 4, -9.$$

26. If $$y = 3x^2 - 4x,$$

$$3x^2 - 4x - 11\sqrt{3x^2 - 4x} + 28 = 0 \text{ becomes}$$

$$y - 11y^{\frac{1}{2}} + 28 = 0.$$

$$\left(y^{\frac{1}{2}} - 7\right)\left(y^{\frac{1}{2}} - 4\right) = 0.$$

$$y = 49, 16.$$

Then $$3x^2 - 4x - 49 = 0.$$

$$x = \frac{4 \pm \sqrt{16 + 588}}{6} = \frac{2 \pm \sqrt{151}}{3}.$$

Or $$3x^2 - 4x - 16 = 0.$$

$$x = \frac{4 \pm \sqrt{16 + 192}}{6} = \frac{2 \pm 2\sqrt{13}}{3}.$$

27. If $$y = 2x^2 - 3x - 1,$$

$$2x^2 - 3x - 4 - \sqrt{2x^2 - 3x - 1} + 1 = 0, \text{ or}$$

$$2x^2 - 3x - 1 - \sqrt{2x^2 - 3x - 1} - 2 = 0, \text{ becomes}$$

$$y - y^{\frac{1}{2}} - 2 = 0.$$

$$\left(y^{\frac{1}{2}} - 2\right)\left(y^{\frac{1}{2}} + 1\right) = 0.$$

$$y = 4, 1.$$

But $y = 1$ does not satisfy $y - y^{\frac{1}{2}} - 2 = 0.$

Therefore $$2x^2 - 3x - 1 = 4 \text{ only}.$$

$$2x^2 - 3x - 5 = 0.$$

$$x = \frac{3 \pm \sqrt{9 + 40}}{4}$$

$$= \tfrac{5}{2}, -1.$$

28. If $$y = x^2 - 2x - 4,$$

$$x^2 - 2x - 5\sqrt{x^2 - 2x - 4} + 2 = 0, \text{ or}$$

$$x^2 - 2x - 4 - 5\sqrt{x^2 - 2x - 4} + 6 = 0, \text{ becomes}$$

$$y - 5y^{\frac{1}{2}} + 6 = 0.$$

$$\left(y^{\frac{1}{2}} - 2\right)\left(y^{\frac{1}{2}} - 3\right) = 0.$$

$$y = 4, 9.$$

Then $x^2 - 2x - 4 = 4,$
$x^2 - 2x - 8 = 0,$
$(x-4)(x+2) = 0,$
$x = 4, -2,$
and
$x^2 - 2x - 4 = 9,$
$x^2 - 2x - 13 = 0,$
$x = \dfrac{2 \pm \sqrt{56}}{2}$
$= 1 \pm \sqrt{14}.$

29.
$$12x^{\frac{4}{3}} - 27x^{\frac{2}{3}} = 20x^{\frac{2}{3}} - 45.$$
$$12x^{\frac{4}{3}} - 47x^{\frac{2}{3}} + 45 = 0.$$
$$x^{\frac{2}{3}} = \frac{47 \pm \sqrt{2209 - 2160}}{24}$$
$$= \frac{47 \pm 7}{24}$$
$$= \tfrac{9}{4}, \tfrac{5}{3}.$$
$$x = \pm \tfrac{27}{8}, \pm \tfrac{5}{9}\sqrt{15}.$$

30. If
$$y = x^2 + 9,$$
$$15 - 2\sqrt{x^2+9} = x^2 + 9, \text{ or}$$
$$x^2 + 9 + 2\sqrt{x^2+9} - 15 = 0, \text{ becomes}$$
$$y + 2y^{\frac{1}{2}} - 15 = 0.$$
$$\left(y^{\frac{1}{2}} + 5\right)\left(y^{\frac{1}{2}} - 3\right) = 0.$$
$$y^{\frac{1}{2}} = 3, -5.$$

But $y = 25$ does not satisfy $y + 2y^{\frac{1}{2}} - 15 = 0.$

Therefore $x^2 + 9 = 9$ only,

and $x = 0.$

31.
$$\frac{6\sqrt{x} - 8}{\sqrt{2x} - \sqrt{8}} = \sqrt{2}.$$
$$6\sqrt{x} - 8 = 2\sqrt{x} - 4.$$
$$4\sqrt{x} = 4.$$
$$\sqrt{x} = 1.$$
$$x = 1.$$

32.
$$\frac{2\sqrt{3}}{\sqrt{2x-3}} = \frac{\sqrt{2x+12}}{3\sqrt{3}}.$$
$$18 = \sqrt{4x^2 + 18x - 36}.$$
$$324 = 4x^2 + 18x - 36.$$
$$4x^2 + 18x - 360 = 0.$$
$$2x^2 + 9x - 180 = (2x - 15)(x + 12) = 0.$$
$$x = \tfrac{15}{2}, -12. \qquad \text{(Root } -12 \text{ rejected.)}$$

33. If $y = x - 5,$

$$4(x-5) - 2(x-5)^{\frac{1}{2}} - 2 = 0, \text{ or}$$

$$2(x-5) - (x-5)^{\frac{1}{2}} - 1 = 0, \text{ becomes}$$

$$2y - y^{\frac{1}{2}} - 1 = 0.$$

$$\left(2y^{\frac{1}{2}} + 1\right)\left(y^{\frac{1}{2}} - 1\right) = 0.$$

$$y^{\frac{1}{2}} = 1, -\tfrac{1}{2}.$$

$$y = 1, \tfrac{1}{4}.$$

But $y = \frac{1}{4}$ does not satisfy $2y - y^{\frac{1}{2}} - 1 = 0.$

Therefore $x - 5 = 1$ only.

$$x = 6.$$

34. $$4x\sqrt{x} = (x+3)\sqrt{9x}.$$

$$4x\sqrt{x} = x\sqrt{9x} + 3\sqrt{9x}.$$

$$4x\sqrt{x} = 3x\sqrt{x} + 9\sqrt{x}.$$

$$(x-9)\sqrt{x} = 0.$$

$$x = 9, 0.$$

35. $$\frac{\sqrt{x+2}}{\sqrt{8-x}} - \frac{\sqrt{8-x}}{\sqrt{x+2}} = \frac{8}{3}.$$

$$\frac{x+2}{8-x} - 2 + \frac{8-x}{x+2} = \frac{64}{9}.$$

$$\frac{x+2}{8-x} + \frac{8-x}{x+2} = \frac{82}{9}.$$

$$9(x+2)^2 + 9(8-x)^2 = 82(x+2)(8-x).$$

Expanding and combining,

$$x^2 - 6x - 7 = 0.$$

$$x = 7, -1. \qquad \text{(Root } -1 \text{ rejected.)}$$

36. $$t = \pi\sqrt{\frac{l}{g}}.$$

$$t^2 = \pi^2\frac{l}{g}.$$

Whence $$l = \frac{gt^2}{\pi^2},$$

and then $$g = \frac{\pi^2 l}{t^2}.$$

37. $$r = \tfrac{1}{2}x\sqrt{3},$$

and $$A = 2\pi r^2.$$

Then $$A = 2\pi\left[\frac{1}{2}x\sqrt{3}\right]^2 = \frac{3\pi x^2}{2}.$$

38. $a^2 = 2r^2,$

and $C = 2\pi r.$

Then $r^2 = \frac{a^2}{2},$

and $r = \frac{a}{2}\sqrt{2}.$

Therefore $C = 2\pi \cdot \frac{a}{2}\sqrt{2} = \pi a\sqrt{2}.$

39. $A = \frac{3r^2}{2}\sqrt{3},$

and $x = \frac{1}{2}r\sqrt{3}.$

Then $r = \frac{2x}{\sqrt{3}} = \frac{2}{3}x\sqrt{3}.$

Therefore $A = \frac{3\sqrt{3}}{2}\left[\frac{2}{3}x\sqrt{3}\right]^2 = 2x^2\sqrt{3}.$

40. $K = 3r^2,$

and $a = \frac{1}{2}r\sqrt{3+\sqrt{2}}.$

Then $r = \frac{2a}{\sqrt{3+\sqrt{2}}},$

and $r^2 = \frac{4a^2}{3+\sqrt{2}} = \frac{4a^2(3-\sqrt{2})}{7}.$

Therefore $K = \frac{12a^2}{7}(3-\sqrt{2}).$

Page 388

1. $3\sqrt{-1} + 2\sqrt{-1} = 5\sqrt{-1}.$
2. $4\sqrt{-1} + \sqrt{-4} = 4\sqrt{-1} + 2\sqrt{-1} = 6\sqrt{-1}.$
3. $\sqrt{-25} - \sqrt{-16} = 5\sqrt{-1} - 4\sqrt{-1} = \sqrt{-1}.$
4. $\sqrt{-9} + \sqrt{-4} = 3\sqrt{-1} + 2\sqrt{-1} = 5\sqrt{-1}.$
5. $\sqrt{-4} + \sqrt{-16} = 2\sqrt{-1} + 4\sqrt{-1} = 6\sqrt{-1}.$
6. $(-8)^{\frac{1}{2}} + (-32)^{\frac{1}{2}} = \sqrt{-8} + \sqrt{-32}$
   $= 2\sqrt{2}\cdot i + 4\sqrt{2}\cdot i$
   $= 6\sqrt{2}\cdot i$
   $= 6\sqrt{-2}.$
7. $\sqrt{-18} + \sqrt{-8} = 3\sqrt{2}\cdot i + 2\sqrt{2}\cdot i = 5\sqrt{2}\cdot i = 5\sqrt{-2}.$
8. $4\sqrt{-25x^2} - 2\sqrt{-36x^2} = 4\cdot 5x\sqrt{-1} - 2\cdot 6x\sqrt{-1}$
   $= (20x - 12x)\sqrt{-1}$
   $= 8x\sqrt{-1}.$

9. $2+3\sqrt{-1}+6-5\sqrt{-1}=8-2\sqrt{-1}.$

10. $7\sqrt{-a^2}-5a+4\sqrt{-a^2}=11\sqrt{-a^2}-5a=11a\sqrt{-1}-5a.$

11. $(3a-6ib)+(a+ib)=4a-5ib.$

12. $4-8i+16-3\sqrt{-9}=4-8i+16-3\cdot 3i=20-17i.$

13. $5+3\sqrt{-49x^2}-6\sqrt{-16x^2}+4=9+3\cdot 7x\cdot\sqrt{-1}-6\cdot 4x\cdot\sqrt{-1}$
$=9-3x\sqrt{-1}.$

14. $18-3(-1)^{\frac{1}{2}}+6(-25)^{\frac{1}{2}}+4=22-3\sqrt{-1}+6\cdot 5\cdot\sqrt{-1}$
$=22+27\sqrt{-1}.$

15. $5\sqrt{-3}+3\sqrt{-2}-\sqrt{-27}+2\sqrt{-8}$
$=5\sqrt{-3}+3\sqrt{-2}-3\sqrt{-3}+4\sqrt{-2}$
$=2\sqrt{-3}+7\sqrt{-2}.$

16. $(8-5\sqrt{-16})-(7+3\sqrt{-25})=8-20\sqrt{-1}-7-15\sqrt{-1}$
$=1-35\sqrt{-1}.$

17. $4\sqrt{-9a^4}-6a^2\sqrt{-16}+3\sqrt{-6}+5\sqrt{-54}$
$=12a^2\sqrt{-1}-24a^2\sqrt{-1}+3\sqrt{-6}+15\sqrt{-6}$
$=18\sqrt{-6}-12a^2\sqrt{-1}.$

18. $(5x-6iy)-(3x+2iy)=2x-8iy.$

Page 390

1. $(-1)^5=(-1)^4(-1)=-1.$

2. $(-1)^6=1.$

3. $(-1)^7=(-1)^6(-1)=-1.$

4. $(-1)^8=1.$

5. $(\sqrt{-1})^{-4}=\dfrac{1}{(\sqrt{-1})^4}=\dfrac{1}{1}=1.$

6. $(\sqrt{-1})^7=(\sqrt{-1})^4\cdot(\sqrt{-1})^3=(1)(-\sqrt{-1})=-\sqrt{-1}.$

7. $(\sqrt{-2})^3=-2\sqrt{-2}.$

8. $(\sqrt{-3})^5=(\sqrt{-3})^4\cdot\sqrt{-3}=9\sqrt{-3}.$

9. $(2\sqrt{-5})^7=(2)^7\cdot(\sqrt{-5})^6\cdot(\sqrt{-5})$
$=(128)(-125)\sqrt{-5}$
$=-16{,}000\sqrt{-5}.$

10. $\sqrt{-4}\cdot\sqrt{-25}=2i\cdot 5i=10i^2=-10.$

11. $\sqrt{-4}\cdot(-\sqrt{-9})=(2i)(-1)(3i)=-6i^2=6.$

12. $\sqrt{-7}\cdot(-\sqrt{-6})=(\sqrt{7}i)(-1)(\sqrt{6}i)=-\sqrt{42}i^2=\sqrt{42}.$

13. $\sqrt{-16}\cdot\sqrt{-10}=4i\cdot\sqrt{10}i=4\sqrt{10}i^2=-4\sqrt{10}.$

14. $2\sqrt{-3}\cdot 3\sqrt{-2}=6\sqrt{3}i\sqrt{2}i=6\sqrt{6}i^2=-6\sqrt{6}.$

15. $\sqrt{-a}\cdot\sqrt{-b}=i\sqrt{a}\cdot i\sqrt{b}=\sqrt{ab}\cdot i^2=-\sqrt{ab}.$

16. $3\sqrt{-7}(-2\sqrt{-5})=-6i\sqrt{7}i\sqrt{5}=-6i^2\sqrt{35}=6\sqrt{35}.$

17. $\sqrt{m+n}\cdot\sqrt{-m-n}=\sqrt{m+n}\cdot\sqrt{m+n}\cdot i=(m+n)\sqrt{-1}.$

18. $(3+\sqrt{-1})(3-\sqrt{-1})=3^2-(\sqrt{-1})^2=9+1=10.$

19. $(5+\sqrt{-3})(5-\sqrt{-3})=5^2-(\sqrt{-3})^2=25+3=28.$

20. $(3-4\sqrt{2}\,i)(3+2\sqrt{2}\,i)=9+6\sqrt{2}\,i-12\sqrt{2}\,i+16=25-6\sqrt{-2}.$

21. $(4+\sqrt{-1})(5-\sqrt{-3})=20+5\sqrt{-1}-4\sqrt{-3}+\sqrt{3}$
$=20+\sqrt{3}+5\sqrt{-1}-4\sqrt{-3}.$

22. $(5-3i)(6-5\sqrt{2}\,i)=30-25\sqrt{2}\,i-18i-15\sqrt{2}$
$=(30-15\sqrt{2})-(18+25\sqrt{2})i.$

23. $(a+ib)(c+id)=ac+adi+bci-bd=(ac-bd)+(ad+bc)i.$

24. $(a+ib)^2=a^2+2abi-b^2=a^2-b^2+2abi.$

25. $(a+ib)(a-ib)=a^2+b^2.$

26. $(-\frac{1}{2}+\frac{1}{2}\sqrt{-3})^2=\frac{1}{4}-\frac{1}{2}\sqrt{-3}-\frac{3}{4}=-\frac{1}{2}-\frac{1}{2}\sqrt{-3}.$

27. $(-\frac{1}{2}-\frac{1}{2}\sqrt{-3})^2=\frac{1}{4}+\frac{1}{2}\sqrt{-3}-\frac{3}{4}=-\frac{1}{2}+\frac{1}{2}\sqrt{-3}.$

28. $(-2+2\sqrt{-3})^3=-8+3\cdot 8\sqrt{-3}-3\cdot 8(-3)+8(-3\sqrt{-3})$
$=-8+72$
$=64.$

29. $(-2-2\sqrt{-3})^3=-8-3\cdot 8\sqrt{-3}-3\cdot 8(-3)-8(-3\sqrt{-3})$
$=-8+72$
$=64.$

30. $(x-iy)^3=x^3-3x^2yi+3xy^2i^2-y^3i^3=(x^3-3xy^2)+(y^3-3x^2y)i.$

31. $(a+ib)^2-(a-ib)^2=(a^2+2abi+i^2b^2)-(a^2-2abi+i^2b^2)=4abi.$

32. $(2+2\sqrt{-3})^3-(2-2\sqrt{-3})^3$
$=(8+24\sqrt{-3}-72-24\sqrt{-3})-(8-24\sqrt{-3}-72+24\sqrt{-3})$
$=-64+64=0.$

33. $(a+i\sqrt{1-b^2})(a-i\sqrt{1-b^2})=a^2-i^2(1-b^2)=a^2-b^2+1.$

34. $(5+6\sqrt{-2})(5-6\sqrt{-2})=5^2-(6\sqrt{-2})^2$
$=25+72$
$=97$, a real number.
$(5+6\sqrt{-2})+(5-6\sqrt{-2})=10$, a real number.

35. $(2+\sqrt{-3})(2-\sqrt{-3})=2^2-(\sqrt{-3})^2=4+3=7$, a real number.
$(2+\sqrt{-3})+(2-\sqrt{-3})=4$, a real number.

Page 392 (First set)

1. $\dfrac{\sqrt{-12}}{\sqrt{2}}=\dfrac{\sqrt{-12}\cdot\sqrt{2}}{2}=\dfrac{2\sqrt{-3}\cdot\sqrt{2}}{2}=\sqrt{-6}.$

2. $\dfrac{\sqrt{6}}{\sqrt{-2}}=\dfrac{\sqrt{6}\cdot\sqrt{-2}}{-2}=\dfrac{\sqrt{-12}}{-2}=\dfrac{2\sqrt{-3}}{-2}=-\sqrt{-3}.$

3. $\dfrac{2\sqrt{5}}{4\sqrt{-1}}=\dfrac{2\sqrt{5}\sqrt{-1}}{-4}=-\dfrac{1}{2}\sqrt{-5}.$

4. $\dfrac{\sqrt{-9}}{\sqrt{-1}}=\dfrac{\sqrt{-9}\cdot\sqrt{-1}}{-1}=\dfrac{-3}{-1}=3.$

5. $\dfrac{1}{\sqrt{-3}}=\dfrac{\sqrt{-3}}{-3}=-\dfrac{1}{3}\sqrt{-3}.$

6. $\dfrac{4}{\sqrt{-5}}=\dfrac{4\sqrt{-5}}{-5}=-\dfrac{4}{5}\sqrt{-5}.$

7. $\dfrac{\sqrt{8}}{\sqrt{-2}}=\dfrac{2\sqrt{2}\cdot\sqrt{-2}}{-2}=-\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{-1}=-2\sqrt{-1}.$

8. $\dfrac{(-49)^{\frac{1}{2}}}{(-64)^{\frac{1}{2}}}=\dfrac{\sqrt{49}\sqrt{-1}}{\sqrt{64}\sqrt{-1}}=\dfrac{7}{8}.$

9. $\dfrac{\sqrt{by}}{\sqrt{-y}}=\dfrac{\sqrt{by}\sqrt{-y}}{-y}=\dfrac{y\sqrt{-b}}{-y}=-\sqrt{-b}.$

10. $\dfrac{\sqrt{-m}}{\sqrt{-n}}=\dfrac{\sqrt{-m}\sqrt{-1}}{\sqrt{-n}\sqrt{-1}}=\dfrac{-\sqrt{m}}{-\sqrt{n}}=\sqrt{\dfrac{m}{n}}=\dfrac{\sqrt{mn}}{n}.$

11. $\dfrac{(-6bx)^{\frac{1}{2}}}{(-5x)^{\frac{1}{2}}}=\dfrac{\sqrt{-6bx}\cdot\sqrt{-1}}{\sqrt{-5x}\cdot\sqrt{-1}}=\dfrac{-\sqrt{6bx}}{-\sqrt{5x}}=\sqrt{\dfrac{6b}{5}}=\dfrac{1}{5}\sqrt{30b}.$

12. $\dfrac{(-x^6)^{\frac{1}{2}}-(-x^2)^{\frac{1}{2}}}{(-x)^{\frac{1}{2}}}=\dfrac{(\sqrt{-x^6})(\sqrt{-x})-(\sqrt{-x^2})(\sqrt{-x})}{\sqrt{-x}\cdot\sqrt{-x}}$

$=\dfrac{-\sqrt{x^7}+\sqrt{x^3}}{-x}=x^2\sqrt{x}-\sqrt{x}=(x^2-1)\sqrt{x}.$

13. $\dfrac{3}{1+\sqrt{-1}}=\dfrac{3(1-\sqrt{-1})}{1+1}=\dfrac{3-3\sqrt{-1}}{2}.$

14. $\dfrac{2}{1-\sqrt{-2}}=\dfrac{2(1+\sqrt{-2})}{1+2}=\dfrac{2+2\sqrt{-2}}{3}.$

15. $\dfrac{2\sqrt{-5}}{\sqrt{-1}+6}=\dfrac{2\sqrt{-5}(\sqrt{-1}-6)}{-1-36}=\dfrac{2\sqrt{5}+12\sqrt{-5}}{37}.$

16. $\dfrac{2\sqrt{-3}}{3\sqrt{-2}+3}=\dfrac{2\sqrt{-3}(3\sqrt{-2}-3)}{-18-9}=\dfrac{-6\sqrt{6}-6\sqrt{-3}}{-27}$

$=\dfrac{2\sqrt{6}+2\sqrt{-3}}{9}.$

17. $\dfrac{-1-\sqrt{-3}}{-1+\sqrt{-3}}=\dfrac{1+2\sqrt{-3}-3}{1+3}=\dfrac{-1+\sqrt{-3}}{2}.$

18. $\dfrac{1+2i}{3-4i}=\dfrac{(1+2i)(3+4i)}{9+16}=\dfrac{10i-5}{25}=\dfrac{2i-1}{5}.$

19. $\dfrac{x}{x+iy}=\dfrac{x(x-iy)}{x^2+y^2}=\dfrac{x^2-xyi}{x^2+y^2}.$

20. $\dfrac{a+ib}{c+id}=\dfrac{(a+ib)(c-id)}{c^2+d^2}=\dfrac{ac+bci-adi+bd}{c^2+d^2}=\dfrac{(ac+bd)+(bc-ad)i}{c^2+d^2}.$

21. $(3+2i)(1-i)\div(3-4i)(1+i)=(3-i+2)\div(3-i+4)$

$$=\frac{5-i}{7-i}$$

$$=\frac{(5-i)(7+i)}{50}$$

$$=\frac{36-2i}{50}$$

$$=\frac{18-i}{25}.$$

22. $(1-\sqrt{-3})^3=1-3\sqrt{-3}+(3)(-3)-(-3)(\sqrt{-3})=-8.$ Yes.

23. Substituting $3\pm\sqrt{-3}$ for x in $x^2-6x+12=0$,

$$(3\pm\sqrt{-3})^2-6(3\pm\sqrt{-3})+12$$

$$=9\pm6\sqrt{-3}-3-18\mp6\sqrt{-3}+12$$

$=0$ with either sign. Yes.

24. Substituting these values in $x^2-xy-12y^2=8$, we get

$$(\tfrac{64}{25})(-10)-(\tfrac{8}{5})(-\tfrac{3}{5})(-10)-12(\tfrac{9}{25})(-10)=8,$$

$$\frac{-128}{5}-\frac{48}{5}+\frac{216}{5}=8.$$

$$\frac{-176+216}{5}=8.$$

$$\tfrac{40}{5}=8,\text{ an identity.}$$

Substituting in $x^2+xy-10y^2=20$,

$$(\tfrac{64}{25})(-10)+(\tfrac{8}{5})(-\tfrac{3}{5})(-10)-10(\tfrac{9}{25})(-10)=20.$$

$$\frac{-128}{5}+\frac{48}{5}+\frac{180}{5}=20.$$

$$\frac{-128+228}{5}=20.$$

$$\tfrac{100}{5}=20,\text{ an identity.}$$

Hence the given pair of values satisfies the given system.

Page 392 (Second set)

1. $x^2+4x+8=0.$

$$x=\frac{-4\pm\sqrt{16-32}}{2}=-2\pm2\sqrt{-1}.$$

2. $x^2-8x+24=0.$

$$x=\frac{8\pm\sqrt{64-96}}{2}=4\pm2\sqrt{-2}.$$

3. $$x^2 + 3x + 9 = 0.$$
$$x = \frac{-3 \pm \sqrt{9-36}}{2} = \frac{-3 \pm 3\sqrt{-3}}{2}.$$

4. $$x^2 - 5x + 16 = 0.$$
$$x = \frac{5 \pm \sqrt{25-64}}{2} = \frac{5 \pm \sqrt{-39}}{2}.$$

5. $$3x^2 + 2x + 4 = 0.$$
$$x = \frac{-2 \pm \sqrt{4-48}}{6} = \frac{-1 \pm \sqrt{-11}}{3}.$$

6. $$x^2 + x + 1 = 0.$$
$$x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}.$$

7. $$x^2 - x + 1 = 0.$$
$$x = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{-3}}{2}.$$

8. $$5x^2 - 6x + 14 = 0.$$
$$x = \frac{6 \pm \sqrt{36-280}}{10} = \frac{3 \pm \sqrt{-61}}{5}.$$

9. $$6x^2 + 10x + 21 = 0.$$
$$x = \frac{-10 \pm \sqrt{100-504}}{12} = \frac{-5 \pm \sqrt{-101}}{6}.$$

10. $$3x^2 + 16x + 21 = 0.$$
$$x = \frac{-16 \pm \sqrt{256-252}}{6}$$
$$= \frac{-16 \pm 2}{6}$$
$$= -3, -\tfrac{7}{3}.$$

11. $$x^3 - 1 = 0.$$
$$(x-1)(x^2+x+1) = 0.$$
$$x = 1, \frac{-1 \pm \sqrt{-3}}{2}.$$ (See Ex. 6.)

12. $$x^3 + 1 = 0.$$
$$(x+1)(x^2-x+1) = 0.$$
$$x = -1, \frac{1 \pm \sqrt{-3}}{2}.$$ (See Ex. 7.)

13. $$x^3 = 8.$$
$$x^3 - 8 = 0.$$
$$(x-2)(x^2+2x+4) = 0.$$
$$x = 2, \frac{-2 \pm \sqrt{4-16}}{2}$$
$$= 2, -1 \pm \sqrt{-3}.$$

14.
$$x^3 = -27.$$
$$x^3 + 27 = 0.$$
$$(x+3)(x^2 - 3x + 9) = 0.$$
$$x = -3,\ \frac{3 \pm \sqrt{9-36}}{2} = -3,\ \frac{3 + 3\sqrt{-3}}{2}.$$

15.
$$x^4 = 1.$$
$$x^4 - 1 = 0.$$
$$(x^2+1)(x+1)(x-1) = 0.$$
$$x = \pm\sqrt{-1},\ \pm 1.$$

16.
$$x^4 = 9.$$
$$x^4 - 9 = 0.$$
$$(x^2-3)(x^2+3) = 0.$$
$$x = \pm\sqrt{3},\ \pm\sqrt{-3}.$$

17.
$$x^6 = 1.$$
$$x^6 - 1 = 0.$$
$$(x+1)(x^2-x+1)(x-1)(x^2+x+1) = 0.$$
$$x = \pm 1,\ \frac{1 \pm \sqrt{-3}}{2},\ \frac{-1 \pm \sqrt{-3}}{2}.$$

18.
$$x^6 = 64.$$
$$x^6 - 64 = 0.$$
$$(x^3+8)(x^3-8) = 0.$$
$$(x+2)(x-2)(x^2-2x+4)(x^2+2x+4) = 0.$$
$$x = \pm 2,\ \frac{2 \pm \sqrt{4-16}}{2},\ \frac{-2 \pm \sqrt{4-16}}{2}$$
$$= \pm 2,\ 1 \pm \sqrt{-3},\ -1 \pm \sqrt{-3}.$$

19.
$$x^3 = 64.$$
$$x^3 - 64 = 0.$$
$$(x-4)(x^2+4x+16) = 0.$$
$$x = 4,\ \frac{-4 \pm \sqrt{16-64}}{2}$$
$$= 4,\ -2 \pm 2\sqrt{-3}.$$

20.
$$x^3 = -125.$$
$$x^3 + 125 = 0.$$
$$(x+5)(x^2-5x+25) = 0.$$
$$x = -5,\ \frac{5 \pm \sqrt{25-100}}{2}$$
$$= -5,\ \frac{5 \pm 5\sqrt{-3}}{2}.$$

21. Two; three; four; six.

22. That it has n nth roots.

23.
$$27x^3 - 8 = 0.$$
$$(3x - 2)(9x^2 + 6x + 4) = 0.$$
$$x = \frac{2}{3}, \frac{-6 \pm \sqrt{36 - 144}}{18}$$
$$= \frac{2}{3}, \frac{-1 \pm \sqrt{-3}}{3}.$$

24.
$$64x^3 + 125 = 0.$$
$$(4x + 5)(16x^2 - 20x + 25) = 0.$$
$$x = -\frac{5}{4}, \frac{20 \pm \sqrt{400 - 1600}}{32}$$
$$= -\frac{5}{4}, \frac{5 \pm 5\sqrt{-3}}{8}.$$

25.
$$x^4 - 2x^2 - 8 = 0.$$
$$(x^2 - 4)(x^2 + 2) = 0.$$
$$x = \pm 2, \pm\sqrt{-2}.$$

26.
$$x^3 + x^2 - 2x - 2 = 0.$$
$$(x^2 - 2)(x + 1) = 0.$$
$$x = -1, \pm\sqrt{2}.$$

27.
$$x^6 + 7x^3 - 8 = 0.$$
$$(x^3 + 8)(x^3 - 1) = 0.$$
$$(x + 2)(x - 1)(x^2 - 2x + 4)(x^2 + x + 1) = 0.$$
$$x = -2, 1, 1 \pm \sqrt{-3}, \frac{-1 \pm \sqrt{-3}}{2}.$$

28.
$$3x^4 + 16x^2 + 21 = 0.$$
$$(3x^2 + 7)(x^2 + 3) = 0.$$
$$x = \pm\sqrt{-\tfrac{7}{3}}, \pm\sqrt{-3}$$
$$= \pm\tfrac{1}{3}\sqrt{-21}, \pm\sqrt{-3}.$$

29.
$$27x^4 - 12x^2 - 64 = 0.$$
$$x^2 = \frac{12 \pm \sqrt{144 + 6912}}{54} = \frac{12 \pm 84}{54}$$
$$= -\tfrac{4}{3}, \tfrac{16}{9}.$$
$$x = \pm\sqrt{-\tfrac{4}{3}}, \pm\tfrac{4}{3}$$
$$= \pm\tfrac{2}{3}\sqrt{-3}, \pm\tfrac{4}{3}.$$

30. $$6x^6 + 21x^3 + 9 = 0.$$
$$2x^6 + 7x^3 + 3 = 0.$$
$$(2x^3 + 1)(x^3 + 3) = 0.$$
$$(x\sqrt[3]{2} + 1)(x^2\sqrt[3]{4} - x\sqrt[3]{2} + 1)(x + \sqrt[3]{3})(x^2 - x\sqrt[3]{3} + \sqrt[3]{9}) = 0.$$ (See § 196.)

From $x\sqrt[3]{2} + 1 = 0$, $\quad x = -\dfrac{1}{\sqrt[3]{2}} = -\dfrac{1}{2}\sqrt[3]{4}$;

and from $x^2\sqrt[3]{4} - x\sqrt[3]{2} + 1 = 0$,
$$x = \frac{\sqrt[3]{2} \pm \sqrt{\sqrt[3]{4} - 4\sqrt[3]{4}}}{2\sqrt[3]{4}}$$
$$= \frac{\sqrt[3]{2} \pm \sqrt{-3\sqrt[3]{4}}}{2\sqrt[3]{4}}$$
$$= \frac{\sqrt[3]{4} \pm \sqrt[3]{2}\cdot\sqrt{-3\sqrt[3]{4}}}{4}$$
$$= \frac{\sqrt[3]{4} \pm \sqrt[6]{-432}}{4};$$

and from $x + \sqrt[3]{3} = 0$, $\quad x = -\sqrt[3]{3}$;

and from $x^2 - x\sqrt[3]{3} + \sqrt[3]{9} = 0$,
$$x = \frac{\sqrt[3]{3} \pm \sqrt{\sqrt[3]{9} - 4\sqrt[3]{9}}}{2}$$
$$= \frac{\sqrt[3]{3} \pm \sqrt{-3\sqrt[3]{9}}}{2}$$
$$= \frac{\sqrt[3]{3} \pm \sqrt[6]{-243}}{2}.$$

Whence $\quad x = -\dfrac{1}{2}\sqrt[3]{4},\ -\sqrt[3]{3},\ \dfrac{\sqrt[3]{4} \pm \sqrt[6]{-432}}{4},\ \dfrac{\sqrt[3]{3} \pm \sqrt[6]{-243}}{2}.$

31. $$25x^4 + 40x^2 + 64 = 0.$$
$$x^2 = \frac{-40 \pm \sqrt{1600 - 6400}}{50} = \frac{-4 \pm 4\sqrt{-3}}{5}.$$
$$x = \pm\sqrt{\frac{-4 \pm 4\sqrt{-3}}{5}} = \pm\frac{2}{5}\sqrt{-5 \pm 5\sqrt{-3}}$$
$$= \pm\tfrac{1}{5}(\sqrt{10} \pm \sqrt{-30}).$$

32. $(x^2 + 4)(x^2 + 3x + 7) = 0.$
$$x = \pm 2\sqrt{-1},\ \frac{-3 \pm \sqrt{-19}}{2}.$$

33. If $$y = x^2 + 2x,$$
$(x^2 + 2x)^2 + 15(x^2 + 2x) + 54 = 0$ becomes
$$(y + 6)(y + 9) = 0.$$
$$y = -6,\ -9.$$

$$\left.\begin{aligned} \text{Then } x^2+2x&=-6,\\ x^2+2x+6&=0,\\ x&=\frac{-2\pm\sqrt{4-24}}{2},\\ x&=-1\pm\sqrt{-5}, \end{aligned}\right\} \text{ and } \left\{\begin{aligned} x^2+2x&=-9,\\ x^2+2x+9&=0,\\ x&=\frac{-2\pm\sqrt{4-36}}{2},\\ x&=-1\pm2\sqrt{-2}. \end{aligned}\right.$$

34. If $\quad y = x^2 + 5x,$

$(x^2 + 5x)^2 + 9(x^2 + 5x) - 112 = 0$ becomes

$$(y-7)(y+16)=0.$$

$$\left.\begin{aligned} \text{Then } x^2+5x-7&=0,\\ x&=\frac{-5\pm\sqrt{53}}{2}, \end{aligned}\right\} \text{ and } \left\{\begin{aligned} x^2+5x+16&=0,\\ x&=\frac{-5\pm\sqrt{-39}}{2}. \end{aligned}\right.$$

35. $\quad x + y = 4, \qquad (1)$

$x^2 - 3xy - y^2 = -39. \qquad (2)$

From (1), $\quad y = 4 - x. \qquad (3)$

In (2),

$$x^2 - 3x(4-x) - (4-x)^2 = -39.$$

$$3x^2 - 4x - 16 = -39.$$

$$3x^2 - 4x + 23 = 0.$$

$$x = \frac{4\pm\sqrt{16-276}}{6} = \frac{2\pm\sqrt{-65}}{3}.$$

$$y = 4 - x = \frac{12}{3} - \frac{2\pm\sqrt{-65}}{3} = \frac{10\mp\sqrt{-65}}{3}.$$

Therefore $\quad x = \dfrac{2+\sqrt{-65}}{3}, \dfrac{2-\sqrt{-65}}{3},$

and $\quad y = \dfrac{10-\sqrt{-65}}{3}, \dfrac{10+\sqrt{-65}}{3}.$

36. $\quad x + 2y = 4, \qquad (1)$

$y^2 - x = 0. \qquad (2)$

In (1), $\quad x = 4 - 2y.$

In (2), $\quad x = y^2.$

Hence $\quad y^2 = 4 - 2y.$

$$y^2 + 2y - 4 = 0.$$

$$y = \frac{-2\pm\sqrt{4+16}}{2} = -1\pm\sqrt{5},$$

and $\quad x = 4 - 2y = 4 - (-2 \pm 2\sqrt{5}) = 6 \mp 2\sqrt{5}.$

Therefore $\quad x = 6 - 2\sqrt{5},\ 6 + 2\sqrt{5},$

and $\quad y = -1 + \sqrt{5},\ -1 - \sqrt{5}.$

37.

$$x^2 + y^2 = 4, \qquad (1)$$
$$x - y = 6. \qquad (2)$$

Squaring (2), $x^2 - 2xy + y^2 = 36.$ (3)

(1) − (3), $2xy = -32.$ (4)

(1) + (4), $(x + y)^2 = -28.$ (5)

Then $x + y = \pm\sqrt{-28} = \pm 2\sqrt{-7}.$ (6)

$$x - y = 6. \qquad (7)$$

From (7) and $x + y = 2\sqrt{-7}$, $2x = 6 + 2\sqrt{-7}.$

Then $x = 3 + \sqrt{-7},$

and $y = x - 6 = -3 + \sqrt{-7}.$

From (7) and $x + y = -2\sqrt{-7}$, $2x = 6 - 2\sqrt{-7}.$

Then $x = 3 - \sqrt{-7},$

and $y = x - 6 = -3 - \sqrt{-7}.$

Page 396

1. $(x - 2)(x - 3) = 0.$
$x^2 - 5x + 6 = 0.$

2. $(x - 3)(x - 7) = 0.$
$x^2 - 10x + 21 = 0.$

3. $(x - 1)(x + 3) = 0.$
$x^2 + 2x - 3 = 0.$

4. $(x + 2)(x + 5) = 0.$
$x^2 + 7x + 10 = 0.$

5. $(x - 2)(x - \frac{2}{5}) = 0.$
$(x - 2)(5x - 2) = 0.$
$5x^2 - 12x + 4 = 0.$

6. $(x - 5)(x - \frac{3}{7}) = 0.$
$(x - 5)(7x - 3) = 0.$
$7x^2 - 38x + 15 = 0.$

7. $(x + \frac{2}{3})(x - \frac{5}{6}) = 0.$
$(3x + 2)(6x - 5) = 0.$
$18x^2 - 3x - 10 = 0.$

8. $(x + \frac{4}{5})(x + \frac{7}{8}) = 0.$
$(5x + 4)(8x + 7) = 0.$
$40x^2 + 67x + 28 = 0.$

9. $(x - 1 - \sqrt{3})(x - 1 + \sqrt{3}) = 0.$
$x^2 - 2x - 2 = 0.$

10. $(x - 3 - \sqrt{7})(x - 3 + \sqrt{7}) = 0.$
$x^2 - 6x + 2 = 0.$

11. $(x - 2 - \sqrt{-5})(x - 2 + \sqrt{-5}) = 0.$
$x^2 - 4x + 9 = 0.$

12. $(x + 7 - \sqrt{-5})(x + 7 + \sqrt{-5}) = 0.$
$x^2 + 14x + 54 = 0.$

13. $(x - \frac{1}{2} - \sqrt{\frac{3}{2}})(x - \frac{1}{2} + \sqrt{\frac{3}{2}}) = 0.$
$x^2 - x - \frac{5}{4} = 0.$
$4x^2 - 4x - 5 = 0.$

14. $$\left(x - \tfrac{1}{2} - \sqrt{-\tfrac{3}{2}}\right)\left(x - \tfrac{1}{2} + \sqrt{-\tfrac{3}{2}}\right) = 0.$$
$$x^2 - x + \tfrac{7}{4} = 0.$$
$$4x^2 - 4x + 7 = 0.$$

15. $$(x-2)(x-3)(x-4) = 0.$$
$$x^3 - 9x^2 + 26x - 24 = 0.$$

16. $$(x-1)(x-3)(x-5) = 0.$$
$$x^3 - 9x^2 + 23x - 15 = 0.$$

17. $$(x-1)(x+1)(x-2) = 0.$$
$$x^3 - 2x^2 - x + 2 = 0.$$

18. $$(x - \tfrac{1}{2})(x - \tfrac{1}{3})(x - \tfrac{1}{4}) = 0.$$
$$(2x-1)(3x-1)(4x-1) = 0.$$
$$24x^3 - 26x^2 + 9x - 1 = 0.$$

19. $$\left(x - \sqrt{-2}\right)\left(x + \sqrt{-2}\right)(x-2) = 0.$$
$$(x^2+2)(x-2) = 0.$$
$$x^3 - 2x^2 + 2x - 4 = 0.$$

20. $$\left(x - 1 - \sqrt{2}\right)\left(x - 1 + \sqrt{2}\right)(x-3) = 0.$$
$$(x^2 - 2x - 1)(x-3) = 0.$$
$$x^3 - 5x^2 + 5x + 3 = 0.$$

21. $$(x-1)(x+1)\left(x - \sqrt{-1}\right)\left(x + \sqrt{-1}\right) = 0.$$
$$(x^2-1)(x^2+1) = 0.$$
$$x^4 - 1 = 0.$$

22. $$\left(x - \sqrt{2} + \sqrt{3}\right)\left(x - \sqrt{2} - \sqrt{3}\right)\left(x + \sqrt{2} + \sqrt{3}\right)\left(x + \sqrt{2} - \sqrt{3}\right) = 0.$$
$$\left(x^2 - 2\sqrt{2}x - 1\right)\left(x^2 + 2\sqrt{2}x - 1\right) = 0.$$
$$x^4 - 10x^2 + 1 = 0.$$

Page 398

2. $-(\tfrac{4}{3} + \tfrac{1}{2}) = -\tfrac{11}{6}$; $(\tfrac{4}{3})(\tfrac{1}{2}) = \tfrac{2}{3}$.
   $x^2 - \tfrac{11}{6}x + \tfrac{2}{3} = 0$, or $6x^2 - 11x + 4 = 0$.
3. $-(\tfrac{1}{4} - \tfrac{2}{3}) = \tfrac{5}{12}$; $(\tfrac{1}{4})(-\tfrac{2}{3}) = -\tfrac{1}{6}$.
   $x^2 + \tfrac{5}{12}x - \tfrac{1}{6} = 0$, or $12x^2 + 5x - 2 = 0$.
4. $-(\tfrac{3}{2} - \tfrac{16}{5}) = \tfrac{17}{10}$; $(\tfrac{3}{2})(-\tfrac{16}{5}) = -\tfrac{24}{5}$.
   $x^2 + \tfrac{17}{10}x - \tfrac{24}{5} = 0$, or $10x^2 + 17x - 48 = 0$.
5. $-(4.41 + 1.59) = -6$; $(4.41)(1.59) = 7.0119$.
   $x^2 - 6x + 7.0119 = 0$.
6. $-\left(2 + 3\sqrt{3} + 2 - 3\sqrt{3}\right) = -4$; $\left(2 + 3\sqrt{3}\right)\left(2 - 3\sqrt{3}\right) = -23$.
   $x^2 - 4x - 23 = 0$.
7. $-\left(4 + \sqrt{-2} + 4 - \sqrt{-2}\right) = -8$; $\left(4 + \sqrt{-2}\right)\left(4 - \sqrt{-2}\right) = 18$.
   $x^2 - 8x + 18 = 0$.

8. $-\left(\frac{3}{2}+\sqrt{7}+\frac{3}{2}-\sqrt{7}\right)=-3$; $\left(\frac{3}{2}+\sqrt{7}\right)\left(\frac{3}{2}-\sqrt{7}\right)=-\frac{19}{4}$.
$x^2-3x-\frac{19}{4}=0$, or $4x^2-12x-19=0$.

9. $-\left(\frac{1}{2}+\frac{1}{2}\sqrt{-3}+\frac{1}{2}-\frac{1}{2}\sqrt{-3}\right)=-1$;
$\left(\frac{1}{2}+\frac{1}{2}\sqrt{-3}\right)\left(\frac{1}{2}-\frac{1}{2}\sqrt{-3}\right)=1$.
$x^2-x+1=0$.

10. $-\left(\frac{5}{3}+\sqrt{3}+\frac{5}{3}-\sqrt{3}\right)=-\frac{10}{3}$; $\left(\frac{5}{3}+\sqrt{3}\right)\left(\frac{5}{3}-\sqrt{3}\right)=-\frac{2}{9}$.
$x^2-\frac{10}{3}x-\frac{2}{9}=0$, or $9x^2-30x-2=0$.

11. $-\left(1+\sqrt{-1}+1-\sqrt{-1}\right)=-2$; $\left(1+\sqrt{-1}\right)\left(1-\sqrt{-1}\right)=2$.
$x^2-2x+2=0$.

12. $-\left(a-\frac{1}{a}\right)=\frac{1-a^2}{a}$; $(a)\left(-\frac{1}{a}\right)=-1$.

$x^2+\frac{(1-a^2)x}{a}-1=0$, or $ax^2+(1-a^2)x-a=0$.

13. $-\left(\frac{3a}{2}+\frac{5a}{2}\right)=-4a$; $\left(\frac{3a}{2}\right)\left(\frac{5a}{2}\right)=\frac{15a^2}{4}$.

$x^2-4ax+\frac{15a^2}{4}=0$, or $4x^2-16ax+15a^2=0$.

14. $-(1+a+1-a)=-2$; $(1+a)(1-a)=1-a^2$.
$x^2-2x+1-a^2=0$.

15. $-\left(\frac{a+b}{a-b}+\frac{a-b}{a+b}\right)=\frac{-2(a^2+b^2)}{a^2-b^2}$; $\left(\frac{a+b}{a-b}\right)\left(\frac{a-b}{a+b}\right)=1$.

$$x^2+\frac{-2(a^2+b^2)}{a^2-b^2}x+1=0,$$

or $$(a^2-b^2)x^2-2(a^2+b^2)x+a^2-b^2=0.$$

16. For the roots 6, 8, and $\frac{1}{2}$, the equation is

$$(x-6)(x-8)\left(x-\tfrac{1}{2}\right)=0. \qquad (1)$$

(1) · 2, $$(x-6)(x-8)(2x-1)=0.$$
$$2x^3-29x^2+110x-48=0.$$

17. $$(x+4)(x-4)\left(x-\tfrac{1}{4}\right)=0.$$
$$(x^2-16)(4x-1)=0.$$
$$4x^3-x^2-64x+16=0.$$

18. $$x^2-5x+6=0.$$
$$x=\frac{5\pm\sqrt{25-24}}{2}=\frac{5\pm1}{2}$$
$$=3, 2.$$

$-(3+2)=-5$; $3\cdot2=6$.

19. $$x^2 - x - 3 = 0.$$
$$x = \frac{1 \pm \sqrt{13}}{2}.$$
$$-\left(\frac{1+\sqrt{13}}{2} + \frac{1-\sqrt{13}}{2}\right) = -1;\ \left(\frac{1}{2} + \frac{1}{2}\sqrt{13}\right)\left(\frac{1}{2} - \frac{1}{2}\sqrt{13}\right) = -3.$$

20. $$x^2 - 2x - 4 = 0.$$
$$x = \frac{2 \pm \sqrt{4+16}}{2} = 1 \pm \sqrt{5}.$$
$$-(1+\sqrt{5}+1-\sqrt{5}) = -2;\ (1+\sqrt{5})(1-\sqrt{5}) = -4.$$

21. $$x^2 - 9x - 10 = 0.$$
$$x = \frac{9 \pm \sqrt{81+40}}{2} = \frac{9 \pm 11}{}$$
$$= 10,\ -1.$$
$$-(10-1) = -9;\ (10)(-1) = -10.$$

22. $$x^2 + 2x + 1 = 0.$$
$$x = \frac{-2 \pm \sqrt{4-4}}{2}$$
$$= -1,\ -1.$$
$$-(-1-1) = 2;\ (-1)(-1) = 1.$$

23. $$x^2 + 8x + 16 = 0.$$
$$x = \frac{-8 \pm \sqrt{64-64}}{2}$$
$$= -4,\ -4.$$
$$-(-4-4) = 8;\ (-4)(-4) = 16.$$

24. $$x^2 + 5x + 5 = 0.$$
$$x = \frac{-5 \pm \sqrt{25-20}}{2} = \frac{-5 \pm \sqrt{5}}{2}.$$
$$-\left(\frac{-5+\sqrt{5}}{2} + \frac{-5-\sqrt{5}}{2}\right) = 5;$$
$$\left(\frac{-5+\sqrt{5}}{2}\right)\left(\frac{-5-\sqrt{5}}{2}\right) = \frac{25-5}{4} = 5.$$

25. $$2x^2 + 3x - 6 = 0.$$
$$x = \frac{-3 \pm \sqrt{9+48}}{4} = \frac{-3 \pm \sqrt{57}}{4}.$$
$$-\left(\frac{-3+\sqrt{57}}{4} + \frac{-3-\sqrt{57}}{4}\right) = \frac{3}{2}.$$
$$\left(\frac{-3+\sqrt{57}}{4}\right)\left(\frac{-3-\sqrt{57}}{4}\right) = \frac{9-57}{16} = -3 = \frac{-6}{2}.$$

26. $3x^2 + 3x - 5 = 0.$

$$x = \frac{-3 \pm \sqrt{9 + 60}}{6} = -\frac{1}{2} \pm \frac{1}{6}\sqrt{69}.$$

$$-(-\tfrac{1}{2} + \tfrac{1}{6}\sqrt{69} - \tfrac{1}{2} - \tfrac{1}{6}\sqrt{69}) = 1 = \tfrac{3}{3};$$

$$(-\tfrac{1}{2} + \tfrac{1}{6}\sqrt{69})(-\tfrac{1}{2} - \tfrac{1}{6}\sqrt{69}) = \tfrac{1}{4} - \tfrac{69}{36} = -\tfrac{60}{36} = -\tfrac{5}{3}.$$

27. $5x^2 - 6x + 10 = 0.$

$$x = \frac{6 \pm \sqrt{36 - 200}}{10} = \frac{3 \pm \sqrt{-41}}{5}.$$

$$-\left(\frac{3 + \sqrt{-41}}{5} + \frac{3 - \sqrt{-41}}{5}\right) = -\frac{6}{5};$$

$$\left(\frac{3 + \sqrt{-41}}{5}\right)\left(\frac{3 - \sqrt{-41}}{5}\right) = \frac{9 + 41}{25} = \frac{50}{25} = \frac{10}{5}.$$

28. $x^2 + x + 1 = 0.$

$$x = \frac{-1 \pm \sqrt{-3}}{2}.$$

$$-\left(\frac{-1 + \sqrt{-3}}{2} + \frac{-1 - \sqrt{-3}}{2}\right) = 1;$$

$$\left(\frac{-1 + \sqrt{-3}}{2}\right)\left(\frac{-1 - \sqrt{-3}}{2}\right) = \frac{1 + 3}{4} = 1.$$

29. $\frac{x^2}{3} + \frac{x}{4} + \frac{1}{5} = 0.$

$20x^2 + 15x + 12 = 0.$

$$x = \frac{-15 \pm \sqrt{225 - 960}}{40} = \frac{-15 \pm \sqrt{-735}}{40}.$$

$$-\left(\frac{-15 + \sqrt{-735}}{40} + \frac{-15 - \sqrt{735}}{40}\right) = \frac{30}{40} = \frac{1}{4} \div \frac{1}{3};$$

$$\left(\frac{-15 + \sqrt{-735}}{40}\right)\left(\frac{-15 - \sqrt{-735}}{40}\right) = \frac{225 + 735}{1600} = \frac{960}{1600} = \frac{1}{5} \div \frac{1}{3}.$$

30. $\frac{x^2}{2} + 4x - 7 = 0.$

$x^2 + 8x - 14 = 0.$

$$x = \frac{-8 \pm \sqrt{64 + 56}}{2} = -4 \pm \sqrt{30}.$$

$$-(-4 + \sqrt{30} - 4 - \sqrt{30}) = 8; \quad (-4 + \sqrt{30})(-4 - \sqrt{30}) = -14.$$

31. $$6x^2 - 4x - 3 = 0.$$

$$x = \frac{4 \pm \sqrt{88}}{12} = \frac{2 \pm \sqrt{22}}{6}.$$

$$-\left(\frac{2+\sqrt{22}}{6} + \frac{2-\sqrt{22}}{6}\right) = -\frac{4}{6};$$

$$\left(\frac{2+\sqrt{22}}{6}\right)\left(\frac{2-\sqrt{22}}{6}\right) = \frac{4-22}{36} = -\frac{3}{6}.$$

32. If one root $= 2$, then $x = 2$ satisfies the equation. Therefore $4 + 4 - c = 0$, or $c = 8$.

33. $36 - 6 - c = 0$, and $c = 30$.

34. $49 - 7c - 70 = 0$; $7c + 21 = 0$; $c = -3$.

35. $16 - 8b + 20 = 0$; $8b - 36 = 0$; $b = \frac{9}{2}$.

36. Let $r =$ the lesser root.
Then $2r =$ the greater root,
and $-(r + 2r) = -3r = -8$.
Whence $r = \frac{8}{3}$,
and $2r = \frac{16}{3}$.
Then $c = \frac{8}{3} \cdot \frac{16}{3} = \frac{128}{9}$.

37. Let $x =$ the lesser root.
Then $x + 2 =$ the greater root.
Then $-(x + x + 2) = 7$.
Whence $x = -\frac{9}{2}$,
and $x + 2 = -\frac{5}{2}$.
Then $c = (-\frac{9}{2})(-\frac{5}{2}) = \frac{45}{4}$.

38. Let $x =$ the greater root.
Then $x - 10 =$ the lesser root.
Then $-(x + x - 10) = 11$.
Whence $x = -\frac{1}{2}$,
and $x - 10 = -\frac{21}{2}$.
Then $b = (-\frac{1}{2})(-\frac{21}{2}) = \frac{21}{4}$.

Page 401

2. $b^2 - 4ac = k^2 - 64 = 0$, and $k = \pm 8$.

3. $b^2 - 4ac = 100 - 4k = 0$, and $k = 25$.

4. $b^2 - 4ac = 64 - 8k = 0$, and $k = 8$.

5. $b^2 - 4ac = 9k^2 - 144 = 0$, and $k = \pm 4$.

6. $b^2 - 4ac = 16k^2 - 144 = 0$, and $k = \pm 3$.

7. $b^2 - 4ac = 900 - 36k - 324 = 0$, and $k = 16$.

8. $b^2 - 4ac = 3600 - 400k = 0$, and $k = 9$.

9. $$b^2 - 4ac = 84^2 - 4 \cdot 9 \cdot 49k^2 = 0.$$
$$12^2 - 4 \cdot 9k^2 = 0.$$
$$12 - 3k^2 = 0, \text{ and } k = \pm 2.$$

10. $b^2 - 4ac = k^2 + 6k + 9 - 784 = 0.$
$$k^2 + 6k - 775 = 0.$$
$$(k - 25)(k + 31) = 0, \text{ and } k = 25, -31.$$

11. $b^2 - 4ac = 900 - 100k^2 - 500 = 0$, and $k = \pm 2$.

12. $$y^2 = 2x, \quad (1)$$
$$y = x + a. \quad (2)$$

Substituting from (2) in (1), $x^2 + 2ax + a^2 = 2x$.
$$x^2 + (2a - 2)x + a^2 = 0.$$
$$b^2 - 4ac = 4a^2 - 8a + 4 - 4a^2 = 0,$$
and $$a = \tfrac{1}{2}.$$

13. $$x^2 + y^2 = a^2, \quad (1)$$
$$y = x + 1. \quad (2)$$

Substituting from (2) in (1), $x^2 + x^2 + 2x + 1 = a^2$.
$$2x^2 + 2x + 1 - a^2 = 0.$$
$$b^2 - 4ac = 4 - 8 + 8a^2 = 0.$$
$$8a^2 - 4 = 0.$$
$$2a^2 - 1 = 0.$$
$$a = \pm \tfrac{1}{2}\sqrt{2}.$$

14. $$xy = a, \quad (1)$$
$$x + y = 1. \quad (2)$$

From (2), $$y = 1 - x. \quad (3)$$

Substituting from (3) in (1), $x(1 - x) = a$.
$$x - x^2 = a.$$
$$x^2 - x + a = 0.$$
$$b^2 - 4ac = 1 - 4a = 0,$$
and $$a = \tfrac{1}{4}.$$

Page 403

1. $b^2 - 4ac = 9 + 160 = 169 = 13^2$.
Hence, rational factors.

2. $b^2 - 4ac = 25 + 56 = 81 = 9^2$.
Hence, rational factors.

3. $b^2 - 4ac = 81 - 504 = -423$, not a perfect square.
Hence, no rational factors.

4. $b^2 - 4ac = 1 + 960 = 961 = 31^2$.
Hence, rational factors.

5. $b^2 - 4ac = 289 - 288 = 1 = 1^2$.
Hence, rational factors.

6. $b^2 - 4ac = 9 + 400 = 409$, not a perfect square.
Hence, no rational factors.

7. $b^2 - 4ac = 81 - 336 = -255$, not a perfect square.
Hence, no rational factors.

8. $b^2 - 4ac = 233^2 + 24 \cdot 33 = 55{,}081$, not a perfect square.
Hence, no rational factors.

9. $b^2 - 4ac = 4a^2 - 4a^2 + 4b^2 = 4b^2 = (2b)^2$.
Hence, rational factors.

10. $b^2 - 4ac = b^4 + 2b^2a^2 + a^4 - 4a^2b^2 = (b^2 - a^2)^2$.
Hence, rational factors.

12. Let $x^2 - 7x - 7 = 0$.

Then $$x = \frac{7 \pm \sqrt{49 + 28}}{2} = \frac{7 \pm \sqrt{77}}{2}.$$

Then $$x^2 - 7x - 7 = \left(x - \frac{7 + \sqrt{77}}{2}\right)\left(x - \frac{7 - \sqrt{77}}{2}\right)$$
$$= \tfrac{1}{4}(2x - 7 - \sqrt{77})(2x - 7 + \sqrt{77}).$$

13. Let $x^2 - 4x - 1 = 0$.

Then $$x = \frac{4 \pm \sqrt{16 + 4}}{2} = 2 \pm \sqrt{5}.$$

Therefore $$x^2 - 4x - 1 = (x - 2 - \sqrt{5})(x - 2 + \sqrt{5}).$$

14. Let $x^2 + 2x + 2 = 0$.

Then $$x = \frac{-2 \pm \sqrt{4 - 8}}{2} = -1 \pm \sqrt{-1}.$$

Therefore $$x^2 + 2x + 2 = (x + 1 - \sqrt{-1})(x + 1 + \sqrt{-1}).$$

15. Let $x^2 + 4x - 9 = 0$.

Then $$x = \frac{-4 \pm \sqrt{16 + 36}}{2} = -2 \pm \sqrt{13}.$$

Therefore $$x^2 + 4x - 9 = (x + 2 - \sqrt{13})(x + 2 + \sqrt{13}).$$

16. Let $4x^2 - 12x - 9 = 0$.

Then $$x = \frac{12 \pm \sqrt{144 + 144}}{8} = \frac{3 \pm 3\sqrt{2}}{2}.$$

Therefore $$4x^2 - 12x - 9 = 4\left(x - \frac{3 + 3\sqrt{2}}{2}\right)\left(x - \frac{3 - 3\sqrt{2}}{2}\right)$$
$$= (2x - 3 - 3\sqrt{2})(2x - 3 + 3\sqrt{2}).$$

17. Let $25x^2 + 20x + 4 = 0.$

Then
$$x = \frac{-20 \pm \sqrt{400 - 400}}{50}$$
$$= -\tfrac{2}{5}, -\tfrac{2}{5}.$$

Therefore $25x^2 + 20x + 40 = (5x + 2)^2.$

18. Let $6x^2 + 14x - 40 = 0.$

Then
$$x = \frac{-14 \pm \sqrt{196 + 960}}{12} = \frac{-14 \pm 34}{12}.$$
$$= -4, \tfrac{5}{3}.$$

Therefore $6x^2 + 14x - 40 = 2(x + 4)(3x - 5).$

19. Let $10 - 9x - 9x^2 = 0.$

Then
$$x = \frac{9 \pm \sqrt{81 + 360}}{-18} = \frac{9 \pm 21}{-18}$$
$$= -\tfrac{5}{3}, \tfrac{2}{3}.$$

Therefore $10 - 9x - 9x^2 = (5 + 3x)(2 - 3x).$

20. Let $10x^2 + 12 - 26x = 0.$

Then
$$x = \frac{26 \pm \sqrt{676 - 480}}{20} = \frac{26 \pm 14}{20}$$
$$= 2, \tfrac{3}{5}.$$

Therefore $10x^2 + 12 - 26x = 2(x - 2)(5x - 3).$

21. Let $x^2 + 7x + 8 = 0.$

Then
$$x = \frac{-7 \pm \sqrt{17}}{2}.$$

Therefore
$$x^2 + 7x + 8 = \left(x - \frac{-7 + \sqrt{17}}{2}\right)\left(x - \frac{-7 - \sqrt{17}}{2}\right)$$
$$= \tfrac{1}{4}(2x + 7 - \sqrt{17})(2x + 7 + \sqrt{17}).$$

22. Let $x^2 + x + 1 = 0.$

Then
$$x = \frac{-1 \pm \sqrt{-3}}{2}.$$

And
$$x^2 + x + 1 = \left(x - \frac{-1 + \sqrt{-3}}{2}\right)\left(x - \frac{-1 - \sqrt{-3}}{2}\right)$$
$$= \tfrac{1}{4}(2x + 1 - \sqrt{-3})(2x + 1 + \sqrt{-3}).$$

23. Let $x^2 + 1 = 0.$

Then $x = \pm\sqrt{-1}.$

Therefore $x^2 + 1 = (x - \sqrt{-1})(x + \sqrt{-1}).$

24. Let $x^2 + 9 = 0.$

Then $x = \pm 3\sqrt{-1}.$

Therefore $x^2 + 9 = (x - 3\sqrt{-1})(x + 3\sqrt{-1}).$

25. Let $x^2 - 2ax + a^2 - b = 0.$

Then $$x = \frac{2a \pm \sqrt{4a^2 - 4a^2 + 4b}}{2} = a \pm \sqrt{b}.$$

Therefore $x^2 - 2ax + a^2 - b = (x - a - \sqrt{b})(x - a + \sqrt{b}).$

26. Let $x^2 + 6ax + 9a^2 - 4b = 0.$

Then $$x = \frac{-6a \pm \sqrt{36a^2 - 36a^2 + 16b}}{2}$$
$$= -3a \pm 2\sqrt{b}.$$

Therefore $x^2 + 6ax + 9a^2 - 4b = (x + 3a - 2\sqrt{b})(x + 3a + 2\sqrt{b}).$

27. Let $4x^2 + 4ax + a^2 - 4c = 0.$

Then $$x = \frac{-4a \pm \sqrt{16a^2 - 16a^2 + 64c}}{8}$$
$$= \frac{-a \pm 2\sqrt{c}}{2}.$$

Therefore $4x^2 + 4ax + a^2 - 4c = (2x + a - 2\sqrt{c})(2x + a + 2\sqrt{c}).$

28. Let $x^2 - 4ax + 4a^2 + c = 0.$

Then $$x = \frac{4a \pm \sqrt{16a^2 - 16a^2 - 4c}}{2}$$
$$= 2a \pm \sqrt{-c}.$$

Therefore $x^2 - 4ax + 4a^2 + c = (x - 2a - \sqrt{-c})(x - 2a + \sqrt{-c}).$

29. Let $ax^2 + bx + c = 0.$

Then $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Therefore
$$ax^2 + bx + c = a\left(x - \frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(x - \frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$
$$= \frac{a}{4a^2}(2ax + b - \sqrt{b^2 - 4ac})(2ax + b + \sqrt{b^2 - 4ac})$$
$$= \frac{1}{4a}(2ax + b - \sqrt{b^2 - 4ac})(2ax + b + \sqrt{b^2 - 4ac}).$$

31. Let $3x^2 - 6xy + 14x - 4y + 8 = 3x^2 - (6y - 14)x - (4y - 8) = 0.$

Then $$x = \frac{6y - 14 \pm \sqrt{36y^2 - 168y + 196 + 48y - 96}}{6}$$
$$= \frac{6y - 14 \pm \sqrt{36y^2 - 120y + 100}}{6}$$
$$= \frac{6y - 14 \pm (6y - 10)}{6}$$
$$= 2y - 4,\ -\tfrac{2}{3}.$$

Therefore $3x^2 - 6xy + 14x - 4y + 8 = (x - 2y + 4)(3x + 2).$

32. Let $x^2 - xy - 2y^2 + 3x - 6y = x^2 - (y-3)x - (2y^2 + 6y) = 0.$

Then
$$x = \frac{y - 3 \pm \sqrt{y^2 - 6y + 9 + 8y^2 + 24y}}{2}$$
$$= \frac{y - 3 \pm \sqrt{9y^2 + 18y + 9}}{2}$$
$$= \frac{y - 3 \pm (3y + 3)}{2}$$
$$= 2y,\ -y - 3.$$

Therefore $x^2 - xy - 2y^2 + 3x - 6y = (x - 2y)(x + y + 3).$

33. Let $x^2 - 4xy - y + 3y^2 - 2 - x = x^2 - (4y+1)x + (3y^2 - y - 2) = 0.$

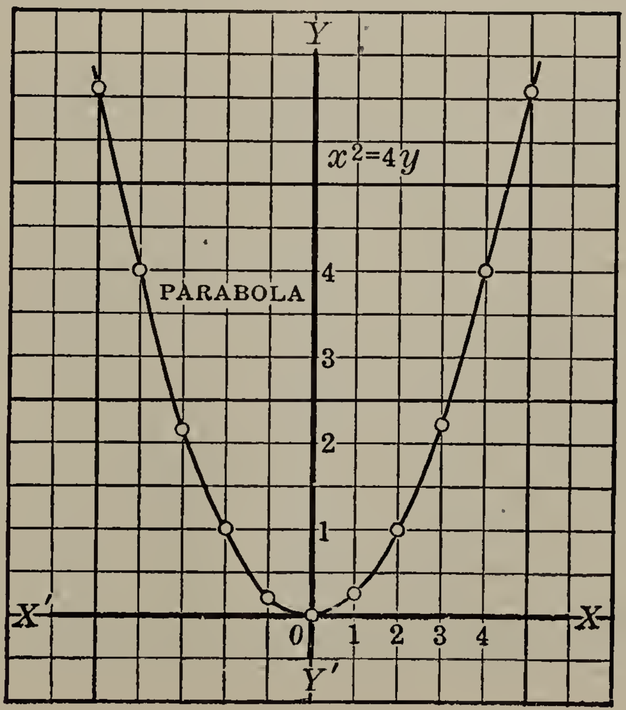
Then
$$x = \frac{4y + 1 \pm \sqrt{16y^2 + 8y + 1 - 12y^2 + 4y + 8}}{2}$$
$$= \frac{4y + 1 \pm \sqrt{4y^2 + 12y + 9}}{2}$$
$$= \frac{4y + 1 \pm (2y + 3)}{2}$$
$$= 3y + 2,\ y - 1.$$

Therefore $x^2 - 4xy - y + 3y^2 - 2 - x = (x - 3y - 2)(x - y + 1).$

Page 409

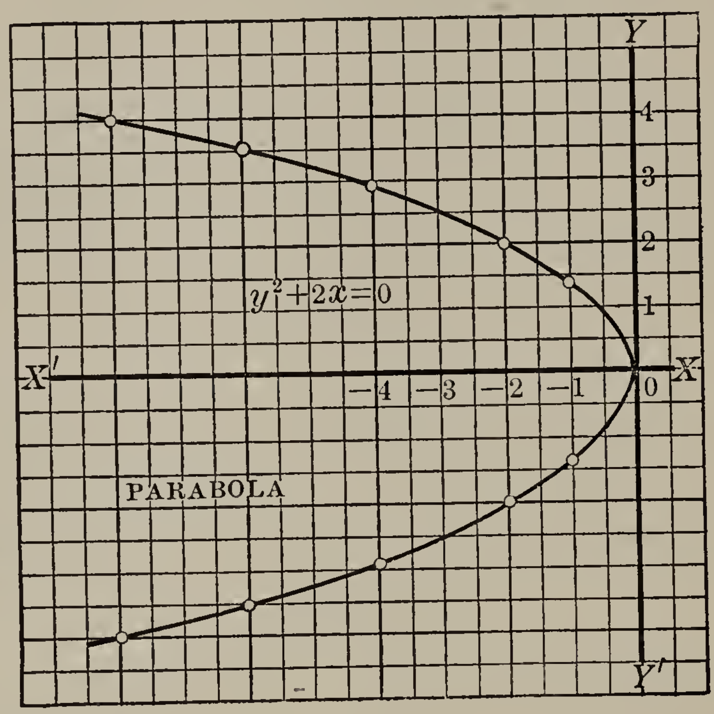
1. $x^2 = 4y$. Solving for y, $y = \dfrac{x^2}{4}$.

If $x =$	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
then $y =$	9	6.25	4	2.25	1	.25	0	.25	1	2.25	4	6.25	9

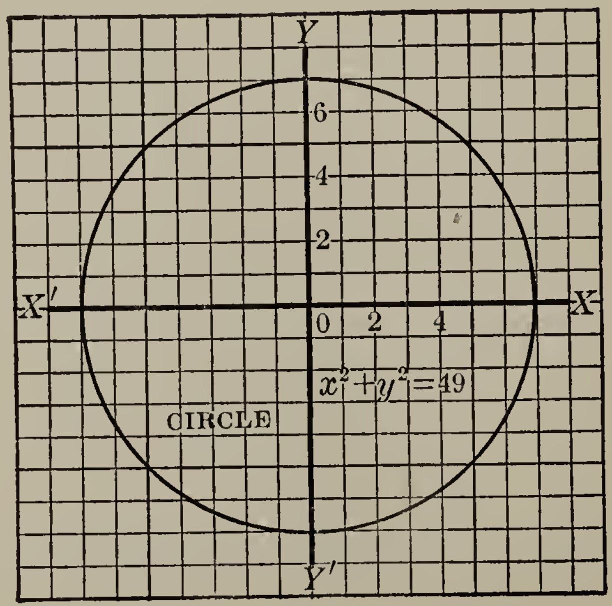


2. $y^2 + 2x = 0$. Solving for y, $y = \pm\sqrt{-2x}$.

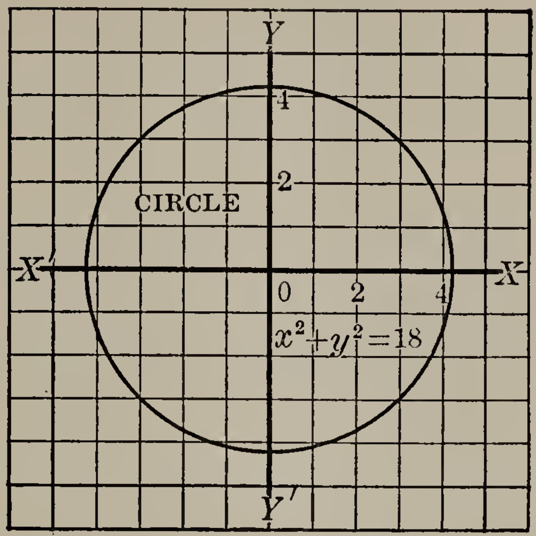
If $x =$	-8	-6	-4	-2	-1	0	Positive values
then $y =$	± 4	± 3.46	± 2.82	± 2	± 1.41	0	Imaginary



3. $x^2 + y^2 = 49$.
Circle, center at (0, 0), and $r = 7$.

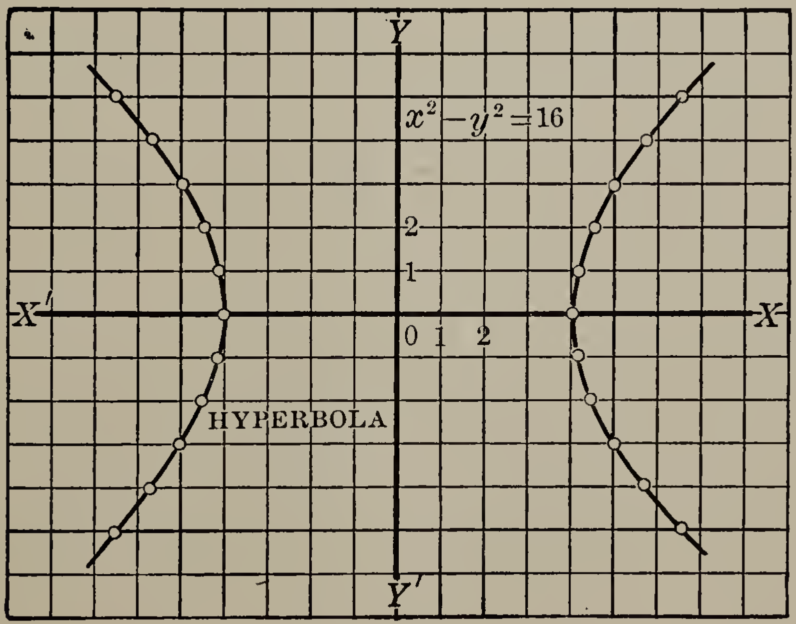


4. $x^2 + y^2 = 18.$
Circle, center at (0, 0), and $r = 4.2$.



5. $x^2 - y^2 = 16$. Solving for x, $x = \pm\sqrt{16 + y^2}$.

If $y =$	± 5	± 4	± 3	± 2	± 1	0	± 1	± 2	± 3	± 4	± 5
then $x =$	-6.4	-5.6	-5	-4.4	-4.123	± 4	4.123	4.4	5	5.6	6.4

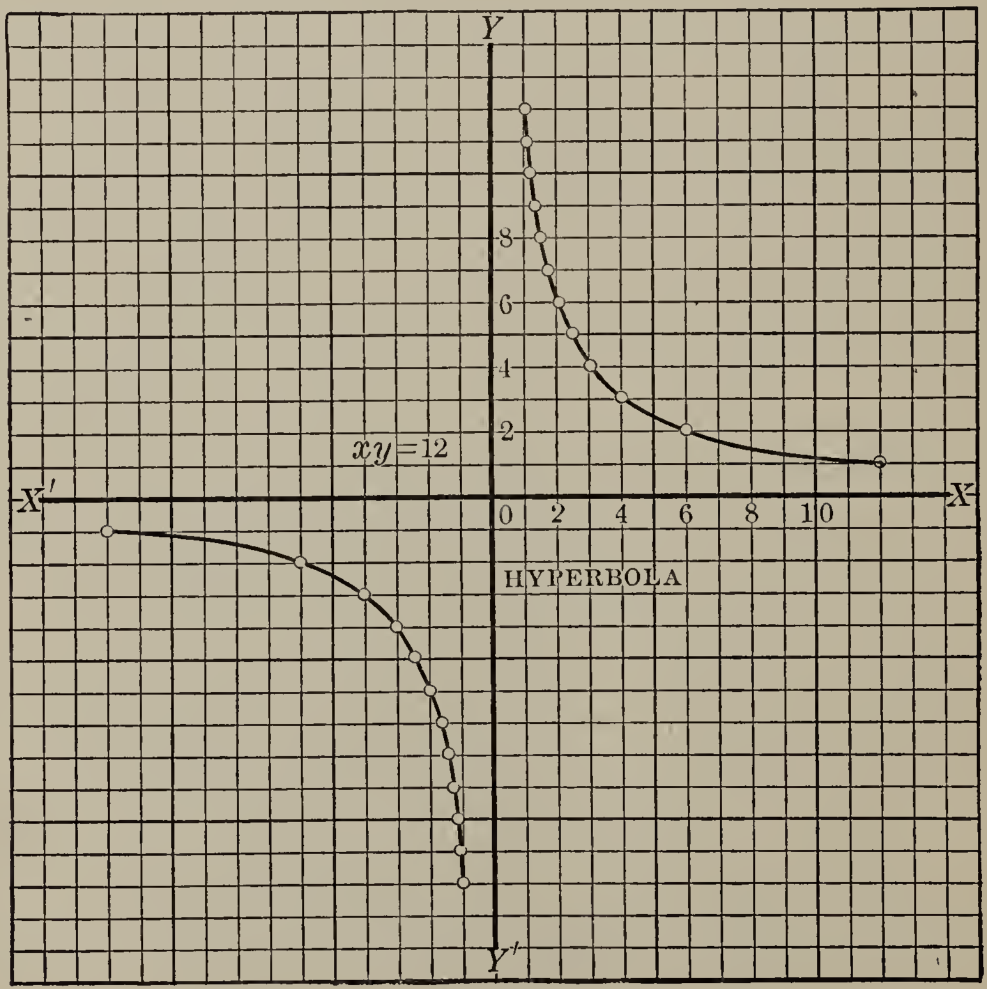


6. $xy = 12$. Solving for x, $x = \dfrac{12}{y}$.

If $y =$	-12	-11	-10	-9	-8	-7
then $x =$	-1	-1.09	-1.2	-1.3	-1.5	-1.7

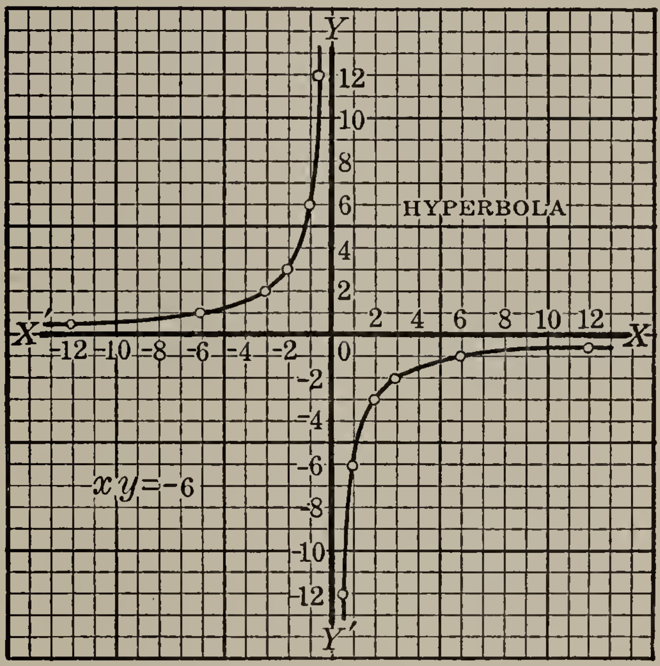
If $y =$	-6	-5	-4	-3	-2	-1
then $x =$	-2	-2.4	-3	-4	-6	-12

If $y =$	1	2	3	4	5	6	7	8	9	10	11	12
then $x =$	12	6	4	3	2.4	2	1.7	1.5	1.3	1.2	1.09	1



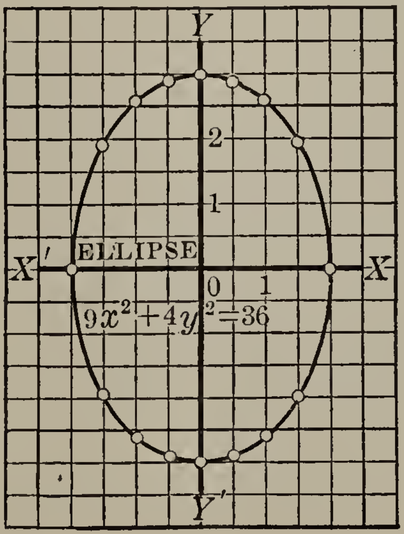
7. $xy = -6$. Solving for y, $y = -\frac{6}{x}$.

If $x =$	12	6	3	2	1	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-2	-3	-6	-12
then $y =$	$-\frac{1}{2}$	-1	-2	-3	-6	-12	12	6	3	2	1	$\frac{1}{2}$



8. $9x^2 + 4y^2 = 36$. Solving for y, $y = \pm \frac{3}{2}\sqrt{4 - x^2}$.

If $x =$	-2	-1.5	-1	0	1	1.5	2
then $y =$	0	± 1.98	± 2.59	± 3	± 2.59	± 1.98	0

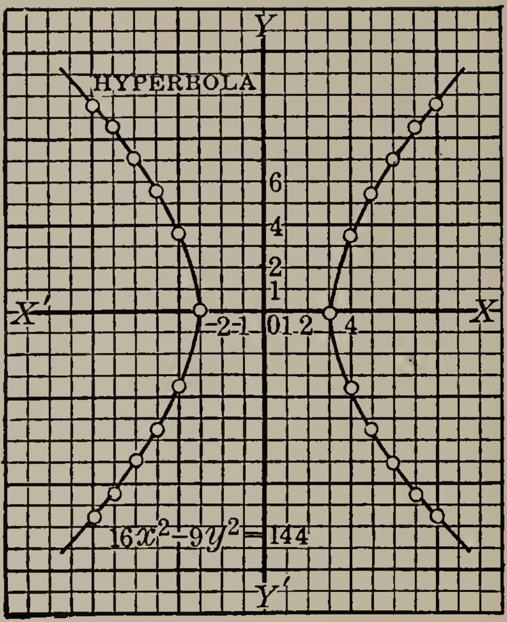


9. $16x^2 - 9y^2 = 144$. Solving for y, $y = \pm \frac{4}{3}\sqrt{x^2 - 9}$.

If $x =$	-8	-7	-6
then $y =$	± 9.88	± 8.43	± 6.92

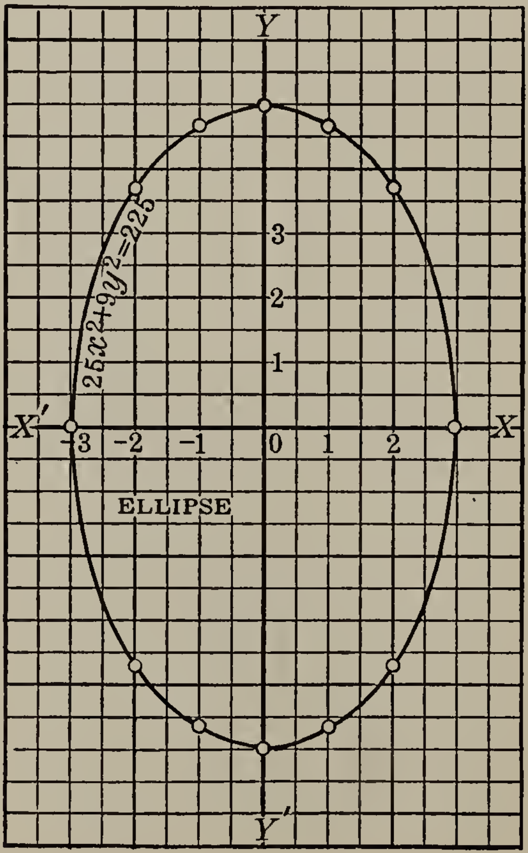
If $x =$	-5	-4	± 3	4
then $y =$	± 5.33	± 3.52	0	± 3.52

If $x =$	5	6	7	8
then $y =$	± 5.33	± 6.92	± 8.43	± 9.88



10. $25x^2 + 9y^2 = 225$. Solving for y, $y = \pm \frac{5}{3}\sqrt{9 - x^2}$.

If $x =$	-3	-2	-1	0	1	2	3
then $y =$	0	± 3.72	± 4.71	± 5	± 4.71	± 3.72	0



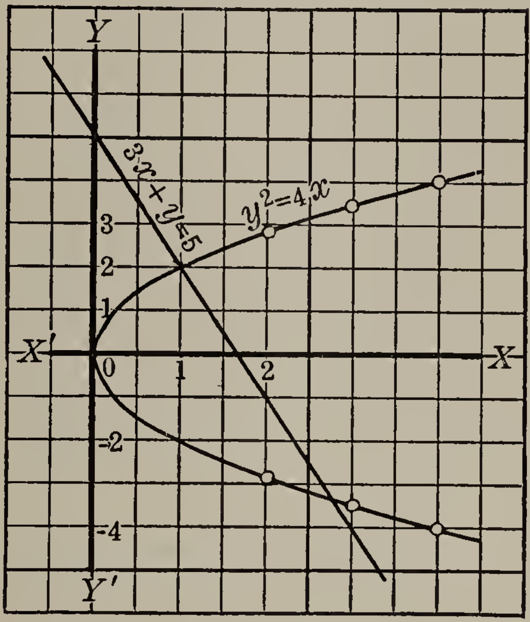
Page 411

1. $y^2 = 4x$. Solving for y, $y = \pm 2\sqrt{x}$.

If $x =$	0	1	2	3	4
then $y =$	0	± 2	± 2.8	± 3.4	± 4

$3x + y = 5$.

If $x =$	0	1
then $y =$	5	2



From the graphs

If $x =$	1	2.8
then $y =$	2	-3.4

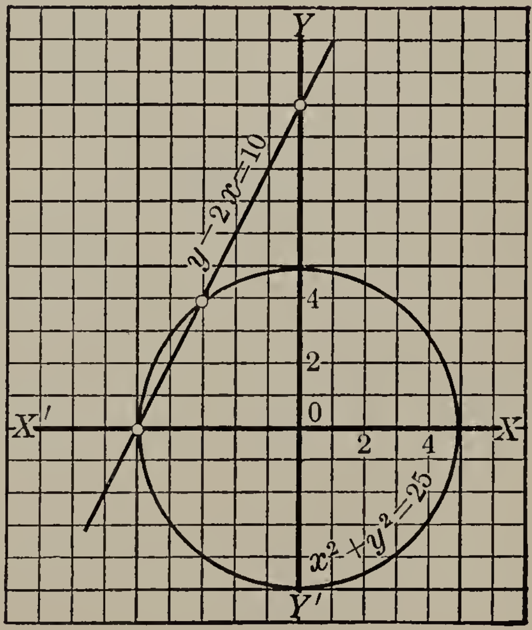
2. $x^2 + y^2 = 25$.
Circle, center (0, 0), and $r = 5$.

$y - 2x = 10$.

If $x =$	0	-5
then $y =$	10	0

From the graphs

If $x =$	-3	-5
then $y =$	4	0



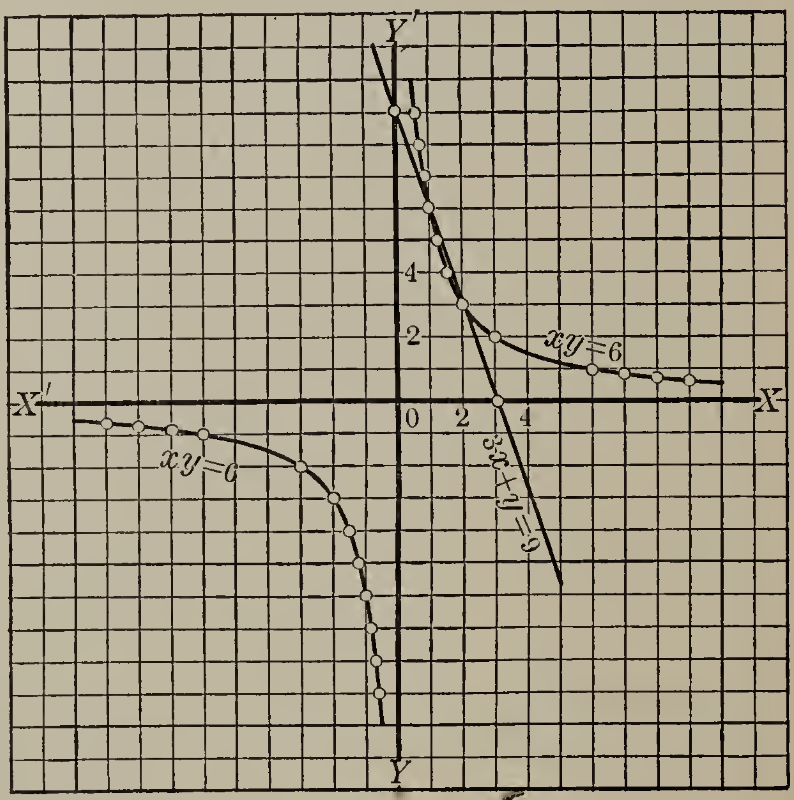
3. $xy = 6$. Solving for y, $y = \frac{6}{x}\cdot$

If $x =$	-9	-8	-6	-3	-2	$-\frac{3}{2}$	-1.2	-1	$-\frac{2}{3}$
then $y =$	$-\frac{2}{3}$	$-\frac{3}{4}$	-1	-2	-3	-4	-5	-6	-9

If $x =$	$\frac{2}{3}$	1	1.2	1.5	2	3	6	8	9
then $y =$	9	6	5	4	3	2	1	$\frac{3}{4}$	$\frac{2}{3}$

$3x + y = 9$.

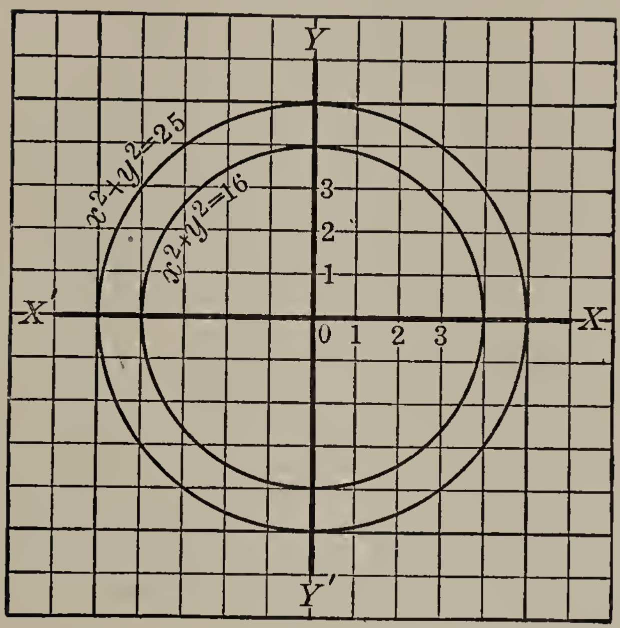
If $x =$	0	3
then $y =$	9	0



From the graphs

If $x =$	2	1
then $y =$	3	6

4. $x^2 + y^2 = 25$.
Circle, center at (0, 0), and $r = 5$.
$x^2 + y^2 = 16$.
Circle, center at (0, 0), and $r = 4$.



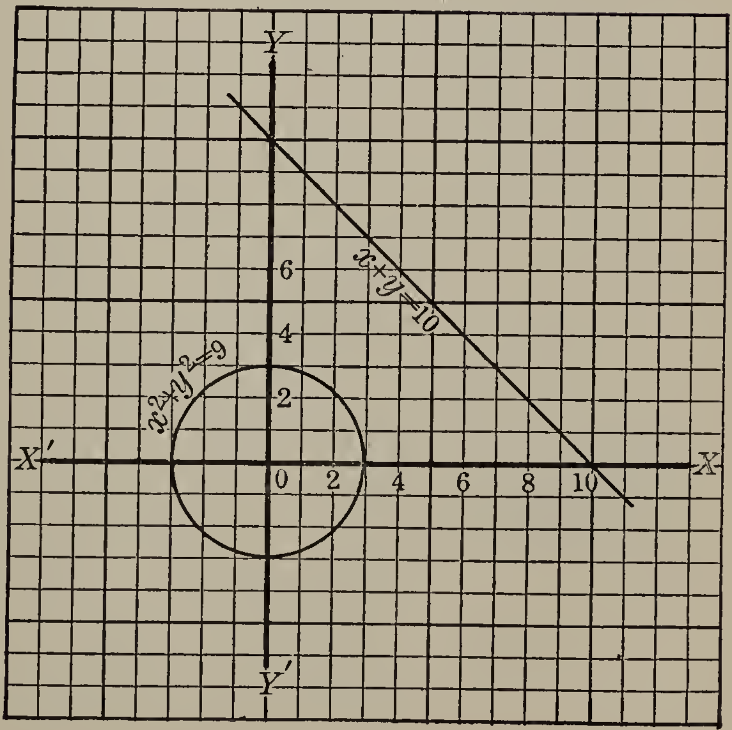
The graphs do not intersect.
Hence, no real roots.

5. $x^2 + y^2 = 9$.
Circle, center at (0, 0), and $r = 3$.

$x + y = 10$.

If $x =$	0	10
then $y =$	10	0

The graphs do not intersect.
Hence, no real roots.

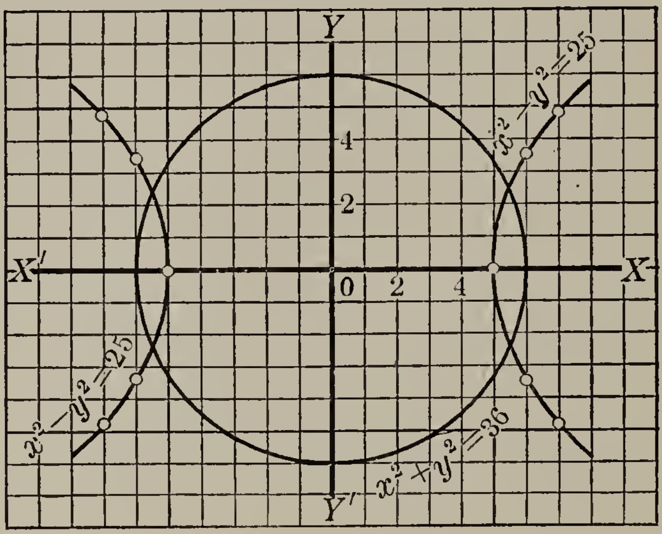


6. $x^2 + y^2 = 36$.

Circle, center at (0, 0), and $r = 6$.

$x^2 - y^2 = 25$. Solving for y, $y = \pm\sqrt{x^2 - 25}$.

If $x =$	-7	-6	-5	5	6	7
then $y =$	± 4.89	± 3.31	0	0	± 3.31	± 4.89



From the graphs

If $x =$	5.5	-5.5	5.5	-5.5
then $y =$	2.5	2.5	-2.5	-2.5

7. $x^2 + y^2 = 9$.

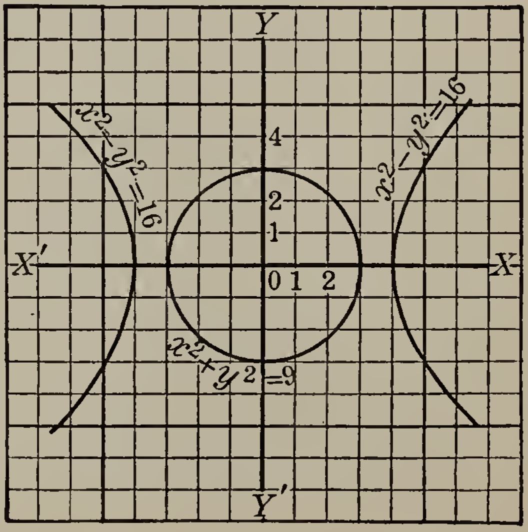
Circle, center at (0, 0), and $r = 3$.

$x^2 - y^2 = 16$. Solving for y, $y = \pm\sqrt{x^2 - 16}$.

If $x =$	-6	-5	-4	4	5	6
then $y =$	± 4.47	± 3	0	0	± 3	± 4.47

The graphs do not intersect.

Hence, no real roots.



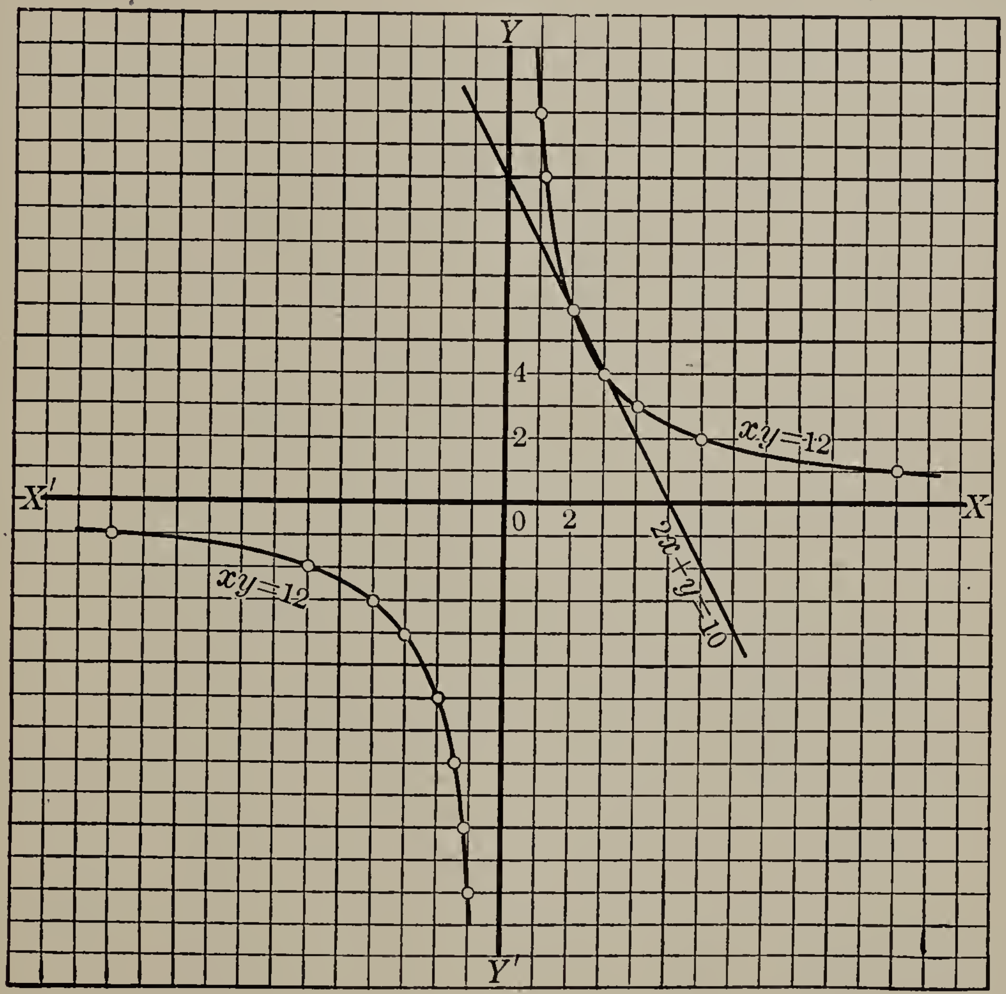
8. $xy = 12$. Solving for y, $y = \frac{12}{x}\cdot$

If $x =$	12	6	4	3	2	1.5	1
then $y =$	1	2	3	4	6	8	12

If $x =$	-1	-1.5	-2	-3	-4	-6	-12
then $y =$	-12	-8	-6	-4	-3	-2	-1

$2x + y = 10$.

If $x =$	0	5
then $y =$	10	0



From the graphs

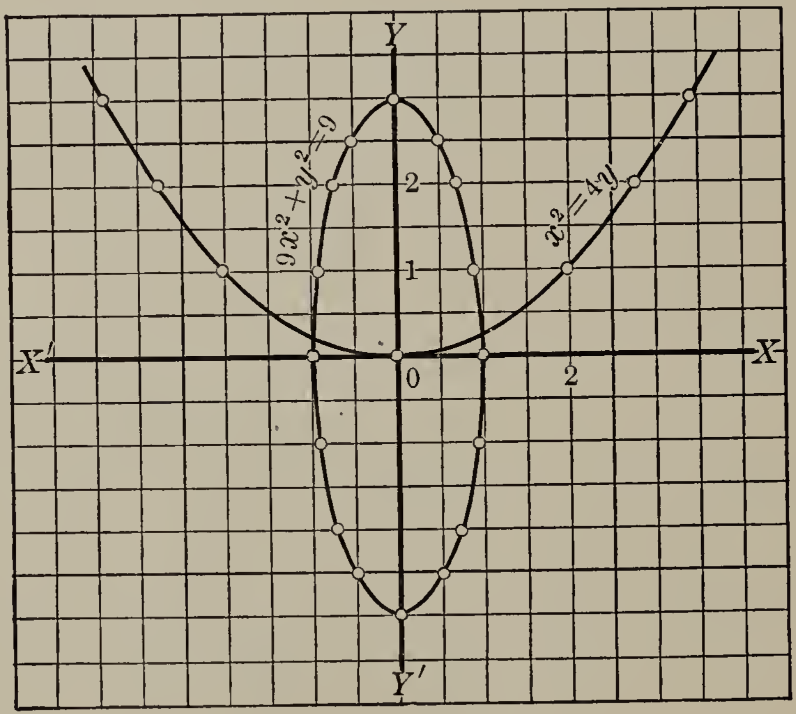
If $x =$	2	3
then $y =$	6	4

9. $x^2 = 4y$. Solving for x, $x = \pm 2\sqrt{y}$.

If $y =$	4	3	2	1	0
then $x =$	± 4	± 3.46	± 2.82	± 2	0

$9x^2 + y^2 = 9$. Solving for x, $x = \pm \frac{1}{3}\sqrt{9 - y^2}$.

If $y =$	-3	-2.5	-2	-1	0	1	2	2.5	3
then $x =$	0	$\pm .55$	$\pm .74$	$\pm .94$	± 1	$\pm .94$	$\pm .74$	$\pm .55$	0



From the graphs

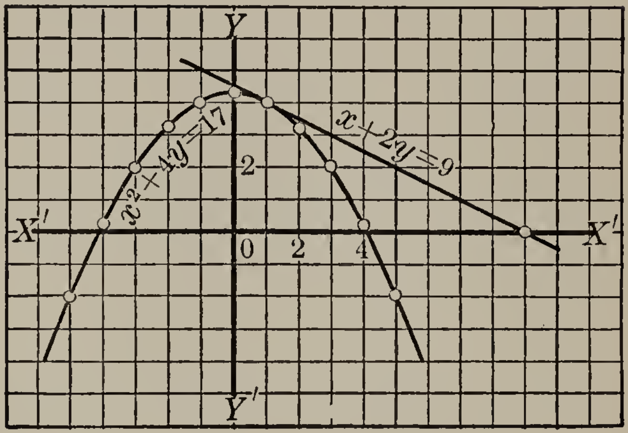
If $x =$	1	-1
then $y =$	$\frac{1}{4}$	$\frac{1}{4}$

10. $x^2 + 4y = 17$. Solving for y, $y = \dfrac{17 - x^2}{4}$.

If $x =$	± 5	± 4	± 3	± 2	± 1	0
then $y =$	-2	.25	2	3.25	4	4.25

$x + 2y = 9$.

If $x =$	9	0
then $y =$	0	$4\frac{1}{2}$



From the graphs

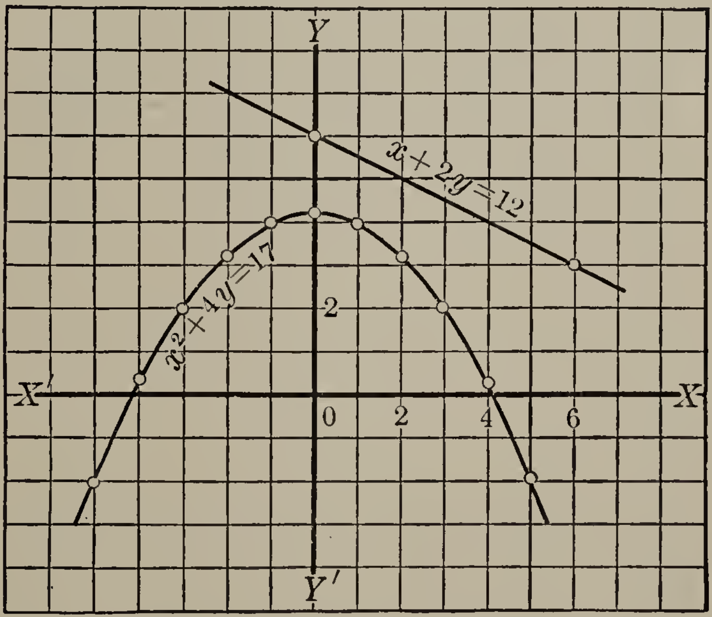
If $x =$	1
then $y =$	4

11. $x^2 + 4y = 17$. Solving for y, $y = \dfrac{17 - x^2}{4}$.

If $x =$	± 5	± 4	± 3	± 2	± 1	0
then $y =$	-2	.25	2	3.25	4	4.25

$x + 2y = 12$.

If $x =$	0	6
then $y =$	6	3



The graphs do not intersect. Hence, no real roots.

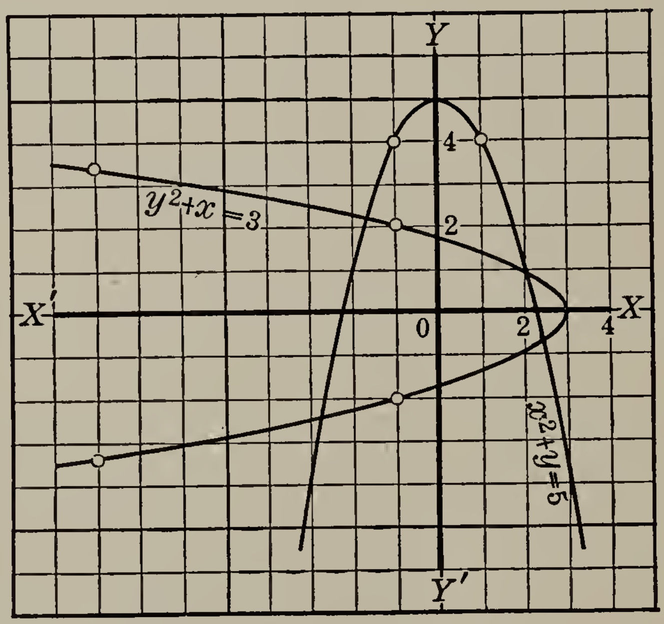
CR

12. $x^2 + y = 5$. Solving for y, $y = 5 - x^2$.

If $x =$	0	± 1	± 2	± 3	± 4
then $y =$	5	4	1	-4	-11

$y^2 + x = 3$. Solving for y, $y = \pm\sqrt{3 - x}$.

If $x =$	-13	-8	-6	-1	2	3
then $y =$	± 4	± 3.3	± 3	± 2	± 1	0



From the graphs

If $x =$	2	2.4	-1.7	-2.7
then $y =$	1	$-.7$	2.2	-2.4

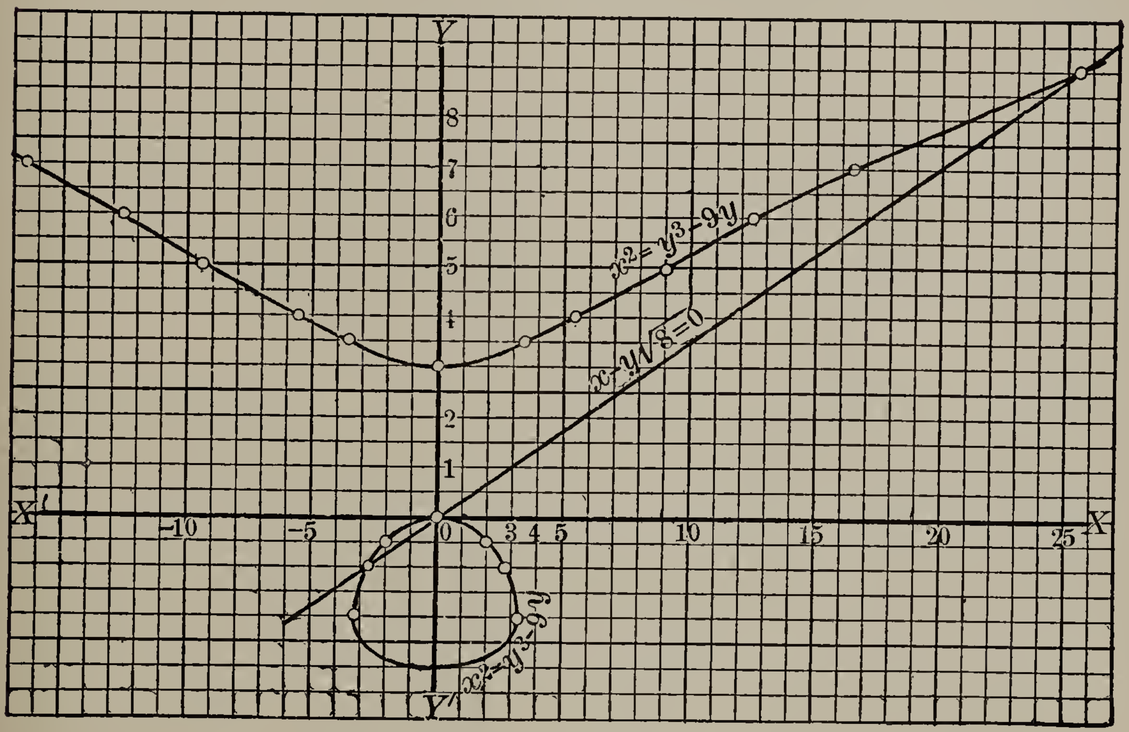
13. $x - y\sqrt{8} = 0$. Solving for x, $x = y\sqrt{8}$.

If $y =$	0	-1	8
then $x =$	0	-2.8	22.6

$x^2 = y^3 - 9y$. Solving for x, $x = \pm\sqrt{y^3 - 9y}$.

If $y =$	-3	$-\frac{5}{2}$	-2	-1	$-\frac{1}{2}$	0	3
then $x =$	0	± 2.6	± 3.16	± 2.82	± 2.09	0	0

If $y =$	3.5	4	5	6	7	8
then $x =$	± 3.37	± 5.29	± 8.94	± 12.72	± 16.7	± 20.9



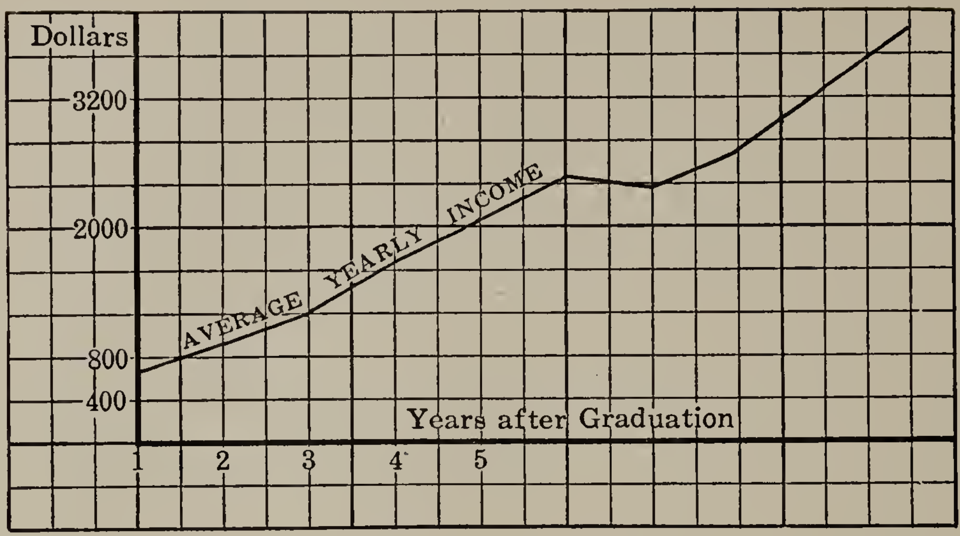
From the graphs

If $x =$	0	-2.8	25.4
then $y =$	0	-1	9

Page 413

1.

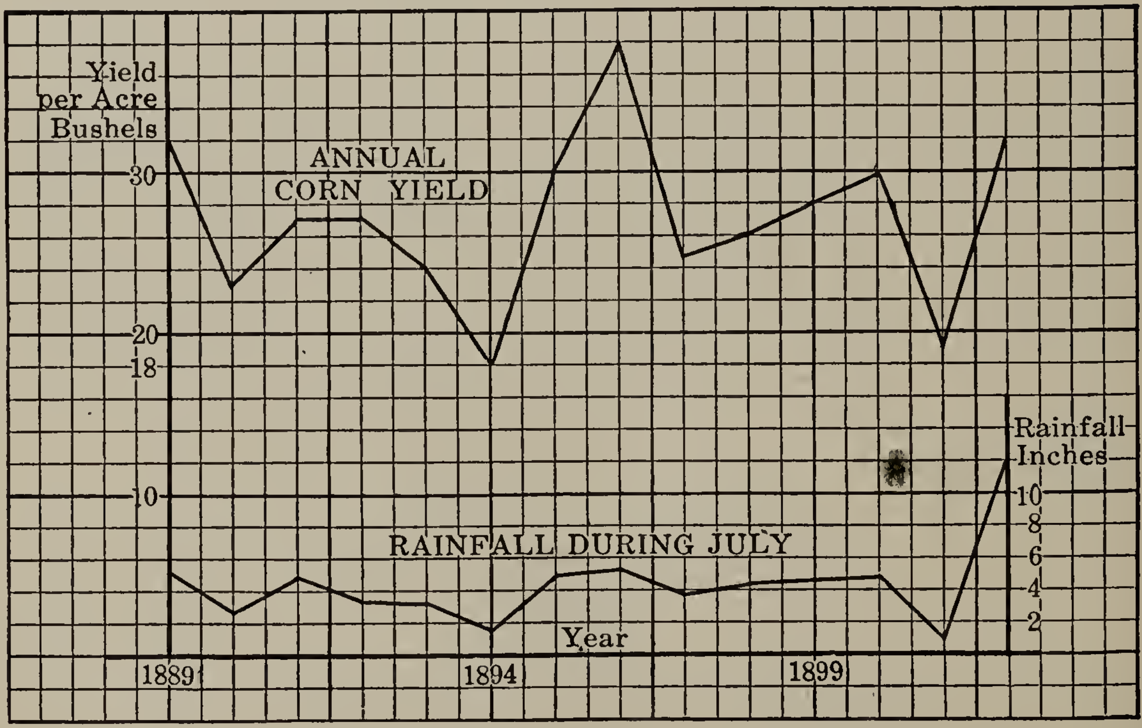
Years after graduation	1	2	3	4	5	6	7	8	9	10
Income in dollars . . .	706	902	1199	1651	2039	2408	2382	2709	3222	3804



The annual income increases almost directly with the years after graduation.

2.

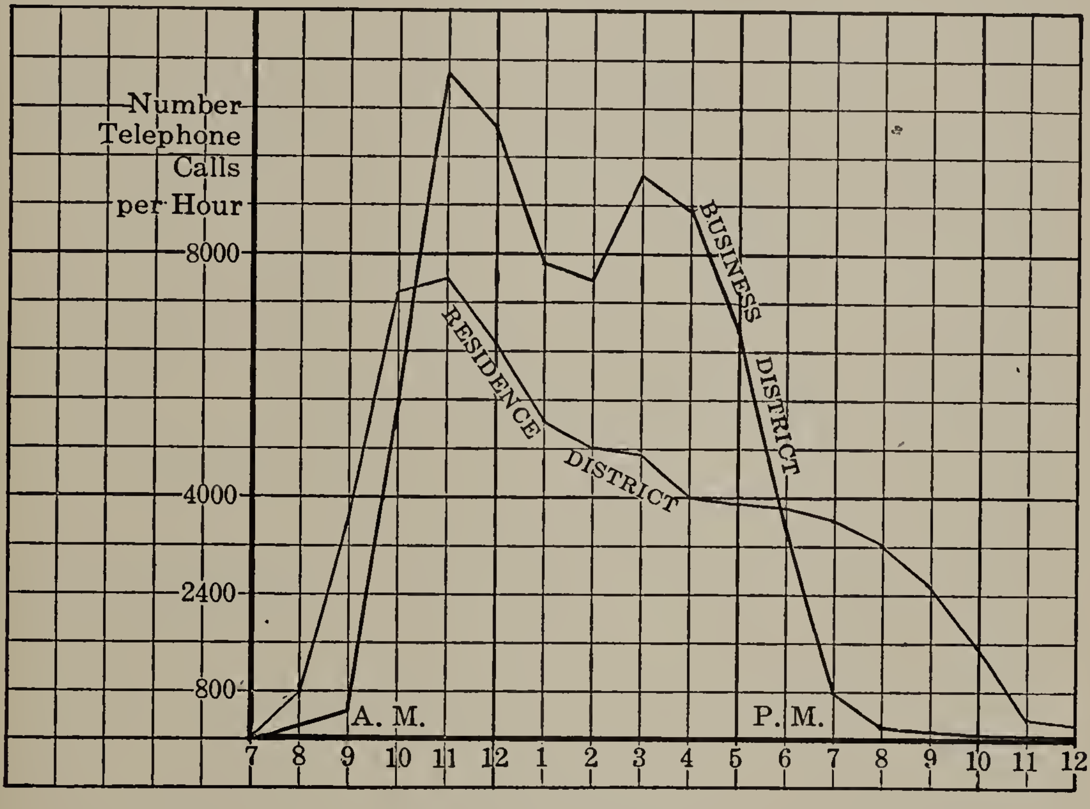
Year	'89	'90	'91	'92	'93	'94	'95	'96	'97	'98	'99	'00	'01	'02
Rainfall in inches	5.4	2.6	5.1	3.7	3.4	1.9	4.8	5.4	3.7	4.3	4.6	4.7	1.2	6.0
Corn yield in bushels	32	23	27	27	24	18	30	38	25	26	28	30	19	32



An increased corn yield accompanies an increased rainfall, and vice versa. The largest crops are harvested in years of greatest July rainfall.

3.

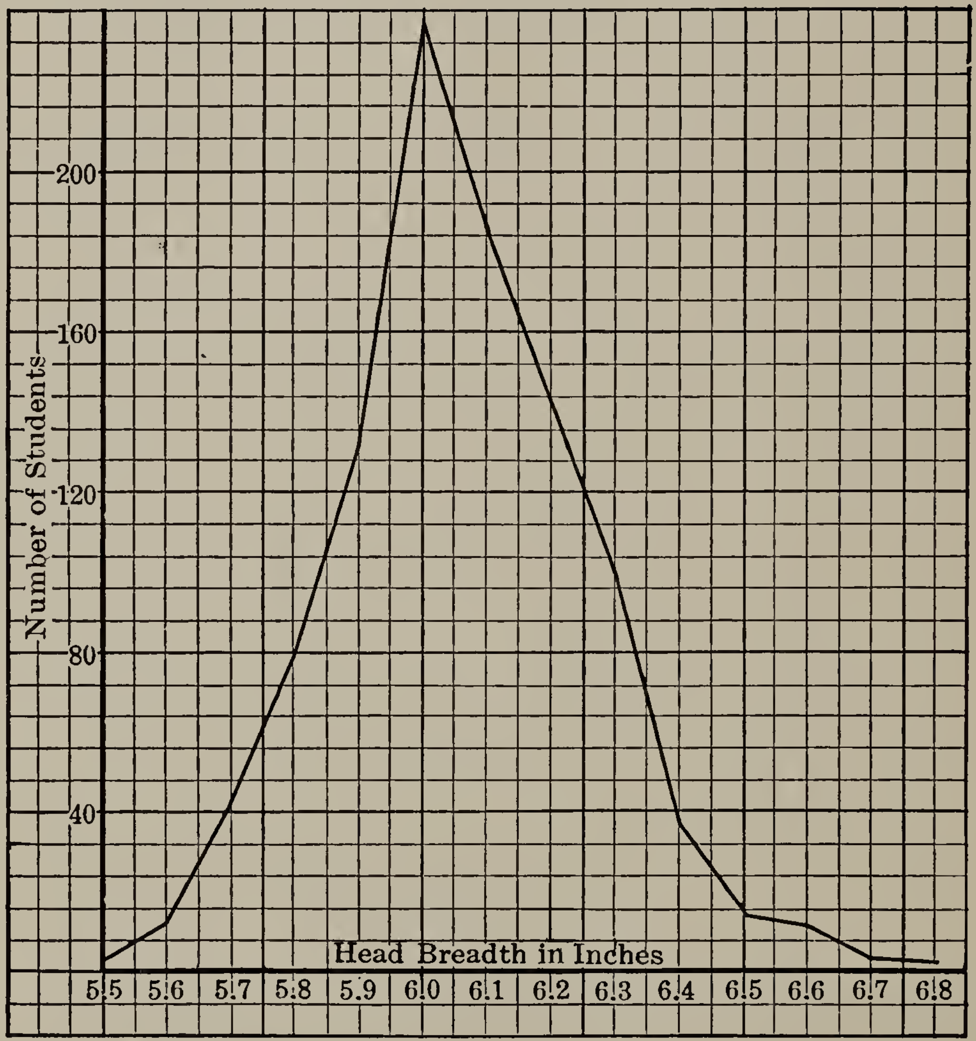
Telephone Calls per Hour in Hundreds																		
Time of day	7	8	9	10	11	Noon	1	2	3	4	5	6	7	8	9	10	11	12
Business district .	1	2	4	55	108	101	78	75	93	87	67	35	8	2	1.5	1	1	1
Residence district	.25	8	35	73	76	65	52	48	47	40	38	38	36	32	25	15	4	3



The number of calls in the residence district is greater than the number in the business district in the early morning and in the evening hours. In the business district the curve shows a decreased number of calls during the lunch hours. In both curves the peak load is at 11 A.M.

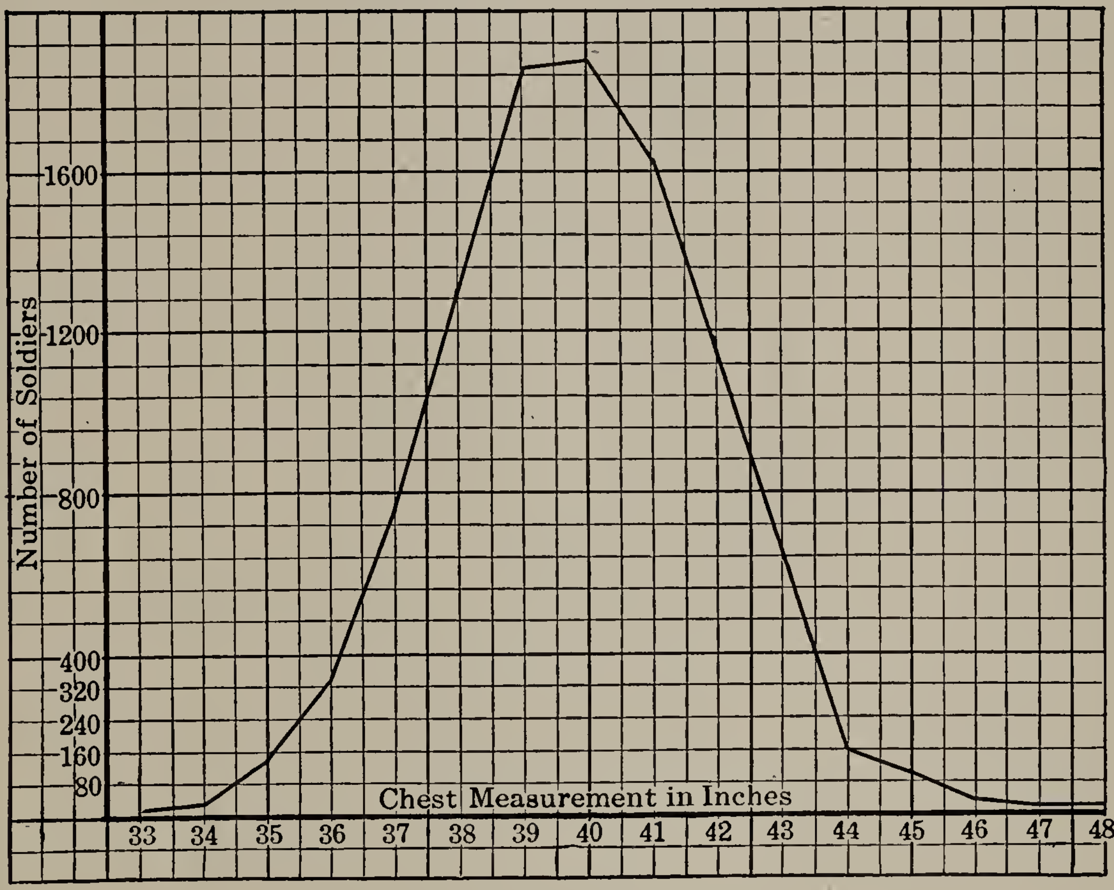
4.

Head breadth in inches	5.5	5.6	5.7	5.8	5.9	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8
Number of students	3	12	43	80	131	236	185	142	99	37	15	12	3	2



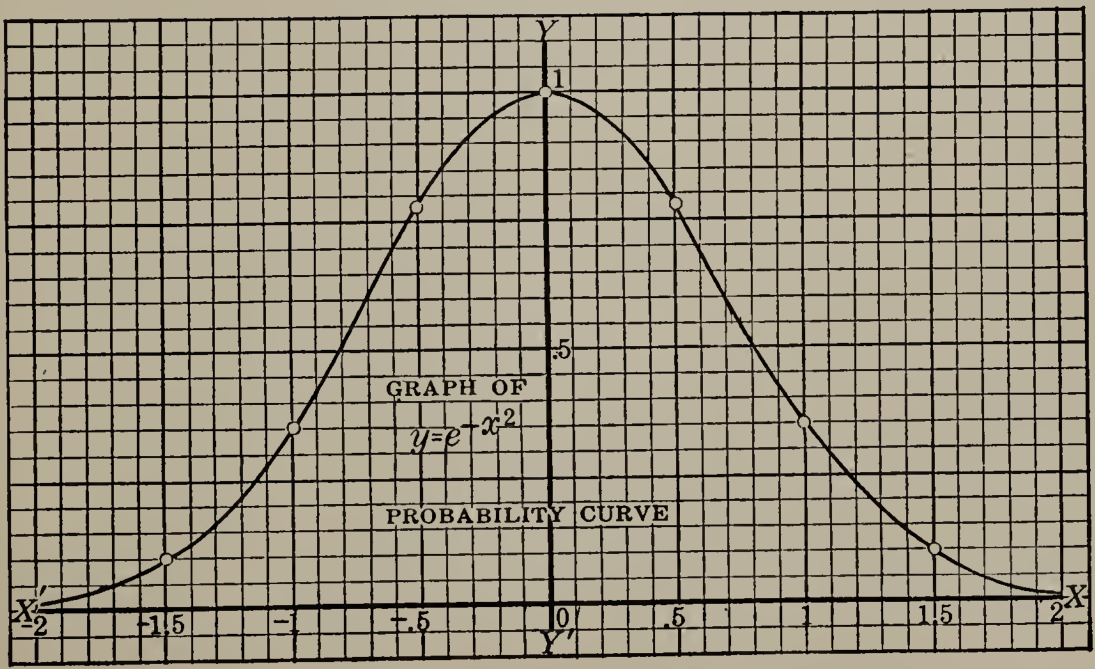
5.

Chest measurement in inches	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
Number of soldiers	5	31	141	322	732	1305	1867	1882	1628	1148	645	160	87	38	7	2



6. For $y = e^{-x^2}$ in which $e = 2.7$.

If $x =$	0	$\pm .5$	± 1	± 1.5	± 2
then $y =$	1	.78	.35	.11	.02



Page 417

1. $$x + y = 5, \quad (1)$$
$$x^2 + y^2 = 13. \quad (2)$$
From (1), $$x = 5 - y. \quad (3)$$
Substituting $(5 - y)$ for x in (2),
$$(5 - y)^2 + y^2 = 13. \quad (4)$$
From (4), $$y^2 - 5y + 6 = 0. \quad (5)$$
From (5), $$y = 3,\ 2. \quad (6)$$
From (6) and (3), $$x = 2,\ 3.$$

2. $$x^2 - y^2 = 8, \quad (1)$$
$$2x + y = 7. \quad (2)$$
From (2), $$y = 7 - 2x. \quad (3)$$
Substituting $(7 - 2x)$ for y in (1),
$$x^2 - (-2x + 7)^2 = 8. \quad (4)$$
From (4), $$3x^2 - 28x + 57 = 0. \quad (5)$$
From (5), $$x = \frac{28 \pm \sqrt{784 - 4(3)(57)}}{6}$$
$$= +\tfrac{19}{3},\ 3. \quad (6)$$
From (6) and (3), $$y = -\tfrac{17}{3},\ 1.$$

3. $$x^2 + xy = 15, \quad (1)$$
$$x + y = 3. \quad (2)$$
From (2), $$y = 3 - x. \quad (3)$$
Substituting $(3 - x)$ for y in (1),
$$x^2 + x(3 - x) = 15. \quad (4)$$
Whence $$x = 5. \quad (5)$$
From (5) and (3), $$y = -2.$$

4. $$x^2 + 2xy = 21, \quad (1)$$
$$x + 2y = 3. \quad (2)$$
From (2), $$y = \frac{3 - x}{2}. \quad (3)$$
Substituting $\frac{3 - x}{2}$ for y in (1),
$$x^2 + 2x\left(\frac{3 - x}{2}\right) = 21. \quad (4)$$
Whence $$x = 7. \quad (5)$$
From (5) and (3), $$y = -2.$$

5.

$$x^2 + 2y = 17, \quad (1)$$
$$2x - y = 2. \quad (2)$$

From (2), $y = 2x - 2.$ (3)

Substituting $(2x - 2)$ for y in (1),

$$x^2 + 2(2x - 2) = 17. \quad (4)$$

Whence $x = -7, 3.$ (5)

From (5) and (3), $y = -16, 4.$

6.

$$y^2 - 10x = 6, \quad (1)$$
$$x + 3y = 13. \quad (2)$$

From (2), $x = 13 - 3y.$ (3)

Substituting $(13 - 3y)$ for x in (1),

$$y^2 - 10(13 - 3y) = 6. \quad (4)$$
$$y^2 + 30y - 136 = 0. \quad (5)$$

Whence $y = \dfrac{-30 \pm \sqrt{900 + 544}}{2}$ (6)

$= 4, -34.$ (7)

From (7) and (3), $x = 1, +115.$

7.

$$y^2 - 2x^2 = 7, \quad (1)$$
$$2y + 3x = 1. \quad (2)$$

From (2), $y = \dfrac{1 - 3x}{2}.$ (3)

Substituting $\dfrac{1 - 3x}{2}$ for y in (1),

$$\left(\frac{1 - 3x}{2}\right)^2 - 2x^2 = 7. \quad (4)$$
$$x^2 - 6x - 27 = 0. \quad (5)$$

Whence $x = -3, +9.$ (6)

From (6) and (3), $y = +5, -13.$

8.

$$2x^2 - xy = 70, \quad (1)$$
$$x + 4y = 23. \quad (2)$$

From (2), $y = \dfrac{23 - x}{4}.$ (3)

Substituting $\dfrac{23 - x}{4}$ for y in (1),

$$2x^2 - x\left(\frac{23 - x}{4}\right) = 70. \quad (4)$$
$$9x^2 - 23x - 280 = 0. \quad (5)$$

Whence $x = \dfrac{23 \pm \sqrt{529 - 4(9)(-280)}}{18}$ (6)

$= 7, -\frac{40}{9}.$ (7)

From (7) and (3), $y = 4, +\frac{247}{36}.$

9.
$$s^2 + 2\,st = 108, \qquad (1)$$
$$4\,s - 3\,t = 6. \qquad (2)$$

From (2),
$$t = \frac{4\,s - 6}{3}. \qquad (3)$$

Substituting $\frac{4\,s-6}{3}$ for t in (1),
$$s^2 + 2\,s\left(\frac{4\,s - 6}{3}\right) = 108. \qquad (4)$$
$$11\,s^2 - 12\,s - 324 = 0. \qquad (5)$$

Whence
$$s = \frac{12 \pm \sqrt{144 - 4(11)(-324)}}{22} \qquad (6)$$
$$= 6,\ -\tfrac{54}{11}. \qquad (7)$$

From (7) and (3),
$$t = 6,\ -\tfrac{94}{11}.$$

10.
$$s^2 - 3\,st = 22, \qquad (1)$$
$$4\,s + 2\,t = 2. \qquad (2)$$

From (2),
$$t = 1 - 2\,s. \qquad (3)$$

Substituting $(1 - 2\,s)$ for t in (1),
$$s^2 - 3\,s(1 - 2\,s) = 22. \qquad (4)$$
$$7\,s^2 - 3\,s - 22 = 0. \qquad (5)$$

Whence
$$s = \frac{3 \pm \sqrt{9 - 4(7)(-22)}}{14} \qquad (6)$$
$$= +2,\ -\tfrac{11}{7}. \qquad (7)$$

From (7) and (3),
$$t = -3,\ +\tfrac{29}{7}.$$

11.
$$s^2 + t^2 = 169, \qquad (1)$$
$$2\,s + t = 22. \qquad (2)$$

From (2),
$$t = 22 - 2\,s. \qquad (3)$$

Substituting $(22 - 2\,s)$ for t in (1),
$$s^2 + (22 - 2\,s)^2 = 169. \qquad (4)$$
$$5\,s^2 - 88\,s + 315 = 0. \qquad (5)$$

Whence
$$s = \frac{88 \pm \sqrt{7744 - 4(5)(315)}}{10} \qquad (6)$$
$$= +\tfrac{63}{5},\ 5. \qquad (7)$$

From (7) and (3),
$$t = -\tfrac{16}{5},\ 12.$$

12.
$$s^2 + 2\,t^2 = 27, \qquad (1)$$
$$4\,t - 2\,s = 18. \qquad (2)$$

From (2),
$$s = 2\,t - 9. \qquad (3)$$

Substituting $(2\,t - 9)$ for s in (1),
$$(2\,t - 9)^2 + 2\,t^2 = 27. \qquad (4)$$
$$t^2 - 6\,t + 9 = 0. \qquad (5)$$

Whence
$$t = 3. \qquad (6)$$

From (6) and (3),
$$s = -3.$$

13. $$s^2 + 3ts + t^2 = 44, \quad (1)$$
$$2s - t = 0. \quad (2)$$
From (2), $$t = 2s. \quad (3)$$
Substituting $2s$ for t in (1),
$$s^2 + 3s(2s) + (2s)^2 = 44. \quad (4)$$
$$s^2 = 4. \quad (5)$$
Whence $$s = 2, -2. \quad (6)$$
From (6) and (3), $$t = 4, -4.$$

14. $$s^2 + ts + t^2 = 12, \quad (1)$$
$$s + t = 2. \quad (2)$$
From (2), $$t = 2 - s. \quad (3)$$
Substituting $(2 - s)$ for t in (1),
$$s^2 + (2 - s)s + (2 - s)^2 = 12. \quad (4)$$
$$s^2 - 2s - 8 = 0. \quad (5)$$
Whence $$s = +4, -2. \quad (6)$$
From (6) and (3), $$t = -2, +4.$$

15. $$2s^2 - ts + t^2 = 16, \quad (1)$$
$$2s - t = 5. \quad (2)$$
From (2), $$t = 2s - 5. \quad (3)$$
Substituting $(2s - 5)$ for t in (1),
$$2s^2 - (2s - 5)s + (2s - 5)^2 = 16. \quad (4)$$
$$4s^2 - 15s + 9 = 0. \quad (5)$$
Whence $$s = +\tfrac{3}{4}, 3. \quad (6)$$
From (6) and (3), $$t = -\tfrac{7}{2}, 1.$$

16. $$xy + 10 = 0, \quad (1)$$
$$4x + y = 6. \quad (2)$$
From (2), $$y = 6 - 4x. \quad (3)$$
Substituting $(6 - 4x)$ for y in (1),
$$x(6 - 4x) + 10 = 0. \quad (4)$$
$$2x^2 - 3x - 5 = 0. \quad (5)$$
Factoring, $$(x + 1)(2x - 5) = 0. \quad (6)$$
Whence $$x = -1, +\tfrac{5}{2}. \quad (7)$$
From (7) and (3), $$y = +10, -4.$$

17. $$4x + y = 28, \quad (1)$$
$$2x^2 + 3xy = 98. \quad (2)$$
From (1), $$y = 28 - 4x. \quad (3)$$
Substituting $(28 - 4x)$ for y in (2),
$$2x^2 + 3x(28 - 4x) = 98. \quad (4)$$
$$5x^2 - 42x + 49 = 0. \quad (5)$$

Whence $$x = \frac{42 \pm \sqrt{1764 - 4(5)(49)}}{10}, \quad (6)$$
$$= 7, \tfrac{7}{5}. \quad (7)$$
From (7) and (3), $$y = 0, \tfrac{112}{5}.$$

18. $$3x_1 + 4x_2 = 5, \quad (1)$$
$$2x_1x_2 - 6x_1 = -3. \quad (2)$$
From (1), $$x_2 = \frac{5 - 3x_1}{4}. \quad (3)$$
Substituting $\frac{5 - 3x_1}{4}$ for x_2 in (2),
$$2x_1\left(\frac{5 - 3x_1}{4}\right) - 6x_1 = -3. \quad (4)$$
$$3x_1^2 + 7x_1 - 6 = 0. \quad (5)$$
Factoring, $$(x_1 + 3)(3x_1 - 2) = 0. \quad (6)$$
Whence $$x_1 = -3, \tfrac{2}{3}. \quad (7)$$
From (7) and (3), $$x_2 = +\tfrac{7}{2}, \tfrac{3}{4}.$$

19. $$\frac{x}{y} + \frac{y}{x} = 2, \quad (1)$$
$$3x + 2y = 5. \quad (2)$$
From (1), $$x^2 + y^2 = 2xy. \quad (3)$$
From (2), $$y = \frac{5 - 3x}{2}. \quad (4)$$
Substituting $\frac{5 - 3x}{2}$ for y in (3),
$$x^2 + \left(\frac{5 - 3x}{2}\right)^2 = 2x\left(\frac{5 - 3x}{2}\right). \quad (5)$$
$$x^2 - 2x + 1 = 0. \quad (6)$$
Whence $$x = 1. \quad (7)$$
From (7) and (4), $$y = 1.$$

20. $$\frac{1}{x} - \frac{1}{y} = \frac{1}{4}, \quad (1)$$
$$3x - y = 2. \quad (2)$$
From (1), $$4y - 4x = xy. \quad (3)$$
From (2), $$y = 3x - 2. \quad (4)$$
Substituting $(3x - 2)$ for y in (3),
$$4(3x - 2) - 4x = x(3x - 2). \quad (5)$$
$$3x^2 - 10x + 8 = 0. \quad (6)$$
Factoring, $$(x - 2)(3x - 4) = 0. \quad (7)$$
Whence $$x = 2, \tfrac{4}{3}. \quad (8)$$
From (8) and (4), $$y = 4, 2.$$

21. $$\frac{x}{4}+\frac{y}{5}=6, \tag{1}$$

$$\frac{10}{y}-\frac{10}{x}=\frac{3}{2}. \tag{2}$$

From (2), $$20x-20y=3xy. \tag{3}$$

From (1), $$y=\frac{120-5x}{4}. \tag{4}$$

Substituting $\frac{120-5x}{4}$ for y in (3),

$$20x-20\left(\frac{120-5x}{4}\right)=3x\left(\frac{120-5x}{4}\right). \tag{5}$$

$$x^2-12x-160=0. \tag{6}$$

Whence $$x=20,\ -8. \tag{7}$$

From (7) and (4), $$y=5,\ +40.$$

22. $$\frac{1}{x}+\frac{y}{2}=3, \tag{1}$$

$$\frac{1}{y}+\frac{x}{2}=\frac{5}{4}. \tag{2}$$

From (1), $$2+xy=6x. \tag{3}$$

From (2), $$4+2xy=5y. \tag{4}$$

$2\cdot(3)-(4)$, $$12x-5y=0. \tag{5}$$

From (5), $$y=\frac{12x}{5}. \tag{6}$$

Substituting $\frac{12x}{5}$ for y in (3),

$$2+x\left(\frac{12x}{5}\right)=6x. \tag{7}$$

$$6x^2-15x+5=0. \tag{8}$$

Whence $$x=\frac{15\pm\sqrt{225-4(6)(5)}}{12} \tag{9}$$

$$=\frac{15+\sqrt{105}}{12},\ \frac{15-\sqrt{105}}{12}. \tag{10}$$

From (10) and (6), $$y=\frac{15+\sqrt{105}}{5},\ \frac{15-\sqrt{105}}{5}.$$

23. $$\frac{s}{3}+\frac{t}{4}=5, \tag{1}$$

$$\frac{6}{s}-\frac{4}{t}=\frac{2}{3}. \tag{2}$$

From (1), $$4s+3t=60. \tag{3}$$

From (2), $$9t-6s=st. \tag{4}$$

From (3), $$t = \frac{60 - 4s}{3}. \qquad (5)$$

Substituting $\frac{60 - 4s}{3}$ for t in (4),

$$9\left(\frac{60 - 4s}{3}\right) - 6s = s\left(\frac{60 - 4s}{3}\right). \qquad (6)$$

$$2s^2 - 57s + 270 = 0. \qquad (7)$$

Whence $$s = \frac{57 \pm \sqrt{3249 - 4(2)(270)}}{4}. \qquad (8)$$

$$= + \tfrac{45}{2},\ 6. \qquad (9)$$

From (9) and (5), $$t = -\ 10,\ 12.$$

24. $$\frac{t + 1}{s + 1} = \frac{3}{2}, \qquad (1)$$

$$\frac{t + s^2}{s + t^2} = \frac{1}{2}. \qquad (2)$$

From (1), $$2t + 2 = 3s + 3. \qquad (3)$$

From (2), $$2t + 2s^2 = s + t^2. \qquad (4)$$

From (3), $$t = \frac{3s + 1}{2}. \qquad (5)$$

Substituting $\frac{3s + 1}{2}$ for t in (4),

$$2s^2 - \frac{9s^2 + 6s + 1}{4} + 3s + 1 - s = 0. \qquad (6)$$

$$s^2 - 2s - 3 = 0. \qquad (7)$$

Whence $$s = 3, -1. \qquad (8)$$

From (8) and (5), $$t = 5, -1.$$

25. $$\frac{x}{y} - \frac{y}{x} = \frac{5}{6}, \qquad (1)$$

$$3y + 2x = 2. \qquad (2)$$

From (1), $$6x^2 - 6y^2 = 5xy. \qquad (3)$$

From (2), $$x = \frac{2 - 3y}{2}. \qquad (4)$$

Substituting $\frac{2 - 3y}{2}$ for x in (3),

$$6\left(\frac{2 - 3y}{2}\right)^2 - 6y^2 = 5\left(\frac{2 - 3y}{2}\right)y. \qquad (5)$$

$$15y^2 - 23y + 6 = 0. \qquad (6)$$

Factoring, $$(5y - 6)(3y - 1) = 0. \qquad (7)$$

Whence $$y = +\tfrac{6}{5},\ \tfrac{1}{3}. \qquad (8)$$

From (8) and (4), $$x = -\tfrac{4}{5},\ \tfrac{1}{2}.$$

26.
$$s^2 + t^2 + 2t = 40, \qquad (1)$$
$$s + t + 2 = 0. \qquad (2)$$
From (2), $$s = -t - 2. \qquad (3)$$
Substituting $(-t-2)$ for s in (1),
$$(-t-2)^2 + t^2 + 2t = 40. \qquad (4)$$
$$t^2 + 3t - 18 = 0. \qquad (5)$$
Whence $$t = -6, +3. \qquad (6)$$
From (6) and (3), $$s = +4, -5.$$

27.
$$2x^2 + xy = 8, \qquad (1)$$
$$y - x = \sqrt{2}. \qquad (2)$$
From (2), $$y = x + \sqrt{2}. \qquad (3)$$
Substituting $(x + \sqrt{2})$ for y in (1),
$$2x^2 + x(x + \sqrt{2}) = 8. \qquad (4)$$
$$3x^2 + \sqrt{2}\,x - 8 = 0. \qquad (5)$$
Whence $$x = \frac{-\sqrt{2} \pm \sqrt{2 - 4(3)(-8)}}{6} \qquad (6)$$
$$= \sqrt{2}, -\frac{4\sqrt{2}}{3}. \qquad (7)$$
From (7) and (3), $$y = 2\sqrt{2}, -\frac{\sqrt{2}}{3}.$$

28.
$$x^2 + y^2 + 4x + 6y = 40, \qquad (1)$$
$$x - 10 = y. \qquad (2)$$
From (2), $$x = y + 10. \qquad (3)$$
Substituting $(y + 10)$ for x in (1),
$$(y+10)^2 + y^2 + 4(y+10) + 6y = 40. \qquad (4)$$
$$y^2 + 15y + 50 = 0. \qquad (5)$$
Whence $$y = -5, -10. \qquad (6)$$
From (6) and (3), $$x = +5, \quad 0.$$

29.
$$y + x\sqrt{15} = 0, \qquad (1)$$
$$y^2 + x^3 = 16x. \qquad (2)$$
From (1), $$y = -x\sqrt{15}. \qquad (3)$$
Substituting $-x\sqrt{15}$ for y in (2),
$$15x^2 + x^3 = 16x. \qquad (4)$$
Factoring, $$x(x+16)(x-1) = 0. \qquad (5)$$
Whence $$x = 0, \quad -16, \quad +1. \qquad (6)$$
From (6) and (3), $$y = 0, +16\sqrt{15}, -\sqrt{15}.$$

Page 421

1.
$$x^2 + y^2 = 20, \tag{1}$$
$$3y^2 + x^2 = 28. \tag{2}$$
(2) – (1),
$$2y^2 = 8. \tag{3}$$
From (3),
$$y = \pm 2. \tag{4}$$
From (1) and (4),
$$x^2 + 4 = 20. \tag{5}$$
From (5),
$$x = \pm 4. \tag{6}$$
Therefore
$$x = 4, +4, -4, -4,$$
and
$$y = 2, -2, +2, -2.$$

2.
$$3x^2 + 2xy = 7, \tag{1}$$
$$2x^2 - 3xy = -4. \tag{2}$$
(1) · 3,
$$9x^2 + 6xy = 21. \tag{3}$$
(2) · 2,
$$4x^2 - 6xy = -8. \tag{4}$$
(3) + (4),
$$13x^2 = 13. \tag{5}$$
From (5),
$$x = 1, -1. \tag{6}$$
From (1) and (6),
$$y = 2, -2.$$

3.
$$3y^2 - xy = 2, \tag{1}$$
$$2y^2 + 3xy = 38. \tag{2}$$
(1) · 3,
$$9y^2 - 3xy = 6. \tag{3}$$
(2) + (3),
$$11y^2 = 44. \tag{4}$$
From (4),
$$y = 2, -2. \tag{5}$$
From (1) and (5),
$$x = 5, -5.$$

4.
$$y^2 - xy = 2x^2, \tag{1}$$
$$2x^2 + xy = 16. \tag{2}$$
From (1),
$$x = \frac{-y \pm \sqrt{y^2 + 8y^2}}{4} \tag{3}$$
$$= \frac{y}{2}, -y. \tag{4}$$
Substituting $\frac{y}{2}$ for x in (2),
$$2\left(\frac{y}{2}\right)^2 + \left(\frac{y}{2}\right)y = 16. \tag{5}$$
Whence
$$y = 4, -4. \tag{6}$$
Then from $x = \frac{y}{2}$ in (4),
$$x = 2, -2.$$
Substituting $(-y)$ for x in (2),
$$2y^2 - y^2 = 16.$$
Whence
$$y = +4, -4.$$
Then from $x = -y$ in (4),
$$x = -4, +4.$$
Therefore
$$x = 2, -2, -4, +4,$$
and
$$y = 4, -4, +4, -4.$$

5.
$$x^2 - 3y + 3 = 0, \quad (1)$$
$$y^2 + x^2 = 25. \quad (2)$$

From (1), $x^2 = 3y - 3$. (3)

Substituting $(3y - 3)$ for x^2 in (2),
$$y^2 + 3y - 3 = 25. \quad (4)$$

Whence $y = -7, 4$. (5)

From (5) and (2), $49 + x^2 = 25$. (6)

Whence $x = \pm 2\sqrt{-6}$. (7)

Also from (5) and (2), $16 + x^2 = 25$. (8)

Whence $x = \pm 3$. (9)

Therefore $x = 2\sqrt{-6},\ -2\sqrt{-6},\ 3,\ -3,$

and $y = \quad -7, \quad -7,\ 4,\ +4.$

6.
$$3x^2 + 2y^2 = 7xy, \quad (1)$$
$$x^2 + y^2 - 5x = 3. \quad (2)$$

From (1), $x = 2y, \frac{y}{3}$. (3)

Substituting $2y$ for x in (2),
$$5y^2 - 10y - 3 = 0. \quad (4)$$

Whence $y = \frac{5 \pm 2\sqrt{10}}{5}$. (5)

From (3) and (5), $x = \frac{10 \pm 4\sqrt{10}}{5}$. (6)

Substituting $\frac{y}{3}$ for x in (2),
$$10y^2 - 15y - 27 = 0. \quad (7)$$

Whence $y = \frac{15 \pm 3\sqrt{145}}{20}$. (8)

From (3) and (8), $x = \frac{5 \pm \sqrt{145}}{20}$.

Therefore
$$x = \frac{10 + 4\sqrt{10}}{5},\ \frac{10 - 4\sqrt{10}}{5},\ \frac{5 + \sqrt{145}}{20},\ \frac{5 - \sqrt{145}}{20},$$

and
$$y = \frac{5 + 2\sqrt{10}}{5},\ \frac{5 - 2\sqrt{10}}{5},\ \frac{15 + 3\sqrt{145}}{20},\ \frac{15 - 3\sqrt{145}}{20}.$$

7.
$$x^2 + 2xy - y^2 = 32, \quad (1)$$
$$2x^2 - 3xy + y^2 = 0. \quad (2)$$

From (2), $x = y$, (3)

and $x = \frac{y}{2}$. (4)

Substituting y for x in (1),

$$y^2 + 2y^2 - y^2 = 32. \quad (5)$$

Whence $y = \pm 4.$ (6)

From (3) and (6), $x = \pm 4.$ (7)

Substituting $\frac{y}{2}$ for x in (1),

$$\left(\frac{y}{2}\right)^2 + 2\left(\frac{y}{2}\right)y - y^2 = 32. \quad (8)$$

Whence $y = \pm 8\sqrt{2}.$ (9)

From (4) and (9), $x = \pm 4\sqrt{2}.$

Therefore $x = 4,\ -4,\ 4\sqrt{2},\ -4\sqrt{2},$

and $y = 4,\ -4,\ 8\sqrt{2},\ -8\sqrt{2}.$

8. $2x^2 - xy + 2y^2 = 12,$ (1)

$2x^2 + xy + 2y^2 = 8.$ (2)

$3 \cdot (2) - 2 \cdot (1),$ $2x^2 + 5xy + 2y^2 = 0.$ (3)

From (3), $x = -\frac{y}{2},\ -2y.$

Substituting $\left(-\frac{y}{2}\right)$ for x in (2) and solving,

$y = +2,\ -2.$

Whence $x = -1,\ +1.$

Substituting $(-2y)$ for x in (2) and solving,

$y = +1,\ -1.$

Whence $x = -2,\ +2.$

Therefore $x = -1,\ +1,\ -2,\ +2,$

and $y = +2,\ -2,\ +1,\ -1.$

9. $2x^2 + y^2 = 3xy,$ (1)

$x^2 + 3xy = 16.$ (2)

Solving (1) for y, $y = 2x,\ x.$

Substituting $2x$ for y in (2) and solving,

$x = \pm \frac{4}{7}\sqrt{7}.$

Whence $y = \pm \frac{8}{7}\sqrt{7}.$

Substituting x for y in (2) and solving,

$x = \pm 2.$

Whence $y = \pm 2.$

Therefore $x = \frac{4}{7}\sqrt{7},\ -\frac{4}{7}\sqrt{7},\ 2,\ -2,$

and $y = \frac{8}{7}\sqrt{7},\ -\frac{8}{7}\sqrt{7},\ 2,\ -2.$

10.
$$s^2 - 3st = 4, \quad (1)$$
$$t^2 + s^2 = 4. \quad (2)$$
(2) – (1), $t^2 + 3st = 0.$ (3)

Solving for t, $t = 0, -3s.$

Substituting 0 for t in (2) and solving,
$$s = \pm 2.$$
Substituting $(-3s)$ for t in (2) and solving,
$$s = \pm \tfrac{1}{5}\sqrt{10}.$$
Whence $t = \mp \tfrac{3}{5}\sqrt{10}.$

Therefore $s = 2, -2, +\tfrac{1}{5}\sqrt{10}, -\tfrac{1}{5}\sqrt{10},$

and $t = 0, \quad 0, -\tfrac{3}{5}\sqrt{10}, +\tfrac{3}{5}\sqrt{10}.$

11.
$$s^2 + 2st + 4t^2 = 13, \quad (1)$$
$$t^2 + 8 = -2st. \quad (2)$$
(1) · 8 + (2) · 13, $8s^2 + 42st + 45t^2 = 0.$ (3)

Solving for s, $s = -\dfrac{3t}{2}, -\dfrac{15t}{4}.$

Substituting $\left(-\dfrac{3t}{2}\right)$ for s in (2) and solving,
$$t = +2, -2.$$
Whence $s = -3, +3.$

Substituting $\left(-\dfrac{15t}{4}\right)$ for s in (2) and solving,
$$t = +\tfrac{4}{13}\sqrt{13}, -\tfrac{4}{13}\sqrt{13}.$$
Whence $s = -\tfrac{15}{13}\sqrt{13}, +\tfrac{15}{13}\sqrt{13}.$

Therefore $s = -3, +3, -\tfrac{15}{13}\sqrt{13}, +\tfrac{15}{13}\sqrt{13},$

and $t = +2, -2, +\tfrac{4}{13}\sqrt{13}, -\tfrac{4}{13}\sqrt{13}.$

Page 422

1. Solving the four systems given in the Hints,
$$x = 6, 1, -1, -6,$$
and $y = 1, 6, -6, -1.$

2.
$$x^2 + y^2 = 100, \quad (1)$$
$$xy = 48. \quad (2)$$
(1) + (2) · 2, $x^2 + 2xy + y^2 = 196.$ (3)

Whence $x + y = \pm 14.$ (4), (5)

(1) – (2) · 2, $x^2 - 2xy + y^2 = 4.$ (6)

Whence $x - y = \pm 2.$ (7), (8)

Solving (4), (5), (7), and (8), $x = 8, 6, -6, -8,$

and $y = 6, 8, -8, -6.$

3. $$x^2 + xy + y^2 = 19, \qquad (1)$$
$$xy = 6. \qquad (2)$$
(1) + (2), $$x^2 + 2xy + y^2 = 25. \qquad (3)$$
Whence $$x + y = \pm 5. \qquad (4), (5)$$
(1) − (2) · 3, $$x^2 - 2xy + y^2 = 1. \qquad (6)$$
Whence $$x - y = \pm 1. \qquad (7), (8)$$
Solving (4), (5), (7), and (8), $x = 3, 2, -2, -3,$
and $y = 2, 3, -3, -2.$

4. $$x^2 + 4y^2 = 20, \qquad (1)$$
$$xy = 4. \qquad (2)$$
(1) + (2) · 4, $$x^2 + 4xy + 4y^2 = 36. \qquad (3)$$
Whence $$x + 2y = \pm 6. \qquad (4), (5)$$
(1) − (2) · 4, $$x^2 - 4xy + 4y^2 = 4. \qquad (6)$$
Whence $$x - 2y = \pm 2. \qquad (7), (8)$$
Solving (4), (5), (7), and (8), $x = 4, 2, -2, -4,$
and $y = 1, 2, -2, -1.$

5. $$9x^2 + y^2 = 61, \qquad (1)$$
$$xy = 10. \qquad (2)$$
(1) + (2) · 6, $$9x^2 + 6xy + y^2 = 121. \qquad (3)$$
Whence $$3x + y = \pm 11. \qquad (4), (5)$$
(1) − (2) · 6, $$9x^2 - 6xy + y^2 = 1. \qquad (6)$$
Whence $$3x - y = \pm 1. \qquad (7), (8)$$
Solving (4), (5), (7), and (8), $x = 2, \frac{5}{3}, -\frac{5}{3}, -2,$
and $y = 5, 6, -6, -5.$

6. $$4x^2 + xy + y^2 = 85, \qquad (1)$$
$$xy = 12. \qquad (2)$$
(1) + (2) · 3, $$4x^2 + 4xy + y^2 = 121. \qquad (3)$$
Whence $$2x + y = \pm 11. \qquad (4), (5)$$
(1) − (2) · 5, $$4x^2 - 4xy + y^2 = 25. \qquad (6)$$
Whence $$2x - y = \pm 5. \qquad (7), (8)$$
Solving (4), (5), (7), and (8), $x = 4, \frac{3}{2}, -\frac{3}{2}, -4,$
and $y = 3, 8, -8, -3.$

7. $$4x^2 + 25y^2 = 41, \qquad (1)$$
$$xy = 2. \qquad (2)$$
(1) + (2) · 20, $$4x^2 + 20xy + 25y^2 = 81. \qquad (3)$$
Whence $$2x + 5y = \pm 9. \qquad (4), (5)$$
(1) − (2) · 20, $$4x^2 - 20xy + 25y^2 = 1. \qquad (6)$$
Whence $$2x - 5y = \pm 1. \qquad (7), (8)$$
Solving (4), (5), (7), and (8), $x = \frac{5}{2}, 2, -2, -\frac{5}{2},$
and $y = \frac{4}{5}, 1, -1, -\frac{4}{5}.$

8. $$x^2 - xy + y^2 = 8, \quad (1)$$
$$xy + 1 = 0. \quad (2)$$
From (2), $$xy = -1. \quad (3)$$
(1) − (3), $$x^2 - 2xy + y^2 = 9. \quad (4)$$
Whence $$x - y = \pm 3. \quad (5), (6)$$
(1) + (3) · 3, $$x^2 + 2xy + y^2 = 5. \quad (7)$$
Whence $$x + y = \pm\sqrt{5}. \quad (8), (9)$$
Solving (5), (6), (8), and (9), $$x = \frac{3+\sqrt{5}}{2}, \frac{\sqrt{5}-3}{2}, \frac{3-\sqrt{5}}{2}, \frac{-3-\sqrt{5}}{2},$$
and $$y = \frac{\sqrt{5}-3}{2}, \frac{\sqrt{5}+3}{2}, \frac{-\sqrt{5}-3}{2}, \frac{3-\sqrt{5}}{2}.$$

9. $$\frac{x^2}{3} - xy + \frac{4y^2}{3} = \frac{7}{3}, \quad (1)$$
$$x = \frac{6}{y}. \quad (2)$$
From (1), $$x^2 - 3xy + 4y^2 = 7. \quad (3)$$
From (2), $$xy = 6. \quad (4)$$
(3) − (4), $$x^2 - 4xy + 4y^2 = 1. \quad (5)$$
Whence $$x - 2y = \pm 1. \quad (6), (7)$$
(3) + (4) · 7, $$x^2 + 4xy + 4y^2 = 49. \quad (8)$$
Whence $$x + 2y = \pm 7. \quad (9), (10)$$
Solving (6), (7), (9), and (10), $$x = 4, 3, -3, -4,$$
and $$y = \tfrac{3}{2}, 2, -2, -\tfrac{3}{2}.$$

10. $$\frac{1}{x^2} + \frac{1}{y^2} = 13. \quad (1)$$
$$\frac{1}{xy} = 6. \quad (2)$$
(1) + (2) · 2, $$\frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 25. \quad (3)$$
Whence $$\frac{1}{x} + \frac{1}{y} = \pm 5. \quad (4), (5)$$
(1) − (2) · 2, $$\frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1. \quad (6)$$
Whence $$\frac{1}{x} - \frac{1}{y} = \pm 1. \quad (7), (8)$$
Solving (4), (5), (7), and (8), $$x = \tfrac{1}{3}, \tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{3},$$
and $$y = \tfrac{1}{2}, \tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{1}{2}.$$

11. $$\frac{1}{x^2} + \frac{1}{y^2} = 25, \quad (1)$$
$$\frac{1}{xy} = 12. \quad (2)$$

(1) + (2) · 2, $$\frac{1}{x^2}+\frac{2}{xy}+\frac{1}{y^2}=49. \qquad (3)$$

Whence $$\frac{1}{x}+\frac{1}{y}=\pm 7. \qquad (4), (5)$$

(1) − (2) · 2, $$\frac{1}{x^2}-\frac{2}{xy}+\frac{1}{y^2}=1. \qquad (6)$$

Whence $$\frac{1}{x}-\frac{1}{y}=\pm 1. \qquad (7), (8)$$

Solving (4), (5), (7), and (8), $x=\frac{1}{4}, \frac{1}{3}, -\frac{1}{3}, -\frac{1}{4}$,

and $y=\frac{1}{3}, \frac{1}{4}, -\frac{1}{4}, -\frac{1}{3}$.

12. $$\frac{1}{x^2}+\frac{1}{y^2}=\frac{5}{4}, \qquad (1)$$

$$\frac{1}{x}-\frac{1}{y}=\frac{1}{2}. \qquad (2)$$

(2) squared, $$\frac{1}{x^2}-\frac{2}{xy}+\frac{1}{y^2}=\frac{1}{4}. \qquad (3)$$

(1) − (3), $$\frac{2}{xy}=1. \qquad (4)$$

(1) + (4), $$\frac{1}{x^2}+\frac{2}{xy}+\frac{1}{y^2}=\frac{9}{4}. \qquad (5)$$

Whence $$\frac{1}{x}+\frac{1}{y}=\pm\frac{3}{2}. \qquad (6), (7)$$

Solving (2), (6), and (7), $x=1, -2$,

and $y=2, -1$.

13. $$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}, \qquad (1)$$

$$\frac{1}{9x^2}+\frac{1}{4y^2}=\frac{97}{36}. \qquad (2)$$

Squaring (1), $$\frac{1}{9x^2}+\frac{2}{6xy}+\frac{1}{4y^2}=\frac{169}{36}. \qquad (3)$$

(3) − (2), $$\frac{2}{6xy}=\frac{72}{36}. \qquad (4)$$

(2) − (4), $$\frac{1}{9x^2}-\frac{2}{6xy}+\frac{1}{4y^2}=\frac{25}{36}. \qquad (5)$$

Whence $$\frac{1}{3x}-\frac{1}{2y}=\pm\frac{5}{6}. \qquad (6), (7)$$

Solving (1), (6), and (7), $x=\frac{1}{2}, \frac{2}{9}$,

and $y=\frac{1}{3}, \frac{3}{4}$.

14. $$\frac{1}{x^2} - \frac{1}{xy} + \frac{1}{y^2} = 7, \qquad (1)$$

$$\frac{1}{xy} = 6. \qquad (2)$$

(1) + (2) · 3, $$\frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 25. \qquad (3)$$

Whence $$\frac{1}{x} + \frac{1}{y} = \pm 5. \qquad (4), (5)$$

(1) – (2), $$\frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1. \qquad (6)$$

Whence $$\frac{1}{x} - \frac{1}{y} = \pm 1. \qquad (7), (8)$$

Solving (4), (5), (7), and (8), $$x = \tfrac{1}{3}, \tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{3},$$
and $$y = \tfrac{1}{2}, \tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{1}{2}.$$

15. $$\frac{1}{x^2} - \frac{1}{xy} + \frac{1}{y^2} = 13, \qquad (1)$$

$$\frac{1}{x} + \frac{1}{y} = 7. \qquad (2)$$

Squaring (2), $$\frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 49. \qquad (3)$$

(3) – (1), $$\frac{3}{xy} = 36. \qquad (4)$$

(4) ÷ 3, $$\frac{1}{xy} = 12. \qquad (5)$$

(1) – (5), $$\frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1. \qquad (6)$$

Whence $$\frac{1}{x} - \frac{1}{y} = \pm 1. \qquad (7), (8)$$

Solving (2), (7), and (8), $$x = \tfrac{1}{4}, \tfrac{1}{3},$$
and $$y = \tfrac{1}{3}, \tfrac{1}{4}.$$

Page 424

1. Solving the system given in the Hint,

$$x = 6,$$
$$y = -2.$$

2. $$x^2 - y^2 = 77, \qquad (1)$$
$$x - y = 7. \qquad (2)$$

(1) ÷ (2), $$x + y = 11. \qquad (3)$$

From (2) and (3), $$x = 9,$$
and $$y = 2.$$

3. $x^2 - 2xy = 187, \quad (1)$
$2y - x = -17. \quad (2)$
$(1) \div (2), \quad x = 11. \quad (3)$
From (2) and (3), $y = -3$.

4. $4x^2 - y^2 = 16, \quad (1)$
$2x + y = 8. \quad (2)$
$(1) \div (2), \quad 2x - y = 2. \quad (3)$
From (2) and (3), $x = \frac{5}{2}$,
and $y = 3$.

5. $R^2h - 75 = 0, \quad (1)$
$Rh = 15. \quad (2)$
$(1) \div (2), \quad R = 5.$
Whence $h = 3$.

6. $\frac{1}{x^2} - \frac{1}{y^2} = 15, \quad (1)$
$\frac{1}{x} + \frac{1}{y} = 3. \quad (2)$
$(1) \div (2), \quad \frac{1}{x} - \frac{1}{y} = 5. \quad (3)$
Solving (2) and (3),
$x = \frac{1}{4}$,
and $y = -1$.

7. $x^3 + y^3 = 28, \quad (1)$
$x + y = 4. \quad (2)$
$(1) \div (2)$,
$x^2 - xy + y^2 = 7. \quad (3)$
Squaring (2),
$x^2 + 2xy + y^2 = 16. \quad (4)$
$(4) - (3), \quad 3xy = 9. \quad (5)$
$(5) \div 3, \quad xy = 3. \quad (6)$
$(3) - (6)$,
$x^2 - 2xy + y^2 = 4. \quad (7)$
Whence $x - y = \pm 2. \quad (8), (9)$
Solving (2), (8), and (9),
$x = 3, 1$,
and $y = 1, 3$.

8. $x^3 - 8y^3 = 35, \quad (1)$
$x - 2y = 5. \quad (2)$
$(1) \div (2)$,
$x^2 + 2xy + 4y^2 = 7. \quad (3)$
Squaring (2),
$x^2 - 4xy + 4y^2 = 25. \quad (4)$
$(3) - (4), \quad 6xy = -18. \quad (5)$
$(5) \div 3, \quad 2xy = -6. \quad (6)$
$(3) + (6)$,
$x^2 + 4xy + 4y^2 = 1. \quad (7)$
Whence $x + 2y = \pm 1. \quad (8), (9)$
Solving (2), (8), and (9),
$x = +3, +2$,
and $y = -1, -\frac{3}{2}$.

9. $\frac{1}{x} + \frac{1}{y} = 1, \quad (1)$
$\frac{1}{y^2} - \frac{1}{x^2} = 5. \quad (2)$
$(2) \div (1), \quad \frac{1}{y} - \frac{1}{x} = 5. \quad (3)$
Solving (1) and (3),
$x = -\frac{1}{2}$,
and $y = \frac{1}{3}$.

10. $\frac{1}{x^3} + \frac{1}{y^3} = 152, \quad (1)$
$\frac{1}{x} + \frac{1}{y} = 8. \quad (2)$
$(1) \div (2), \quad \frac{1}{x^2} - \frac{1}{xy} + \frac{1}{y^2} = 19. \quad (3)$
Squaring (2),
$\frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 64. \quad (4)$
$(4) - (3), \quad \frac{3}{xy} = 45. \quad (5)$
$(5) \div 3, \quad \frac{1}{xy} = 15. \quad (6)$
$(3) - (6), \quad \frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 4. \quad (7)$
Whence $\frac{1}{x} - \frac{1}{y} = \pm 2. \quad (8), (9)$
Solving (2), (8), and (9),
$x = \frac{1}{5}, \frac{1}{3}$,
and $y = \frac{1}{3}, \frac{1}{5}$.

Page 425

1. $$y^2 + xy = 8, \quad (1)$$
$$x + y = 4. \quad (2)$$

From (2), $x = 4 - y.$ (3)

Substituting in (1) and solving, $y = 2.$

Whence $x = 2.$

2. $$x^2 - 9y^2 = 7, \quad (1)$$
$$x - 3y = 1. \quad (2)$$

(1) ÷ (2), $x + 3y = 7.$ (3)

Solving (2) and (3), $x = 4,$

and $y = 1.$

3. $$x^2 - x - 2y = 0, \quad (1)$$
$$y^2 - 4y + x = 0. \quad (2)$$

(1) – (2), $x^2 - 2x + 2y - y^2 = 0.$ (3)

From (3), $x = 2 - y,$ (4)

and $x = y.$ (5)

From (4) and (1),

$$4 - 4y + y^2 - 2 + y - 2y = 0. \quad (6)$$

Whence $y = \dfrac{5 \pm \sqrt{17}}{2}.$ (7)

From (4) and (7), $x = \dfrac{-1 \mp \sqrt{17}}{2}.$ (8)

From (1) and (5), $y^2 - y - 2y = 0.$ (9)

Whence $y = 0, 3.$ (10)

From (5) and (10), $x = 0, 3.$

Therefore $x = \dfrac{-1 - \sqrt{17}}{2}, \dfrac{-1 + \sqrt{17}}{2}, 0, 3,$

and $y = \dfrac{5 + \sqrt{17}}{2}, \dfrac{5 - \sqrt{17}}{2}, 0, 3.$

4. $$x^2 + y^2 = 25, \quad (1)$$
$$x + y = 7. \quad (2)$$

From (2), $x = 7 - y.$ (3)

Substituting in (1) and solving, $y = 4, 3.$ (4)

From (3) and (4), $x = 3, 4.$

CR

5. $$\frac{x}{y} + 3 = 0, \qquad (1)$$

$$xy + 12 = 0. \qquad (2)$$

Solving (1) for x, $x = -3y$.

Substituting in (2) and solving, $y = +2, -2,$

and $x = -6, +6.$

6. $$(x + y)^2 = 9, \qquad (1)$$

$$(x - y)^2 = 49. \qquad (2)$$

From (1), $x + y = \pm 3.$ (3), (4)

From (2), $x - y = \pm 7.$ (5), (6)

Solving (3), (4), (5), and (6), $x = +5, -2, +2, -5,$

and $y = -2, +5, -5, +2.$

7. $$2x^2 + y^2 = 33, \qquad (1)$$

$$x^2 + 2y^2 = 54. \qquad (2)$$

$(1) \cdot 2 - (2)$, $3x^2 = 12.$ (3)

Whence $x = \pm 2.$

Substituting in (2), $4 + 2y^2 = 54.$ (4)

Whence $y = \pm 5.$

Therefore $x = 2, +2, -2, -2,$

and $y = 5, -5, +5, -5.$

8. $$3h^2 - 8k^2 = 40, \qquad (1)$$

$$5h^2 + k^2 = 81. \qquad (2)$$

$(1) + (2) \cdot 8$, $43h^2 = 688.$

Whence $h = \pm 4.$

Substituting in (2), $80 + k^2 = 81.$

Whence $k = \pm 1.$

Therefore $h = 4, +4, -4, -4,$

and $k = 1, -1, +1, -1.$

9. $$4R_1^2 + 3 = 9R_2^2, \qquad (1)$$

$$12R_1^2 + R_2^2 = \tfrac{31}{9}. \qquad (2)$$

$(1) + (2) \cdot 9$, $112R_1^2 = 28.$

Whence $R_1 = \pm \frac{1}{2}.$

Substituting in (1), $1 + 3 = 9R_2^2.$

Whence $R_2 = \pm \frac{2}{3}.$

Therefore $R_1 = \frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2},$

and $R_2 = \frac{2}{3}, -\frac{2}{3}, +\frac{2}{3}, -\frac{2}{3}.$

10. $xy + x = 18,$ (1)

$xy + y = 20.$ (2)

(2) – (1), $y - x = 2.$ (3)

Solving for x in (3), $x = y - 2.$ (4)

Substituting in (2), $y^2 - 2y + y = 20.$

Whence $y = 5, -4.$

From (4), $x = 3, -6.$

11. $x^2 + y^2 = 169,$ (1)

$xy = 60.$ (2)

(1) + (2) · 2, $x^2 + 2xy + y^2 = 289.$ (3)

Whence $x + y = \pm 17.$ (4), (5)

(1) – (2) · 2, $x^2 - 2xy + y^2 = 49.$ (6)

Whence $x - y = \pm 7.$ (7), (8)

Solving (4), (5), (7), and (8), $x = 12, 5, -5, -12,$

and $y = 5, 12, -12, -5.$

12. $x^2 = y,$ (1)

$xy = 8.$ (2)

Substituting x^2 for y in (2), $x^2 \cdot x = 8.$ (3)

Whence $x = 2, -1 \pm \sqrt{-3}.$ (4)

From (2) and (4), $y = 4, -2 \mp 2\sqrt{-3}.$

Therefore $x = 2, -1 + \sqrt{-3}, -1 - \sqrt{-3},$

and $y = 4, -2 - 2\sqrt{-3}, -2 + 2\sqrt{-3}.$

13. $x - xy = 5,$ (1)

$2y + xy = 6.$ (2)

(1) + (2), $x + 2y = 11.$ (3)

Solving for y, $y = \dfrac{11 - x}{2}.$ (4)

Substituting in (1), $x - \dfrac{11x - x^2}{2} = 5.$

Whence $x = 10, -1.$

From (4), $y = \frac{1}{2}, +6.$

14. $x^3 - y^3 = 19,$ (1)

$x^2 + xy + y^2 = 19.$ (2)

(1) ÷ (2), $x - y = 1.$ (3)

Squaring (3), $x^2 - 2xy + y^2 = 1.$ (4)

[(2) – (4)] ÷ 3, $xy = 6.$ (5)

(2) + (5), $x^2 + 2xy + y^2 = 25.$ (6)

Whence $x + y = \pm 5.$ (7), (8)

Solving (3), (7), and (8), $x = 3, -2,$

and $y = 2, -3.$

15. $4n^2 + 7m^2 = 9,$ (1)

$2n^2 - \frac{9}{2} = m^2.$ (2)

(1) − (2) · 2, $9m^2 = 0.$

Whence $m = 0.$

Substituting in (1), $4n^2 = 9.$

Whence $n = \frac{3}{2}, -\frac{3}{2},$

and $m = 0, 0.$

16. $5W_1^2 - 6.8W_2^2 = 99.55,$ (1)

$W_1^2 - W_2^2 = 20.$ (2)

(2) · 5 − (1), $1.8W_2^2 = .45.$

Whence $W_2 = \pm .5.$

Substituting in (2), $W_1^2 - .25 = 20.$

Whence $W_1 = \pm 4.5.$

Therefore $W_1 = 4.5, -4.5, +4.5, -4.5,$

and $W_2 = .5, +.5, -.5, -.5.$

17. $xy + 2y^2 = 2,$ (1)

$3xy + 5y^2 = 2.$ (2)

(2) − (1), $3y^2 + 2xy = 0.$ (3)

Solving (3), $y = -\frac{2x}{3}, 0.$ (4)

Substituting $\left(-\frac{2x}{3}\right)$ for y in (1),

$$\frac{-2x^2}{3} + \frac{8x^2}{9} = 2.$$

Whence $x = +3, -3.$

From (4), $y = -2, +2.$

Substituting 0 for y in (1) or (2), x is infinite.

18. $x^2 + 2xy + 2y^2 = 10,$ (1)

$3x^2 - xy - y^2 = 51.$ (2)

(1) + (2) · 2, $7x^2 = 112.$

Whence $x = \pm 4.$

Substituting (+ 4) for x in (1),

$$16 + 8y + 2y^2 = 10.$$

Whence $y = -3, -1.$

Substituting (− 4) for x in (1),

$$16 - 8y + 2y^2 = 10.$$

Whence $y = 3, 1.$

Therefore $x = +4, +4, -4, -4,$

and $y = -3, -1, +3, +1.$

19.

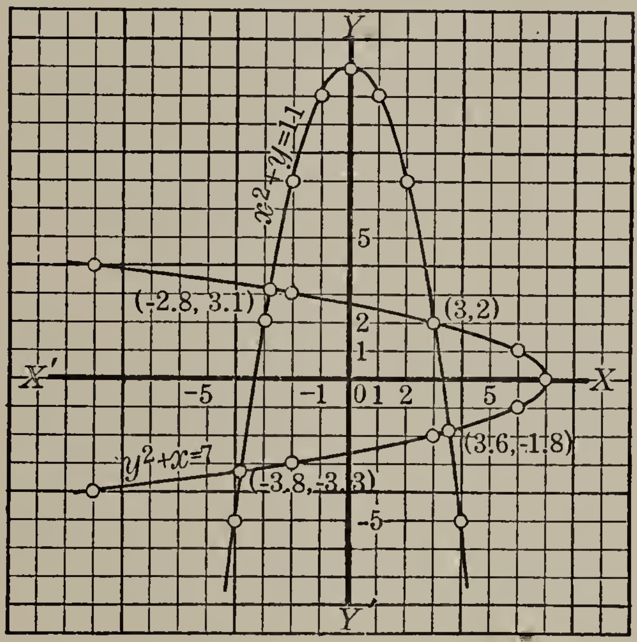
$$y^2 + x = 7,$$
$$x^2 + y = 11.$$

$y^2 + x = 7$. Then $y = \pm\sqrt{7 - x}$.

If $x =$	7	6	3	-2	-9	0
then $y =$	0	± 1	± 2	± 3	± 4	± 2.64

$x^2 + y = 11$. Then $y = 11 - x^2$.

If $x =$	0	± 1	± 2	± 3	± 4	± 5
then $y =$	11	10	7	2	-5	-14



The sets of roots are

x	-2.8	3	3.6	-3.8
y	3.1	2	-1.8	-3.3

20.

$x^2 + xy + y^2 = 7,$ (1)

$x^2 + y^2 = 10.$ (2)

(1) − (2), $xy = -3.$ (3)

(1) + (3), $x^2 + 2xy + y^2 = 4.$ (4)

Whence $x + y = \pm 2.$ (5), (6)

(1) − (3) · 3, $x^2 - 2xy + y^2 = 16.$ (7)

Whence $x - y = \pm 4.$ (8), (9)

Solving (5), (6), (8), and (9), $x = +3, -1, +1, -3,$

and $y = -1, +3, -3, +1.$

21.

$$x^2 + xy + y = 0, \quad (1)$$
$$x^2 + xy + x = 0. \quad (2)$$

(1) − (2), $$y = x. \quad (3)$$

Substituting in (1), $$x^2 + x^2 + x = 0.$$

Whence $$x = 0, -\tfrac{1}{2}.$$

From (3), $$y = 0, -\tfrac{1}{2}.$$

22.

$$\frac{1}{x^2} + \frac{1}{y^2} = 13, \quad (1)$$
$$\frac{1}{x} - \frac{1}{y} = 1. \quad (2)$$

Squaring (2), $$\frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1. \quad (3)$$

(1) − (3), $$\frac{2}{xy} = 12. \quad (4)$$

(1) + (4), $$\frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 25. \quad (5)$$

Whence $$\frac{1}{x} + \frac{1}{y} = \pm 5. \quad (6), (7)$$

Solving (2), (6), and (7), $$x = \tfrac{1}{3}, -\tfrac{1}{2},$$
and $$y = \tfrac{1}{2}, -\tfrac{1}{3}.$$

23.

$$\frac{1}{x^3} - \frac{1}{y^3} = 7, \quad (1)$$
$$\frac{1}{x} - \frac{1}{y} = 1. \quad (2)$$

(1) ÷ (2), $$\frac{1}{x^2} + \frac{1}{xy} + \frac{1}{y^2} = 7. \quad (3)$$

Squaring (2), $$\frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1. \quad (4)$$

[(3) − (4)] ÷ 3, $$\frac{1}{xy} = 2. \quad (5)$$

(3) + (5), $$\frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 9. \quad (6)$$

Whence $$\frac{1}{x} + \frac{1}{y} = \pm 3. \quad (7), (8)$$

Solving (2), (7), and (8), $$x = \tfrac{1}{2}, -1,$$
and $$y = 1, -\tfrac{1}{2}.$$

24. $$4x^2 + y^2 = 289, \quad (1)$$
$$xy = 60. \quad (2)$$

(1) + (2) · 4, $$4x^2 + 4xy + y^2 = 529. \quad (3)$$

Whence $$2x + y = \pm 23. \quad (4), (5)$$

(1) − (2) · 4, $$4x^2 - 4xy + y^2 = 49. \quad (6)$$

Whence $$2x - y = \pm 7. \quad (7), (8)$$

Solving (4), (5), (7), and (8), $x = \frac{15}{2}, 4, -4, -\frac{15}{2}$,

and $y = 8, 15, -15, -8$.

25. $$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 9, \quad (1)$$
$$x^{\frac{1}{2}}y^{\frac{1}{2}} = 20. \quad (2)$$

Squaring (1),
$$x + 2\sqrt{xy} + y = 81. \quad (3)$$

(3) − (2) · 4,
$$x - 2\sqrt{xy} + y = 1. \quad (4)$$

From (4), $\sqrt{x} - \sqrt{y} = \pm 1.$ (5)

From (5) and (1), $x = 25, 16$,

and $y = 16, 25$.

26. $$x^2 + y^2 + x + y = 58, \quad (1)$$
$$x + y = 5. \quad (2)$$

(1) − (2), $x^2 + y^2 = 53.$ (3)

Solving (2) for x, $x = 5 - y.$ (4)

Substituting in (3),
$$(5 - y)^2 + y^2 = 53. \quad (5)$$

Whence $y = +7, -2$.

From (4), $x = -2, +7$.

27. $$\frac{9x}{y} = 18 = xy. \quad (1),(2)$$

From (1), $x = 2y.$ (3)

$xy = 18.$ (2)

Substituting in (2),
$$2y^2 = 18. \quad (4)$$

Whence $y = 3, -3$.

From (3), $x = 6, -6$.

28. $$4x + \frac{1}{y} = 46. \quad (1)$$
$$\frac{26x}{5} - \frac{1}{y} = 46. \quad (2)$$

(1) + (2), $$\frac{46x}{5} = 92. \quad (3)$$
$$x = 10. \quad (4)$$

From (1) and (4),
$$40 + \frac{1}{y} = 46.$$

Whence $y = \frac{1}{6}$.

29. $$\frac{1}{x-2} + \frac{1}{y-2} = \frac{3}{4}, \quad (1)$$
$$\frac{1}{x} - \frac{1}{y} = \frac{1}{12}. \quad (2)$$

From (2), $$12y - 12x = xy. \quad (3)$$

From (1), $$10x + 10y = 3xy + 28. \quad (4)$$

(3) · 3, $$-36x + 36y = 3xy. \quad (5)$$

(4) − (5), $$46x - 26y = 28. \quad (6)$$

Whence $$x = \frac{13y + 14}{23}. \quad (7)$$

From (7) and (3),
$$12y - \frac{156y + 168}{23} = \frac{13y^2 + 14y}{23}. \quad (8)$$

From (8), $$y = 6, \tfrac{28}{13}. \quad (9)$$

From (9) and (7), $$x = 4, \tfrac{42}{23}.$$

30. $$3xy = x^2y^2 - 88, \qquad (1)$$
$$x - y = 6. \qquad (2)$$
From (1), $(xy - 11)(xy + 8) = 0.$ (3)

From (3) and (2),
$$(y^2 + 6y - 11)(y^2 + 6y + 8) = 0. \qquad (4)$$
Whence $y = -3 + 2\sqrt{5},\ -3 - 2\sqrt{5},\ -2,\ -4.$

From (2) and (4), $x = +3 + 2\sqrt{5},\ +3 - 2\sqrt{5},\ +4,\ +2.$

31. $$xy = c, \qquad (1)$$
$$x + y = a. \qquad (2)$$
From (2), $x = a - y.$ (3)

From (1) and (3), $ay - y^2 = c.$ (4)

Whence $$y = \frac{a \pm \sqrt{a^2 - 4c}}{2}. \qquad (5)$$
From (3) and (5), $$x = \frac{a \mp \sqrt{a^2 - 4c}}{2}.$$
Therefore $$x = \frac{a + \sqrt{a^2 - 4c}}{2},\ \frac{a - \sqrt{a^2 - 4c}}{2},$$
and $$y = \frac{a - \sqrt{a^2 - 4c}}{2},\ \frac{a + \sqrt{a^2 - 4c}}{2}.$$

32. $x^{-2} - y^{-2} = 6,$ (1)

$x^{-1} + y^{-1} = 2.$ (2)

(1) ÷ (2), $x^{-1} - y^{-1} = 3.$ (3)

(3) + (2), $2x^{-1} = 5.$

Whence $x = \frac{2}{5}.$

(2) − (3), $2y^{-1} = -1.$

Whence $y = -2.$

33. $x - y = 16,$ (1)

$x^{\frac{1}{2}} - y^{\frac{1}{2}} = 2.$ (2)

(1) ÷ (2), $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 8.$ (3)

(2) + (3), $2x^{\frac{1}{2}} = 10.$

Whence $x = 25.$

(3) − (2), $2y^{\frac{1}{2}} = 6.$

Whence $y = 9.$

34. $$\frac{x-1}{y-1} = 3, \qquad (1)$$
$$\frac{y^2 + y + 1}{x^2 - x + 1} = \frac{13}{43}. \qquad (2)$$
From (1), $x = 3y - 2.$ (3)

From (2), $43y^2 + 43y + 43 = 13x^2 - 13x + 13.$ (4)

From (3) and (4),
$$43y^2 + 43y + 30 = 117y^2 - 156y + 52 - 39y + 26. \qquad (5)$$
From (5), $37y^2 - 119y + 24 = 0.$ (6)

From (6), $y = 3,\ +\frac{8}{37}.$ (7)

From (3) and (7), $x = 7,\ -\frac{50}{37}.$

35. $4x^2 - 13xy + 9y^2 = 9,$ (1)

$xy - y^2 = 3.$ (2)

(2) · 3, $3xy - 3y^2 = 9.$ (3)

(1) – (3), $4x^2 - 16xy + 12y^2 = 0.$ (4)

From (4), $x = 3y.$ (5)

$x = y.$ (6)

From (2) and (5), $3y^2 - y^2 = 3.$ (7)

From (7), $y = \pm \frac{1}{2}\sqrt{6}.$ (8)

From (5), $x = \pm \frac{3}{2}\sqrt{6}.$ (9)

No results can be obtained by using (6) and (1) or (6) and (2).

Therefore $x = \frac{3}{2}\sqrt{6}, -\frac{3}{2}\sqrt{6},$

and $y = \frac{1}{2}\sqrt{6}, -\frac{1}{2}\sqrt{6}.$

36. $x^2 + 2xy = 16,$ (1)

$3x^2 - 4xy + 2y^2 = 6.$ (2)

(1) · 3, $3x^2 + 6xy = 48.$ (3)

(2) · 8, $24x^2 - 32xy + 16y^2 = 48.$ (4)

(4) – (3), $16y^2 - 38xy + 21x^2 = 0.$ (5)

From (5), $y = \frac{3}{2}x,$ (6)

and $y = \frac{7}{8}x.$ (7)

From (1) and (6), $x^2 + 3x^2 = 16.$ (8)

From (8), $x = \pm 2.$ (9)

From (6) and (9), $y = \pm 3.$ (10)

From (1) and (7), $x^2 + \dfrac{7x^2}{4} = 16.$ (11)

From (11), $x = \pm \frac{8}{11}\sqrt{11}.$ (12)

From (7) and (12), $y = \pm \frac{7}{11}\sqrt{11}.$

Therefore $x = 2, -2, \frac{8}{11}\sqrt{11}, -\frac{8}{11}\sqrt{11},$

and $y = 3, -3, \frac{7}{11}\sqrt{11}, -\frac{7}{11}\sqrt{11}.$

37. $x^3 = y^3 + 37,$ (1)

$x^2y = xy^2 + 12.$ (2)

From (1), $x^3 - y^3 = 37.$ (3)

From (2), $x^2y - xy^2 = 12.$ (4)

(3) ÷ (4), $\dfrac{x^2 + xy + y^2}{xy} = \dfrac{37}{12}.$ (5)

Clearing and solving for x, $x = \dfrac{4y}{3},$ (6)

and $x = \dfrac{3y}{4}.$ (7)

From (2) and (7), $$\frac{9y^3}{16} = \frac{3y^3}{4} + 12. \tag{8}$$

Whence $$y = -4,\ 2 \pm 2\sqrt{-3}. \tag{9}$$

From (7) and (9), $$x = -3,\ \frac{3 \pm 3\sqrt{-3}}{2}. \tag{10}$$

From (2) and (6), $$\frac{16y^3}{9} = \frac{4y^3}{3} + 12. \tag{11}$$

Whence $$y = 3,\ \frac{-3 \pm 3\sqrt{-3}}{2}. \tag{12}$$

From (6) and (12), $$x = 4,\ -2 \pm 2\sqrt{-3}.$$

Therefore
$$x = 4,\ -3,\ -2 + 2\sqrt{-3},\ -2 - 2\sqrt{-3},\ \frac{3 + 3\sqrt{-3}}{2},\ \frac{3 - 3\sqrt{-3}}{2},$$
and $$y = 3,\ -4,\ \frac{-3 + 3\sqrt{-3}}{2},\ \frac{-3 - 3\sqrt{-3}}{2},\ 2 + 2\sqrt{-3},\ 2 - 2\sqrt{-3}.$$

Page 427

1. Let x equal one number and y the other.

Then $$x - y = 6, \tag{1}$$
and $$x^2 - y^2 = 120. \tag{2}$$
(2) ÷ (1), $$x + y = 20. \tag{3}$$
From (1) and (3), $$x = 13,\ y = 7.$$

2. Let x equal one number and y the other.

Then $$x + y = 20, \tag{1}$$
and $$x^2 + y^2 = 202. \tag{2}$$
From (1), $$x = 20 - y. \tag{3}$$
Substituting $(20 - y)$ for x in (2),
$$(20 - y)^2 + y^2 = 202.$$
Whence $$x = 11,\ y = 9.$$

3. Let x equal one number and y the other.

Then $$x + y + xy = 132, \tag{1}$$
and $$\frac{x}{y} = 3. \tag{2}$$
From (2), $$x = 3y. \tag{3}$$
Substituting $3y$ for x in (1),
$$3y + y + 3y^2 = 132. \tag{4}$$
From (4) and (3), $$x = 18,\ y = 6.$$

4. Let x and y equal the dimensions of the lot in rods.

Then $$xy = 160, \tag{1}$$
and $$2x + 2y = 56. \tag{2}$$
From (2), $$x = 28 - y. \tag{3}$$
Substituting $(28 - y)$ for x in (1),
$$y(28 - y) = 160. \tag{4}$$
From (4), $$y = 8. \tag{5}$$
From (3) and (5), $$x = 20.$$

5. Let x equal one side and y the other.

Then $$x^2 + y^2 = 1681, \tag{1}$$
and $$\frac{xy}{2} = 180. \tag{2}$$
(1) + (2) · 4, $$x^2 + 2xy + y^2 = 2401. \tag{3}$$
Whence $$x + y = \pm 49. \tag{4), (5}$$
(1) − (2) · 4, $$x^2 - 2xy + y^2 = 961. \tag{6}$$
Whence $$x - y = \pm 31. \tag{7), (8}$$
From (4), (5), (7), and (8), $$x = 40 \text{ feet},$$
and $$y = 9 \text{ feet}.$$

6. Let x and y equal the dimensions in rods.

Then $$xy = 15 \cdot 160, \tag{1}$$
and $$(x + 20)(y + 20) = 30 \cdot 160. \tag{2}$$
From (1), $$xy = 2400. \tag{3}$$
From (2), $$xy + 20x + 20y = 4400. \tag{4}$$
(4) − (3), $$20x + 20y = 2000. \tag{5}$$
Whence $$x = 100 - y. \tag{6}$$
Substituting $(100 - y)$ for x in (3),
$$y(100 - y) = 2400. \tag{7}$$
Whence $$y = 40. \tag{8}$$
From (6) and (8), $$x = 60.$$
Therefore the dimensions are 40 rods by 60 rods.

7. Let x and y equal the sides of the squares, respectively.

Then $$x^2 - y^2 = 495, \tag{1}$$
and $$4x - 4y = 60. \tag{2}$$
From (2), $$x = y + 15. \tag{3}$$
Substituting $(y + 15)$ for x in (1),
$$y^2 + 30y + 225 - y^2 = 495. \tag{4}$$
Whence $$y = 9. \tag{5}$$
From (3) and (5), $$x = 24.$$
Therefore the sides are 24 feet and 9 feet.

8. Let x and y equal the dimensions in rods.

Then $xy = 6912,$ (1)

and $x^2 + y^2 = 14{,}400.$ (2)

(2) + (1) · 2, $x^2 + 2xy + y^2 = 28{,}224.$ (3)

Whence $x + y = \pm 168.$ (4), (5)

(2) − (1) · 2, $x^2 - 2xy + y^2 = 576.$ (6)

Whence $x - y = \pm 24.$ (7), (8)

From (4), (5), (7), and (8), $x = 96.$

$y = 72.$

Therefore the perimeter $2x + 2y$ is 336 rods.

9. Let $x =$ the numerator,

and $y =$ the denominator.

Then $$\frac{x}{y} = \frac{2}{3},\quad (1)$$

and $$\frac{x^2 - 44}{y^2 - 44} = \frac{5}{14}.\quad (2)$$

From (1), $$x = \frac{2y}{3}.\quad (3)$$

Substituting $\frac{2y}{3}$ for x in (2),

$$14 \cdot \frac{4y^2}{9} - 396 = 5y^2.\quad (4)$$

From (4), $y = \pm 18.$ (5)

From (3) and (5), $x = \pm 12.$

Therefore the fraction is $\frac{12}{18}$.

10. Let x and y equal the number of days, respectively, required by each of the two men when working alone.

Then $$\frac{1}{x} + \frac{1}{y} = \frac{5}{24},\quad (1)$$

and $x - y = 4.$ (2)

From (2), $x = y + 4.$ (3)

Substituting $(y + 4)$ for x in (1),

$$\frac{1}{y+4} + \frac{1}{y} = \frac{5}{24}.\quad (4)$$

Whence $y = 8.$ (5)

From (3) and (5), $x = 12.$

11. Let x and y equal the dimensions in feet.

$$2x + 2y = 250. \quad (1)$$

$$xy = 1926. \quad (2)$$

From (1), $x = 125 - y$. (3)

Substituting $(125 - y)$ for x in (2),

$$y(125 - y) = 1926. \quad (4)$$

Whence $y = 18$. (5)

From (3) and (5), $x = 107$.

Therefore the dimensions are 18 by 107 feet.

12. Let x and y equal the base and altitude in inches.

Then $x = y + 8$, (1)

and $\dfrac{xy}{2} = 240$. (2)

Substituting $(y + 8)$ for x in (2),

$$\frac{y(y+8)}{2} = 240. \quad (3)$$

Whence $y = -4 + 4\sqrt{31}$. (4)

From (1) and (4), $x = 4 + 4\sqrt{31}$.

Therefore the base is 26.27 inches and the altitude is 18.27 inches.

13. Let x and y equal the edges of the respective cubes.

Then $x^3 - y^3 = 316$, (1)

and $x - y = 4$. (2)

$(1) \div (2)$, $x^2 + xy + y^2 = 79$. (3)

Squaring (2), $x^2 - 2xy + y^2 = 16$. (4)

From (3) and (4), $xy = 21$. (5)

$(3) + (5)$, $x^2 + 2xy + y^2 = 100$. (6)

Whence $x + y = \pm 10$. (7), (8)

From (2), (7), and (8), $x = 7$,

and $y = 3$.

Therefore the edges are 7 inches and 3 inches, respectively.

14. Let x and y equal the sides, respectively.

Then $x + y + \sqrt{x^2 + y^2} = 80$, (1)

and $\dfrac{xy}{2} = 240$. (2)

From (1), $x + y - 80 = -\sqrt{x^2 + y^2}$. (3)

Squaring (3), $80x + 80y - xy = 3200$. (4)

From (2), $xy = 480.$ (5)

(4) + (5), $x = 46 - y.$ (6)

Substituting $(46 - y)$ for x in (2),

$$46y - y^2 = 480. \qquad (7)$$

Whence $y = 16.$ (8)

From (6) and (8), $x = 30.$

$$\sqrt{x^2 + y^2} = 34.$$

Therefore the sides are 16 feet and 30 feet; the hypotenuse, 34 feet.

15. Let x and y equal the dimensions.

Then $xy = a^2,$ (1)

and $2x + 2y = 7a.$ (2)

From (1), $x = \dfrac{a^2}{y}.$ (3)

Substituting $\dfrac{a^2}{y}$ for x in (2),

$$\frac{2a^2}{y} + 2y = 7a. \qquad (4)$$

Whence $y = \dfrac{7a \pm a\sqrt{33}}{4}.$ (5)

From (3) and (5), $x = \dfrac{7a \mp a\sqrt{33}}{4}.$

16. Let x equal the rate of the boat and y equal the rate of the first train.

Then $y + 10$ equals the rate of the second train.

Then $\dfrac{30}{x} + \dfrac{120}{y} = 6,$ (1)

and $\dfrac{120}{y + 10} + \dfrac{30}{x} = 5.$ (2)

(1) – (2), $\dfrac{120}{y} - \dfrac{120}{y + 10} = 1.$ (3)

Whence $y = 30.$ (4)

Then $y + 10 = 40.$

From (1) and (4), $x = 15.$

Therefore the rate of the boat is 15 miles per hour; the rate of the first train, 30 miles per hour; and the rate of the second train, 40 miles per hour.

17. Let x equal the number of reserved seats and y equal the price of each reserved seat.

Then $\quad xy + (x + 1400)(y - 15) = 49{,}000, \qquad (1)$

and $\quad xy = 25{,}000. \qquad (2)$

From (1), $\quad 2xy - 15x + 1400y = 70{,}000. \qquad (3)$

(3) – (2) · 2, $\quad 280y - 3x = 4000. \qquad (4)$

From (2), $\quad x = \dfrac{25000}{y}. \qquad (5)$

Substituting $\dfrac{25000}{y}$ for x in (4),

$$280y - 3 \cdot \frac{25000}{y} = 4000. \qquad (6)$$

Whence $\quad y = 25. \qquad (7)$

$$y - 15 = 10.$$

From (2) and (7), $\quad x = 1000.$

$$x + 1400 = 2400.$$

Therefore there were 1000 reserved seats at 25 cents each and 2400 unreserved seats at 15 cents each.

18. Let x equal the units' digit and y equal the tens' digit.

Then $\quad (x + y)(10y + x) = 324, \qquad (1)$

and $\quad 10y + x + 3y + 3x = 10x + y. \qquad (2)$

From (1), $\quad x^2 + 11xy + 10y^2 = 324. \qquad (3)$

From (2), $\quad 12y = 6x. \qquad (4)$

From (3) and (4), $\quad x^2 + \dfrac{11x^2}{2} + \dfrac{5x^2}{2} = 324. \qquad (5)$

Whence $\quad x = \pm 6. \qquad (6)$

From (4) and (6), $\quad y = \pm 3.$

Therefore the number is 36.

19. Let x and y equal the respective radii.

Then $\quad x + y = 31, \qquad (1)$

and $\quad \pi x^2 - \pi y^2 = 155\pi. \qquad (2)$

From (2), $\quad x^2 - y^2 = 155. \qquad (3)$

From (1), $\quad x = 31 - y. \qquad (4)$

From (4) and (3), $\quad (31 - y)^2 - y^2 = 155. \qquad (5)$

Whence $\quad y = 13. \qquad (6)$

From (4) and (6), $\quad x = 18.$

Therefore the radii of the two circles are 13 inches and 18 inches.

20. Let x equal the sum in dollars and y equal the rate per cent.

Then $\quad \dfrac{y}{100}x = 42, \qquad (1)$

and $\quad \dfrac{(x + 200)(y - 1)}{100} = 48. \qquad (2)$

From (1), $xy = 4200.$ (3)

From (2), $xy + 200y - x - 200 = 4800.$ (4)

From (3) and (4), $x = 200y - 800.$ (5)

From (5) and (3), $200y^2 - 800y = 4200.$ (6)

Whence $y = 7.$ (7)

From (3) and (7), $x = 600.$

Therefore the principal is \$600, and the rate is 7%.

21. Let x equal the rate in miles of the wheelman going north and y equal the rate in miles of the wheelman going east.

Then $$\left(\frac{4x}{3}\right)^2 + \left(3 + \frac{4y}{3}\right)^2 = 289, \qquad (1)$$

and $$\left(\frac{14x}{3}\right)^2 + \left(3 + \frac{14y}{3}\right)^2 = 2809. \qquad (2)$$

Solving (1) and (2), $x = 6,\ y = 9.$

Therefore the rate of the wheelman going north is 6 miles per hour, and the rate of the wheelman going east, 9 miles per hour.

22.

First. A goes from P to K, and B goes from Q to K.

(A) $r = x$ (in miles per hour).
$t = y$ (in hours).
$d = PK = xy.$

(B) $r = \dfrac{xy - 40}{y}.$
$t = y.$
$d = QK = xy - 40.$

Second. A goes from K to Q, and B goes from K to P.

(A) $r = x.$
$t = \dfrac{xy - 40}{x} = \dfrac{d}{r}.$
$d = xy - 40.$

(B) $r = \dfrac{xy - 40}{y}.$
$t = \dfrac{xy^2}{xy - 40} = \dfrac{d}{r}.$
$d = xy.$

Therefore $$\frac{xy - 40}{x} = 2, \qquad (1)$$

and $$\frac{xy^2}{xy - 40} = 8. \qquad (2)$$

(1) · (2), $y^2 = 16.$ (3)

$y = 4.$ (Root -4 rejected.) (4)

From (4) and (1), $x = 20$, A's rate.

Then $$\frac{xy - 40}{y} = 10, \text{ B's rate,}$$

and $PQ = 2xy - 40 = 120$ miles.

23. 

First. A goes from P to K, and B goes from Q to K.

(A) $r = x$ (in miles per hour).
$t = y$ (in hours).
$d = xy$.

(B) $r = \dfrac{300 - xy}{y}$.
$t = y$.
$d = 300 - xy$.

Second. A goes from K to Q, and B goes from K to P.

(A) $r = x$.
$t = \dfrac{300 - xy}{x}$.
$d = 300 - xy$.

(B) $r = \dfrac{300 - xy}{y}$.
$t = \dfrac{xy^2}{300 - xy}$.
$d = xy$.

Therefore $$\frac{300 - xy}{x} = 6\tfrac{2}{3}, \quad (1)$$

and $$\frac{xy^2}{300 - xy} = 15. \quad (2)$$

(1) · (2), $$y^2 = 100. \quad (3)$$

$$y = 10. \quad \text{(Root } -10 \text{ rejected.)} \quad (4)$$

From (4) and (1), $x = 18$, A's rate.

Therefore $$\frac{300 - xy}{y} = 12, \text{ B's rate.}$$

Page 429

1. $$x^2 + y^2 = 36, \quad (1)$$
$$(10 - x)^2 + y^2 = 64. \quad (2)$$
(2) – (1), $$-20x + 100 = 28. \quad (3)$$
Whence $$x = 3\tfrac{3}{5}. \quad (4)$$
From (1) and (4), $$y = 4\tfrac{4}{5}.$$

2. $$x^2 + y^2 = 64, \quad (1)$$
$$(17 - x)^2 + y^2 = 225. \quad (2)$$
(2) – (1), $$34x - 289 = -161. \quad (3)$$
Whence $$x = 3\tfrac{13}{17}. \quad (4)$$
From (1) and (4), $$y = 7\tfrac{1}{17}.$$
Therefore the altitude is $7\frac{1}{17}$, and the area is 60.

3. $$x^2 + y^2 = 121, \quad (1)$$
$$(20 - x)^2 + y^2 = 169. \quad (2)$$
(2) – (1), $$400 - 40x = 48. \quad (3)$$
Whence $$x = 8\tfrac{4}{5}. \quad (4)$$
From (1) and (4), $$y = 6\tfrac{3}{5}.$$
Therefore the altitude is $6\frac{3}{5}$, and the area is 66.

4. $$x^2 + y^2 = 169, \qquad (1)$$
$$(14 - x)^2 + y^2 = 225. \qquad (2)$$
(2) – (1), $$196 - 28x = 56. \qquad (3)$$
Whence $$x = 5. \qquad (4)$$
From (1) and (4), $$y = 12.$$
Therefore the altitude is 12, and the area is 84.

5. $$x^2 + y^2 = 289, \qquad (1)$$
$$(12 + x)^2 + y^2 = 625. \qquad (2)$$
(2) – (1), $$144 + 24x = 336. \qquad (3)$$
Whence $$x = 8. \qquad (4)$$
From (1) and (4), $$y = 15.$$
Therefore the altitude is 15, and the area is 90.

6. $$x^2 + y^2 = 400, \qquad (1)$$
$$(16 - x)^2 + y^2 = 576. \qquad (2)$$
(2) – (1), $$256 - 32x = 176. \qquad (3)$$
Whence $$x = \tfrac{5}{2}. \qquad (4)$$
From (1) and (4), $$y = 19.84.$$
Therefore the altitude is 19.84.

7. $$x^2 + y^2 = 100, \qquad (1)$$
$$(12 - x)^2 + y^2 = 100. \qquad (2)$$
Solving, $x = 6$, and $y = 8$.
Therefore the altitude is 8.

8. $$x^2 + y^2 = 100, \qquad (1)$$
$$(21 - x)^2 + y^2 = 289. \qquad (2)$$
Solving, $x = 6$, and $y = 8$.
Therefore the altitude is 8.

9. $$x^2 + y^2 = 900, \qquad (1)$$
$$(5 - x)^2 + y^2 = 841. \qquad (2)$$
Solving, $x = 8.4$, and $y = 28.8$.
Therefore the altitude is 28.8, and the area is 504.

10. $$x^2 + y^2 = 49, \qquad (1)$$
$$(15 - x)^2 + y^2 = 400. \qquad (2)$$
Solving, $x = -4.2$.
(This means that x, or FE in the figure of Exercise 7 of the text, is laid off on FB to the right of F instead of to the left.)
Then $y = 5.6$.
Therefore the altitude is 5.6, and the area is 92.4.

11. $$x^2 + y^2 = 729, \quad (1)$$
$$(19 - x)^2 + y^2 = 900. \quad (2)$$
Solving, $y = 26.53 +.$
Therefore the altitude is 26.53, and the area is 809.2.

12. $x^2 + (100 - 23)^2 = \overline{85}^2.$
Solving, $x = 36.$
Therefore the area is $\frac{1}{2} \cdot 36 \cdot 123$, or 2214.

13. Let x equal the third side, $x + 14$ the second, and y the projection of the third side on the first.

Now $$\frac{16 \times \text{first side}}{2} = 144.$$
Whence the first side is 18.

Then $$x^2 - y^2 = 256, \quad (1)$$
$$(14 + x)^2 = 256 + (18 - y)^2. \quad (2)$$
From (1) and (2), $7x + 9y = 32. \quad (3)$
From (3) and (1), $x^2 + 14x - 680 = 0.$
Whence $x = -34, 20.$ (Root -34 rejected.)
$x + 14 = 34.$
Therefore the three sides are 18, 34, and 20.

14. From the Hints, $$\frac{x}{80 - y} = \frac{y}{102 - x},$$
$$x^2 + y^2 = 100.$$

Page 434

2. $l = 1 + 7 \cdot 3 = 22.$

3. $l = 15 + 10 \cdot (-6) = -45.$

4. $l = a + 20 \cdot 3a = 61a.$

5. $l = 9x + 15(-8)x = -111x.$

6. $l = \frac{10}{3} + 5(-\frac{4}{3}) = -\frac{10}{3}.$

7. $l = -\frac{1}{7} + 10 \cdot \frac{4}{7} = \frac{39}{7}.$

8. $l = \sqrt{3} + 5 \cdot 3\sqrt{3} = 16\sqrt{3}.$

9. $l = 3\sqrt{5} + 7(-2\sqrt{5}) = -11\sqrt{5}.$

10. $l = 5 + a + 7(2 + 2a) = 19 + 15a.$

11. $l_1 = 0 + 11(-5x + 2) = -55x + 22.$
$l_2 = 0 + 19(-5x + 2) = -95x + 38.$

12. $l = \sqrt{a} + 3 + 9(4\sqrt{a} + 4) = 37\sqrt{a} + 39.$

13. $l_1 = 9\sqrt{a} - 3 + 9(-4\sqrt{a} + 3) = -27\sqrt{a} + 24.$
$l_2 = 9\sqrt{a} - 3 + 19(-4\sqrt{a} + 3) = -67\sqrt{a} + 54.$

14. $l = \frac{\sqrt{x}}{2} + 13 \cdot 2\sqrt{x} = 26\frac{1}{2}\sqrt{x}.$

15. $l_1 = \sqrt{3} + 8\sqrt{3} = 9\sqrt{3}.$
$l_2 = \sqrt{3} + 11\sqrt{3} = 12\sqrt{3}.$

16. $l_1 = \frac{\sqrt{x}}{2} + 1 + 7(\sqrt{x} + 1) = 7\frac{1}{2}\sqrt{x} + 8.$
$l_2 = \frac{\sqrt{x}}{2} + 1 + 15(\sqrt{x} + 1) = 15\frac{1}{2}\sqrt{x} + 16.$

17. $l = \frac{2\sqrt{a}}{3} - 12 + 14\left(7 - \frac{2\sqrt{a}}{3}\right) = 86 - \frac{26\sqrt{a}}{3}.$

18. $l = a + 28d.$

19. $l = a + (m - 1)d.$

20. $l = a + (m - 2)d.$

21. $l = a + (n - 3)5 = a + 5n - 15.$

22. $l = \dfrac{3}{\sqrt{3}} + (n - 6) \cdot 2\sqrt{3} = \sqrt{3} + 2\sqrt{3}\,n - 12\sqrt{3} = 2\sqrt{3}\,n - 11\sqrt{3}.$

23.
$$\begin{aligned} l &= \sqrt{5} - 1 + (n - 4)(\sqrt{5} - 1) \\ &= \sqrt{5} - 1 + (n - 4)\sqrt{5} - n + 4 \\ &= (n - 3)\sqrt{5} - n + 3. \end{aligned}$$

24. $l = \dfrac{1}{a} + (n - 1)\dfrac{a - 2}{a} = \dfrac{1}{a} + \dfrac{na - 2n - a + 2}{a} = \dfrac{(n - 1)a + 3 - 2n}{a}$

25.
$$d = \frac{22 - 2}{2} = 10.$$
$$\text{Seventh term} = 2 + 6 \cdot 10 = 62.$$
$$n\text{th term} = 2 + (n - 1)10 = 10n - 8.$$

26.
$$d = s - r.$$
$$\text{Third term} = r + 2(s - r) = 2s - r.$$
$$n\text{th term} = r + (n - 1)(s - r) = 2r + n(s - r) - s.$$

27. The edges of the box are n, $n + 2$, and $n + 4$, respectively.

Sum of the edges $= 4n + 4(n + 2) + 4(n + 4) = 12n + 24.$

Area of the faces $= 2n(n + 2) + 2n(n + 4) + 2(n + 2)(n + 4)$
$$= 2n^2 + 4n + 2n^2 + 8n + 2n^2 + 12n + 16$$
$$= 6n^2 + 24n + 16.$$

Volume $= n(n + 2)(n + 4) = n^3 + 6n^2 + 8n.$

28.
$$a = 16.$$
$$d = 32.$$
$$l_1 = 16 + 11 \cdot 32 = 368.$$
$$l_2 = 16 + (n - 1)32 = 32n - 16.$$

29. Let the units' digit $= x - d,$

and the tens' digit $= x,$

and the hundreds' digit $= x + d.$

$$x - d + x + x + d = 24.$$
$$x = 8. \quad (1)$$
$$(x - d)^2 + x^2 + (x + d)^2 = 194.$$
$$x^2 - 2dx + d^2 + x^2 + x^2 + 2dx + d^2 = 194.$$
$$3x^2 + 2d^2 = 194. \quad (2)$$

From (1) and (2), $192 + 2d^2 = 194.$
$$d^2 = 1.$$
$$d = \pm 1.$$
$$x - d = 7 \text{ or } 9.$$
$$x + d = 9 \text{ or } 7.$$

Therefore the number is 789 or 987.

Page 436

1. $\quad \text{Mean} = \dfrac{3+15}{2} = 9.$

Progression $= 3, 9, 15.$

2. $\quad \text{Mean} = \dfrac{3+27}{2} = 15.$

Progression $= 3, 15, 27.$

3. $\quad \text{Mean} = \dfrac{h+5h}{2} = 3h.$

Progression $= h, 3h, 5h.$

4. $\quad \text{Mean} = \dfrac{m+n}{2}\cdot$

Progression $= m, \dfrac{m+n}{2}, n.$

5. $$8 = 2 + (4-1)d.$$
$$3d = 6.$$
$$d = 2.$$

Progression $= 2, 4, 6, 8.$

6. $$29 = 5 + (4-1)d.$$
$$3d = 24.$$
$$d = 8.$$

Progression $= 5, 13, 21, 29.$

7. $$217 = 49 + (4-1)d.$$
$$3d = 168.$$
$$d = 56.$$

Progression $= 49, 105, 161, 217.$

8. $$x + 6y = x + (4-1)d.$$
$$3d = 6y.$$
$$d = 2y.$$

Progression $= x, x+2y, x+4y, x+6y.$

9. $$3y = x + (4-1)d.$$
$$3d = 3y - x.$$
$$d = \frac{3y-x}{3}\cdot$$

Progression $= x, \dfrac{3y+2x}{3}, \dfrac{6y+x}{3}, 3y.$

10. $$k = h + (4-1)d.$$
$$3d = k - h.$$
$$d = \frac{k-h}{3}\cdot$$

Progression $= h, \dfrac{2h+k}{3}, \dfrac{h+2k}{3}, k.$

11. $$34 = 10 + (5-1)d.$$
$$d = 6.$$

Progression $= 10, 16, 22, 28, 34.$

12. $$31 = -5 + 4d.$$
$$d = 9.$$

Progression $= -5, 4, 13, 22, 31.$

13. $$44x = -12x + 4d.$$
$$d = 14x.$$

Progression $= -12x, 2x, 16x, 30x, 44x.$

14. $$k = h + 4d.$$
$$d = \frac{k-h}{4}\cdot$$

Progression $= h, \dfrac{3h+k}{4}, \dfrac{h+k}{2}, \dfrac{h+3k}{4}, k.$

15. $$131 = -13 + 8d.$$
$$d = 18.$$

Progression $= -13, 5, 23, 41, 59, 77, 95, 113, 131.$

16. $$\tfrac{46}{3} = -\tfrac{8}{3} + 6d.$$
$$d = 3.$$

Progression $= -\frac{8}{3}, \frac{1}{3}, \frac{10}{3}, \frac{19}{3}, \frac{28}{3}, \frac{37}{3}, \frac{46}{3}.$

17. $$18\sqrt{5} = -2\sqrt{5} + 5d.$$
$$d = 4\sqrt{5}.$$

Progression $= -2\sqrt{5}, 2\sqrt{5}, 6\sqrt{5}, 10\sqrt{5}, 14\sqrt{5}, 18\sqrt{5}.$

18. $$13a + 9b = 7a - 3b + 6d.$$
$$d = a + 2b.$$

Progression $= 7a - 3b,\ 8a - b,\ 9a + b,\ 10a + 3b,\ 11a + 5b,\ 12a + 7b,\ 13a + 9b.$

19. $$\frac{27}{2\sqrt{3}} = \sqrt{3} + 7d.$$
$$d = \frac{\sqrt{3}}{2}.$$

Progression $= \sqrt{3},\ \frac{3\sqrt{3}}{2},\ 2\sqrt{3},\ \frac{5\sqrt{3}}{2},\ 3\sqrt{3},\ \frac{7\sqrt{3}}{2},\ 4\sqrt{3},\ \frac{27}{2\sqrt{3}}.$

20. $$4\sqrt{3} - 6 = \frac{3}{1-\sqrt{3}} + 3d.$$
$$d = \frac{11\sqrt{3} - 9}{6}.$$

Progression $= \frac{3}{1-\sqrt{3}},\ \frac{\sqrt{3} - 9}{3},\ \frac{13\sqrt{3} - 27}{6},\ 4\sqrt{3} - 6.$

21. The average velocity equals the arithmetical mean.
$$m = \frac{25 + 15}{2} = 20 \text{ miles per hour.}$$

Since $\frac{\text{Distance}}{\text{Rate}} = \text{Time},\ \frac{1}{20} = \frac{1}{20}$ hour, or 3 minutes.

22. Mean velocity $= \frac{64 + 96}{2} = 80$ feet per second.

It will fall 80 feet during the third second.

23. Mean velocity $= \frac{0 + 32}{2} = 16$ feet per second.

The body falls 16 feet during the first second, and during the second second it falls $\frac{32 + 64}{2}$, or 48 feet.

24. $$l = 1 + 19 \cdot 2 = 39.$$
$$\frac{1 + 39}{2} = 20 \text{ inches average.}$$

25. $$l = 58 + 14 \cdot 2 = 86.$$
$$\text{Mean} = \frac{58 + 86}{2} = 72 \text{ inches.}$$

26. $$l = 32 + 11 \cdot 32 = 384 \text{ feet.}$$
$$\text{Mean} = \frac{0 + 384}{2} = 192 \text{ feet per second.}$$

27. $$37 = 1 + 9d.$$
$$d = 4.$$

Progression = 1, 5, 9, 13, 17, 21, 25, 29, 33, 37 inches.

Page 439

1. $S = \frac{8}{2}(2 \cdot 2 + 7 \cdot 4) = 128.$
2. $S = \frac{10}{2}(2 \cdot 8 + 9 \cdot 5) = 305.$
3. $S = \frac{9}{2}(2 + 8 \cdot 11) = 405.$
4. $S = \frac{15}{2}(34 + 14 \cdot 3) = 570.$
5. $S = \frac{20}{2}[200 + 19(-5)] = 1050.$
6. $S = \frac{8}{2}(4 + 7 \cdot 2) = 72.$
7. $S = \frac{10}{2}(2 + 9 \cdot 8) = 370.$
8. $S = \frac{10}{2}(-16 + 9 \cdot 4) = 100.$
9. $S = \frac{18}{2}(2 + 17 \cdot 4) = 630.$
10. $S = \frac{20}{2}[20 + 19(-2)] = -180.$
11. $S = \frac{12}{2}(2 + 11 \cdot \frac{1}{2}) = 45.$
12. $S = \frac{12}{2}[30 + 11 \cdot (-\frac{5}{2})] = 15.$
13. $S = \frac{100}{2}(2 + 99 \cdot 1) = 5050.$
14. $S = \frac{100}{2}(4 + 99 \cdot 2) = 10{,}100.$
15. $S = \frac{100}{2}(2 + 99 \cdot 2) = 10{,}000.$
16. $S = \frac{n}{2}[4 + (n-1)2] = n(n+1).$
17. $S = \frac{n}{2}[2 + (n-1)2] = n^2.$

18. $$538 = 248 + (n-1)2.$$

Whence
$$n = 146.$$
$$S = \frac{146}{2}(248 + 538) = 57{,}378.$$

19. $$861 = \frac{n}{2}[2 + (n-1)1].$$
$$n^2 + n = 1722.$$
$$n + \tfrac{1}{2} = \pm \tfrac{83}{2}.$$
$$n = 41 \text{ or } -42. \qquad \text{(Root } -42 \text{ rejected.)}$$

20. $$242 = \frac{n}{2}[14 + (n-1)3].$$
$$3n^2 + 11n - 484 = 0.$$
$$(3n + 44)(n - 11) = 0.$$
$$n = 11 \text{ or } -\tfrac{44}{3}. \qquad \text{(Root } -\tfrac{44}{3} \text{ rejected.)}$$

21. $$372 = \frac{n}{2}[180 + (n-1)2].$$
$$2n^2 + 178n = 744.$$
$$n^2 + 89n - 372 = 0.$$
$$(n + 93)(n - 4) = 0.$$
$$n = 4 \text{ or } -93. \qquad \text{(Root } -93 \text{ rejected.)}$$

The terms are 90, 92, 94, 96.

22. $$22 = -2 + 6d.$$
$$d = 4.$$
$$l = -6 + 10 \cdot 4 = 34.$$

23. $$S = \frac{t}{2}\left(\frac{2}{a} + (t-1)\frac{a-2}{a}\right)$$
$$= \frac{t}{2}\left(\frac{2 + at - a - 2t + 2}{a}\right)$$
$$= \frac{t(at - a - 2t + 4)}{2a}.$$

24. Given $l = 25$, $a = 1$, and $d = 4$; to find n and s.

$$25 = 1 + (n-1)4.$$
$$n = 7.$$
$$s = \frac{n}{2}(1 + 25).$$
$$s = 91.$$

25. Given $a = -20$, $d = 11$, and $s = 216$; to find n and l.

$$216 = \frac{n}{2}(-20 + l), \text{ or } l = \frac{432 + 20n}{n}. \qquad (1)$$
$$l = -20 + (n-1)11, \text{ or } l = 11n - 31. \qquad (2)$$

From (1) and (2),
$$\frac{432 + 20n}{n} = 11n - 31.$$
$$11n^2 - 51n - 432 = 0.$$
$$(11n + 48)(n - 9) = 0.$$
$$n = 9 \text{ or } -\tfrac{48}{11}. \qquad \text{(Root } -\tfrac{48}{11} \text{ rejected.)}$$

Then from (2), $l = 68.$

26. Given $d = -9$, $n = 15$, $s = 0$; to find a and l.

$$0 = \tfrac{15}{2}(a + l), \text{ or } a = -l. \qquad (1)$$
$$l = a + 14 \cdot (-9) \text{ or } a = l + 126. \qquad (2)$$

From (1) and (2), $-l = l + 126.$
$$l = -63.$$

From (1), $a = 63.$

27. Given $a = h$, $n = n$, and $d = k - h$; to find S_n.

$$S_n = \frac{n}{2}[2h + (n-1)(k-h)]$$
$$= \frac{3nh}{2} + \frac{kn^2}{2} - \frac{kn}{2} - \frac{hn^2}{2}.$$

28. Given $s = 9h$, $a = 12 - 10h$, and $n = 9$; to find l and d.

$$9h = \tfrac{9}{2}(12 - 10h + l). \qquad (1)$$

From (1), $l = 12h - 12. \qquad (2)$

Again, $l = 12 - 10h + 8d. \qquad (3)$

Then from (2) and (3),
$$12 - 10h + 8d = 12h - 12. \qquad (4)$$
$$8d = 22h - 24.$$
$$d = \frac{11h - 12}{4}.$$

From (2), $l = 12h - 12.$

29. Given $s = 66\sqrt{3}$, $a = -4\sqrt{3}$, and $d = 2\sqrt{3}$; to find n and l.

$$66\sqrt{3} = \frac{n}{2}(-4\sqrt{3} + l) \text{ or } l = \frac{132\sqrt{3} + 4\sqrt{3}\,n}{n}.$$

$$l = -4\sqrt{3} + (n-1)2\sqrt{3}.$$

Then $$\frac{132\sqrt{3} + 4\sqrt{3}\,n}{n} = -4\sqrt{3} + (n-1)2\sqrt{3}.$$

$$n^2 - 5n - 66 = 0.$$

$$(n-11)(n+6) = 0.$$

$$n = 11 \text{ and } -6. \qquad \text{(Root } -6 \text{ rejected.)}$$

$$l = -4\sqrt{3} + 10 \cdot 2\sqrt{3} = 16\sqrt{3}.$$

30.
$$S = \tfrac{12}{2}(1 + 12)$$
$$= 78 \text{ in a 12-hour day.}$$
$$2 \times 78 = 156 \text{ in a 24-hour day.}$$

31. (*a*) 15 miles per hour equals 22 feet per second.

$$l = a + (n-1)d.$$
$$\tfrac{1}{2} = 22 + (n-1)(-\tfrac{1}{2}).$$
$$\tfrac{1}{2}n = 22, \; n = 44.$$

(*b*)
$$S = \tfrac{44}{2}(22 + \tfrac{1}{2})$$
$$= 495 \text{ feet.}$$

32. (*a*) Given $a = 3$, $d = 8$, and $n = 30$.

$l = 3 + 29 \cdot 8 = 235$ inches passed over during the thirtieth second.

The rate is, however, $235 + 4$ inches per second, or $19\tfrac{11}{12}$ feet per second at this time, for to gain 8 inches during the last second the car must be going 4 inches faster at the close of the second than its average rate during that second.

(*b*)
$$S = \tfrac{30}{2}(3 + 235)$$
$$= 3570 \text{ inches, or } 297\tfrac{1}{2} \text{ feet.}$$

33. Given $a = 24$, $d = -\tfrac{1}{2}$, and $l = \tfrac{1}{2}$.

$$\tfrac{1}{2} = 24 + (n-1)(-\tfrac{1}{2}).$$
$$1 = 48 + 1 - n.$$
$$n = 48.$$
$$S = \tfrac{48}{2}(24 + \tfrac{1}{2})$$
$$= 24 \cdot 24\tfrac{1}{2} = 588 \text{ inches fallen.}$$

In rebounding the ball covers 564 inches, making a total distance passed through of 1152 inches, or 96 feet.

34. Let the digits be 3, $3+d$, and $3+2d$.

$$100\cdot 3+10(3+d)+3+2d=20\tfrac{1}{2}(9+3d).$$
$$666+24d=369+123d.$$
$$99d=297.$$
$$d=3.$$
$$3+d=6.$$
$$3+2d=9.$$

The number is 369.

35. Given $a=60\times 12$, $d=75$, and $n=10$.

$$l=a+(n-1)d.$$
$$l=720+9\cdot 75=1395, \text{ or } \$1395.$$
$$S=\tfrac{10}{2}(720+1395)$$
$$=10{,}575, \text{ or } \$10{,}575.$$

36. Given $a=200$, $d=.06$ of 200, and $n=6$.

$$S=\tfrac{6}{2}[400+5(\tfrac{6}{100}\times 200)]$$
$$=1380, \text{ or } \$1380.$$

37. Given $S=555$ feet, $a=16$ feet, and $d=32$ feet.

$$S=\frac{n}{2}[2a+(n-1)d].$$
$$555=\frac{n}{2}[32+(n-1)32].$$
$$n^2=\tfrac{555}{16}.$$
$$n=\pm 5.8896. \quad \text{(Root } -5.8896 \text{ rejected.)}$$
$$l=16+4.8896\cdot 32=172.45+.$$

Velocity at end of time = distance passed during last second + 16 feet = 188.45 feet per second.

38. Given $S=256$, $a=16$, and $d=32$.

$$256=\frac{n}{2}[32+(n-1)32].$$
$$32n^2=512.$$
$$n=\pm 4. \qquad \text{(Root } -4 \text{ rejected.)}$$
$$l=16+3(32)=112.$$

From Exercise 37 the final velocity will be 128 feet per second.

39. In $S=\dfrac{n}{2}[2a+(n-1)d]$, here $n=t$, $d=32=g$, and $a=16=\dfrac{g}{2}$.

Substituting these values in the above, we have

$$S=\frac{t}{2}[g+(t-1)g]=\frac{t}{2}(g+tg-g)=\frac{t}{2}(tg)=\frac{1}{2}gt^2.$$

40. Given $n = \frac{6}{2} = 3$, $a = 16$, and $d = 32$.

$$l = 16 + 2 \cdot 32 = 80.$$

$$S = \tfrac{3}{2}(16 + 80) = 144.$$

Velocity $= 80 + 16 = 96$ feet per second.

41. A's rate $= 20$ miles per hour.

B's rate $= 30, 26, 22, 18$, etc., miles per hour.

Let $x =$ number of hours which must elapse.

Then $20x =$ number of miles traveled by A,

and $\frac{x}{2}[60 + (x-1)(-4)] =$ number of miles traveled by B.

Therefore
$$20x = \frac{x}{2}[60 + (x-1)(-4)].$$

$$20x = 30x - 2x^2 + 2x.$$

$$2x^2 - 12x = 0.$$

$$x = 0 \text{ and } 6.$$

They are together at the start and again 6 hours later.

Page 444

2. Given $a = 1$, $r = 2$, and $n - 1 = 7$.

Then $t_8 = 1 \cdot 2^7 = 128.$

3. Given $a = 3$, $r = 2$, and $n - 1 = 9$.

Then $t_{10} = 3 \cdot 2^9 = 1536.$

4. Given $a = 3$, $r = \frac{2}{3}$, and $n - 1 = 7$.

Then $t_8 = 3 \cdot (\frac{2}{3})^7 = \frac{128}{729}.$

5. Given $a = 7$, $r = -2$, and $n - 1 = 11$.

Then $t_{12} = 7 \cdot (-2)^{11} = -14{,}336.$

6. Here $t_9 = 15 \cdot (-\frac{1}{3})^8 = \frac{5}{2187}.$

7. Here $t_{12} = 12a^2 \cdot (\frac{3}{4})^{11} = \frac{3^{12}}{4^{10}}a^2.$

8. Here $t_6 = -\frac{2c^3}{5} \cdot \left(\frac{5}{2c^3}\right)^5 = -\frac{625}{16c^{12}}.$

9. Here $t_8 = 4\sqrt{2} \cdot \left(\frac{\sqrt{2}}{2}\right)^7 = \frac{64}{128} = \frac{1}{2}.$

10. Here $t_7 = \frac{1}{2\sqrt{2}} \cdot (\sqrt{2})^6 = 2\sqrt{2}.$

11. Here $t_{20} = \$4.12(1.01)(1.01)^{19}$
$= \$4.12(1.01)^{20}.$

12. Given $(n-1)$th term $= ar^{n-2}$.
$(n-2)$th term $= ar^{n-3}$.
$(n-3)$th term $= ar^{n-4}$.
$(n+1)$th term $= ar^{n}$.
$(n+2)$th term $= ar^{n+1}$.

13. Given $a = m$ and $r = \frac{n}{m}$.

Progression $= m,\ n,\ \frac{n^2}{m},\ \frac{n^3}{m^2},\ \cdots$.

Page 446

1. Given $a = 21$, $n = 4$, and $t_4 = 168$.

Then $168 = 21 \cdot r^3$.
$r^3 = 8$.
$r = 2,\ -1 \pm \sqrt{-3}$.

The imaginary roots are rejected.
Progression $= 21, 42, 84, 168$.

2. Given $a = 15$, $n = 4$, and $t_4 = 405$.

Then $405 = 15 \cdot r^3$.
$r^3 = 27$.
$r = 3,\ \frac{-3 \pm \sqrt{-3}}{2}$.

The imaginary roots are rejected.
Progression $= 15, 45, 135, 405$.

3. Given $a = 3$, $n = 5$, and $t_5 = 243$.

Then $243 = 3 \cdot r^4$.
$r^4 = 81$.
$r = \pm 3,\ \pm\sqrt{-9}$.

The imaginary roots are rejected.
Progression $= 3, \pm 9, 27, \pm 81, 243$.

4. Given $a = 9$, $n = 3$, and $t_3 = 81$.

Then $81 = 9 \cdot r^2$.
$r^2 = 9$.
$r = \pm 3$.

Progression $= 9, \pm 27, 81$.

5. Given $a = a^{12}$, $n = 3$, and $t_3 = a^{20}$.

Then $a^{20} = a^{12}r^2$.
$r^2 = a^8$.
$r = \pm a^4$.

Progression $= a^{12}, \pm a^{16}, a^{20}$.

6. Given $a = -9$, $n = 5$, and $t_5 = -144$.

Then $-144 = (-9)r^4$.
$r^4 = \frac{144}{9}$.
$r = \pm 2,\ \pm 2\sqrt{-1}$.

The imaginary roots are rejected.
Progression $= -9, \mp 18, -36, \mp 72, -144$.

7. Given $t_4 = 16$, $n = 5$, and $t_8 = 256$.

Then $256 = 16 r^4$.
$r^4 = 16$.
$r = \pm 2,\ \pm 2\sqrt{-1}$.

The imaginary roots are rejected.
Tenth term $= 256 \cdot (\pm 2)^2$
$= 1024$.

8. $t_2 = ar = 4\sqrt{3}$. (1)
$t_5 = ar^4 = \frac{4}{9}$. (2)

(2) ÷ (1), $r^3 = \frac{\sqrt{3}}{27}$.

$r = \frac{\sqrt[6]{3}}{3}$ and two imaginary roots, which are rejected.

First term $= 4\sqrt{3} \div \frac{\sqrt[6]{3}}{3} = 12\sqrt[3]{3}$.

9. Here $a = a$, $n = 3$, and $t_3 = c$.
$l = c = ar^2$.
$r = \pm\sqrt{\frac{c}{a}} = \pm\frac{1}{a}\sqrt{ac}$.

Therefore the means are

$$a \cdot \frac{\pm 1}{a}\sqrt{ac} = \pm\sqrt{ac}.$$

10. Given $a = a$, $n = 4$, and $t_4 = c$.

Then $t_1 = a = a$, (1)

and $t_4 = ar^3 = c$. (2)

(2) ÷ (1),

$$r^3 = \frac{c}{a}.$$

$$r = \sqrt[3]{\frac{c}{a}} = \frac{1}{a}\sqrt[3]{a^2c}$$

and two imaginary roots, which are rejected.

Progression $= a,\ \sqrt[3]{a^2c},\ \sqrt[3]{ac^2},\ c.$

11. Given $a = h$, $n = 5$, and $t_5 = k$.

$$k = hr^4.$$

$$r^4 = \frac{k}{h}.$$

$$r = \pm\sqrt[4]{\frac{k}{h}} = \pm\frac{1}{h}\sqrt[4]{h^3k}$$

and two imaginary roots, which are rejected.

Progression $= h,\ \pm\sqrt[4]{h^3k},\ \sqrt{hk},\ \pm\sqrt[4]{hk^3},\ k.$

12. Let $a =$ the first term,

and $ar^3 =$ the fourth term.

$$a + ar^3 = 56. \quad (1)$$

$$ar = 6. \quad (2)$$

Substituting from (2) in (1),

$$\frac{6}{r} + 6r^2 = 56.$$

$$3r^3 - 28r + 3 = 0.$$

By factor theorem,

$$(r-3)(3r^2 + 9r - 1) = 0.$$

$$r = 3,\ \frac{-9 \pm \sqrt{93}}{6}.$$

Then $a = 2,\ 27 \pm 3\sqrt{93}.$

The first progression $= 2, 6, 18, 54, \cdots.$

The second and third progressions $= 27 \pm 3\sqrt{93},\ 6,\ -9 \pm \sqrt{93},\ 29 \mp 3\sqrt{93}, \cdots.$

13. (*a*) $AD = \sqrt{4 \cdot 9} = 6.$

(*b*) $DC = BC - BD = 18.$

$AD = \sqrt{3 \cdot 18} = 3\sqrt{6}.$

(*c*) Let $BD = x$,

and $DC = 25 - x.$

$$10 = \sqrt{x(25 - x)}.$$

$$100 = 25x - x^2.$$

$$x^2 - 25x + 100 = 0.$$

$$x = 20 \text{ or } 5.$$

Then $25 - x = 5$ or 20.

14. (*a*) $AB = \sqrt{36 \cdot 4} = 12.$

(*b*) $AC = AD - DC.$

Let $x = AD$;

then $x - 90 = AC.$

$$24 = \sqrt{x(x - 90)}.$$

$$x^2 - 90x - 576 = 0.$$

$$(x - 96)(x + 6) = 0.$$

$$x = 96 \text{ or } -6.$$

(Root -6 rejected.)

$$x - 90 = 6.$$

Page 448

2. Given $a = 1$, $r = 5$, and $n = 9$.

$$S_9 = \frac{1 - 1 \cdot 5^9}{1 - 5} = 488{,}281.$$

3. Given $a = 2$, $r = -2$, and $n = 8$.

$$S_8 = \frac{2 - 2(-2)^8}{1 - (-2)} = \frac{-510}{3} = -170.$$

4. Given $a = 40$, $r = \frac{1}{2}$, and $n = 7$.

$$S_7 = \frac{40 - 40 \cdot (\frac{1}{2})^7}{1 - \frac{1}{2}} = \frac{39\frac{11}{16}}{\frac{1}{2}} = 79\tfrac{3}{8}.$$

5. Given $a = -180$, $r = -\frac{1}{2}$, and $n = 8$.

$$S_8 = \frac{-180-(-180)(-\frac{1}{2})^8}{1-(-\frac{1}{2})} = \frac{-180+\frac{45}{64}}{\frac{3}{2}} = \frac{-179\frac{19}{64}}{\frac{3}{2}} = -119\frac{17}{32}.$$

6. Given $a = \frac{2}{3}$, $r = \frac{3}{2}$, and $n = 6$.

$$S_6 = \frac{\frac{2}{3}-\frac{2}{3}(\frac{3}{2})^6}{1-\frac{3}{2}} = \frac{-\frac{665}{96}}{-\frac{1}{2}} = \frac{665}{48}.$$

7. Given $a = a^3$, $r = a^2$, and $n = 10$.

$$S_{10} = \frac{a^3 - a^3\cdot(a^2)^{10}}{1-a^2} = \frac{a^3-a^{23}}{1-a^2}.$$

8. Given $a = 2\sqrt{3}$, $r = \sqrt{3}$, and $n = 7$.

$$S_7 = \frac{2\sqrt{3}-2\sqrt{3}(\sqrt{3})^7}{1-\sqrt{3}} = \frac{2\sqrt{3}-162}{1-\sqrt{3}} = 78 + 80\sqrt{3}.$$

9. Given $a = 4$, $r = 3$, and $n = n$.

$$S_n = \frac{4-4\cdot 3^n}{1-3} = \frac{4(1-3^n)}{-2} = -2(1-3^n).$$

10. Given $a = 125$, $r = -\frac{\sqrt{5}}{5}$, and $n = n$.

$$S_n = \frac{125\left[1-\left(-\frac{\sqrt{5}}{5}\right)^n\right]}{1+\frac{\sqrt{5}}{5}}.$$

11. Given $a = 3$, $r = 4$, and $n = n-1$.

$$S_{n-1} = \frac{3-3\cdot 4^{n-1}}{1-4} = \frac{3(1-4^{n-1})}{-3} = 4^{n-1}-1.$$

12. Given $a = 3$, $r = -5$, and $n = n$.

$$S_n = \frac{3[1-(-5)^n]}{1+5} = \frac{1-(-5)^n}{2}.$$

13. Given $a = x$, $r = 4x^3$, and $n = n-2$.

$$S_{n-2} = \frac{x[1-(4x^3)^{n-2}]}{1-4x^3}.$$

14. Given $l = ar^{n-1}$ and $rl = ar^n$.

$$S_n = \frac{a-ar^n}{1-r} = \frac{a-rl}{1-r}.$$

15. Given $a = 60$, $r = \frac{1}{2}$, and $n = 6$.

$$t_6 = 60 \cdot (\tfrac{1}{2})^5$$
$$= \tfrac{15}{8} = 1\tfrac{7}{8}.$$
$$S_6 = \frac{60 - \frac{1}{2}(\frac{15}{8})}{1 - \frac{1}{2}} = \frac{59\frac{1}{16}}{\frac{1}{2}} = 118\tfrac{1}{8} \text{ inches.}$$

It rebounds five times in all, or $58\frac{1}{8}$ inches. Total distance covered is $176\frac{1}{4}$ inches.

Page 451

2. $$S_\infty = \frac{a}{1 - r}.$$
$$S_\infty = \frac{1}{1 - \frac{1}{2}} = 2.$$

3. $$S_\infty = \frac{3}{1 - (-\frac{1}{3})} = \frac{3}{\frac{4}{3}} = 2\tfrac{1}{4}.$$

4. $$S_\infty = \frac{5}{1 - (-\frac{2}{5})} = \frac{5}{\frac{7}{5}} = 3\tfrac{4}{7}.$$

5. $$S_\infty = \frac{2}{1 - \frac{\sqrt{2}}{2}} = \frac{2 \cdot 2}{2 - \sqrt{2}} = \frac{8 + 4\sqrt{2}}{2} = 4 + 2\sqrt{2}.$$

6. $$S_\infty = \frac{5a}{1 - \frac{1}{2}} = 10a.$$

7. $$S_\infty = \frac{1}{1 - x}.$$

8. $$S_\infty = \frac{5}{1 - \frac{\sqrt{5}}{5}} = \frac{25}{5 - \sqrt{5}} = \frac{25(5 + \sqrt{5})}{20} = \frac{5(5 + \sqrt{5})}{4}.$$

9. $$S_\infty = \frac{1}{1 - \frac{1}{x}} = \frac{x}{x - 1}.$$

10. $.121212 \cdots = \frac{12}{100} + \frac{12}{10000} + \frac{12}{1000000} \cdots$

$$S_\infty = \frac{\frac{12}{100}}{1 - \frac{1}{100}} = \tfrac{4}{33}.$$

11. $.666 \cdots = \frac{6}{10} + \frac{6}{100} + \frac{6}{1000} \cdots$

$$S_\infty = \frac{\frac{6}{10}}{1 - \frac{1}{10}} = \tfrac{2}{3}.$$

12. $.151515 \cdots = \frac{15}{100} + \frac{15}{10000} + \frac{15}{1000000} \cdots$

$$S_\infty = \frac{\frac{15}{100}}{1 - \frac{1}{100}} = \tfrac{5}{33}.$$

13. $.3232\cdots = \frac{32}{100} + \frac{32}{10000} + \frac{32}{1000000}\cdots.$

$$S_\infty = \frac{\frac{32}{100}}{1 - \frac{1}{100}} = \frac{32}{99}.$$

14. $25.2727\cdots = 25 + \frac{27}{100} + \frac{27}{10000} + \frac{27}{1000000}\cdots.$

$$S_\infty = \frac{\frac{27}{100}}{1 - \frac{1}{100}} = \frac{3}{11}.$$

$$25 + \frac{3}{11} = 25\tfrac{3}{11}.$$

15. $.71515\cdots = \frac{7}{10} + \frac{15}{1000} + \frac{15}{100000} + \frac{15}{10000000}\cdots.$

$$S_\infty = \frac{\frac{15}{1000}}{1 - \frac{1}{100}} = \frac{1}{66}.$$

$$\frac{7}{10} + \frac{1}{66} = \frac{118}{165}.$$

16. $.3108108\cdots = \frac{3}{10} + \frac{108}{10000} + \frac{108}{10000000}\cdots.$

$$S_\infty = \frac{\frac{108}{10000}}{1 - \frac{1}{1000}} = \frac{2}{185}.$$

$$\frac{3}{10} + \frac{2}{185} = \frac{115}{370} = \frac{23}{72}.$$

17. Here $a = 5 \cdot 80$ and $r = \frac{99}{100}.$

$$S_\infty = \frac{400}{1 - \frac{99}{100}} = 40{,}000 \text{ feet.}$$

Page 452

1. Let a, ar, and ar^2 be, respectively, the hundreds', the tens', and the units' digit.

$$a + ar + ar^2 = 13. \qquad (1)$$

$$\frac{100\,a + 10\,ar + ar^2}{13} = 10 + \frac{9}{13}. \qquad (2)$$

From (1), $a(1 + r + r^2) = 13. \qquad (3)$

From (2), $a(100 + 10\,r + r^2) = 139. \qquad (4)$

Dividing (3) by (4) and simplifying,

$$126\,r^2 + 9\,r - 1161 = 0. \qquad (5)$$

$$14\,r^2 + r - 129 = 0. \qquad (6)$$

$$(14\,r + 43)(r - 3) = 0.$$

$$r = 3, -\tfrac{43}{14}. \qquad \text{(Root } -\tfrac{43}{14} \text{ rejected.)}$$

Substituting in (1), $a + 3\,a + 9\,a = 13.$

$$a = 1.$$

The number is 139.

2.
$$t_{11} = ar^{n-1}.$$
$$t_{11} = 1 \cdot 2^{10} = 1024.$$
$$l_{11} = a + (n-1)d.$$
$$l_{11} = 1 + 10 \cdot 100 = 1001.$$
$$\tfrac{1001}{1024} = .977+, \text{ or } 97.7+\%.$$

3.
$$S_n = \frac{a - ar^n}{1-r}.$$
$$S_{10} = \frac{1 - 1 \cdot 2^{10}}{1-2} = \frac{-1023}{-1} = 1023.$$
$$S = \frac{n}{2}[2a + (n-1)d].$$
$$S = 5(2 + 9 \cdot 100) = 4510.$$

The ratio is 1023 : 4510.
$$\tfrac{1023}{4510} = \tfrac{93}{410}, \text{ or } 22.6\%.$$

4. (*a*) The amount of \$500 put at interest for three years at 6%, interest compounded annually.

(*b*) The amount of \$500 at interest for five years at 5%, interest compounded annually.

(*c*) The amount of \$500 at interest for six years at 3%, interest compounded annually.

5. $100 \cdot (1.04)^3 = \$112.49.$
$100 \cdot (1.04)^5 = \$121.67.$

6. $100(1.03)^4 = \$112.55.$
$100(1.03)^6 = \$119.41.$

7. Here $a = 200$, $r = 1.04$, and $n = 5$.
$$S_n = \frac{a - ar^n}{1-r} = \frac{200 - 200(1.04)^5}{1-1.04} = \frac{-43.3305}{-.04} = 1083.26.$$
$$S_{10} = \frac{200 - 200(1.04)^{10}}{1 - 1.04}.$$

9.
$$\frac{Sr(1+r)^n}{(1+r)^n - 1} = p.$$
$$\frac{1000 \times .06(1+.06)^3}{(1+.06)^3 - 1} = \$374.10.$$

10.
$$p = \frac{Sr(1+r)^n}{(1+r)^n - 1}$$
$$= \frac{2000 \times .05(1+.05)^5}{(1+.05)^5 - 1}$$
$$= \frac{100 \cdot 1.27628}{1.27628 - 1}$$
$$= \$461.95.$$

CR

11. Here $a = 10{,}000$, $r = 90\%$, and $n = 7$.

$$t_7 = 10{,}000 \cdot (.90)^6 = \$5314.41.$$

12. Here $a = 1$, $r = \frac{2}{3}$, and $n = 5$.

$$t_5 = 1 \cdot (\tfrac{2}{3})^4 = \tfrac{16}{81}.$$

13. From geometry, $FD = \dfrac{BC}{2}$, $DE = \dfrac{AC}{2}$, $FE = \dfrac{AB}{2}$, $GH = \dfrac{FE}{2}$, etc.

It follows from the above that the perimeter of each successive triangle will be one half that of the triangle just outside it.

Hence the successive perimeters form the terms of a geometrical progression in which r is $\frac{1}{2}$.

If $$AB = 5,$$
then $$\text{perimeter } ABC = 15.$$

The progression is 15, $\frac{15}{2}$, $\frac{15}{4}$, etc.

Then $$S = \frac{15}{1 - \frac{1}{2}} = 30.$$

14. $HG = \frac{1}{2}$ of diagonal AC. (See Exercise 13, above.)

But $AC = DC\sqrt{2}$. (The diagonal of a square equals its side $\times \sqrt{2}$.)

Then $HG = \dfrac{DC\sqrt{2}}{2}$. Also $MN = \dfrac{HG\sqrt{2}}{2}$, etc.

Then $$\text{perimeter } HGFE = \text{perimeter } \frac{ABCD \times \sqrt{2}}{2},$$

and $$\text{perimeter } MNDL = \text{perimeter } \frac{HGFE\sqrt{2}}{2} \text{ etc.}$$

The perimeter of each square after the first equals the perimeter of the preceding square multiplied by $\dfrac{\sqrt{2}}{2}$ or $\dfrac{1}{\sqrt{2}}$. Therefore the successive perimeters form the terms of a geometrical progression in which $r = \dfrac{1}{\sqrt{2}}$.

If $AB = 4$, then

$$S = \frac{16}{1 - \frac{1}{\sqrt{2}}} = \frac{16}{\frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{16\sqrt{2}}{\sqrt{2}-1} = \frac{16\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{32 + 16\sqrt{2}}{1}$$

$$= 32 + 16(1.4142+) = 54.627.$$

Page 457

1. $(a + b)^7 = a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7.$
2. $(a - 1)^6 = a^6 - 6a^5 + 15a^4 - 20a^3 + 15a^2 - 6a + 1.$
3. $(a + 1)^7 = a^7 + 7a^6 + 21a^5 + 35a^4 + 35a^3 + 21a^2 + 7a + 1.$
4. $(a + 2)^5 = a^5 + 10a^4 + 40a^3 + 80a^2 + 80a + 32.$
5. $(a + 3)^6 = a^6 + 18a^5 + 135a^4 + 540a^3 + 1215a^2 + 1458a + 729.$
6. $(2 - a)^7 = 128 - 448a + 672a^2 - 560a^3 + 280a^4 - 84a^5 + 14a^6 - a^7.$

7. $(a+b)^{21} = a^{21} + 21\,a^{20}b + 210\,a^{19}b^2 + 1330\,a^{18}b^3.$

8. $(a+b)^{30} = a^{30} + 30\,a^{29}b + 435\,a^{28}b^2 + 4060\,a^{27}b^3.$

9. $(a+1)^{32} = a^{32} + 32\,a^{31} + 496\,a^{30} + 4960\,a^{29}.$

10. $(a-2)^{25} = a^{25} - 50\,a^{24} + 1200\,a^{23} - 18{,}400\,a^{22}.$

11. $$(a^2+2\,b)^5 = (a^2)^5 + 5\,(a^2)^4(2\,b)^1 + 10\,(a^2)^3(2\,b)^2 + 10\,(a^2)^2(2\,b)^3 + 5\,(a^2)^1(2\,b)^4 + (2\,b)^5$$
$$= a^{10} + 10\,a^8b + 40\,a^6b^2 + 80\,a^4b^3 + 80\,a^2b^4 + 32\,b^5.$$

12. $$(a^2+2)^6 = (a^2)^6 + 6\,(a^2)^5\cdot 2^1 + 15\,(a^2)^4\cdot 2^2 + 20\,(a^2)^3\cdot 2^3 + 15\,(a^2)^2\cdot 2^4 + 6\,(a^2)^1\cdot 2^5 + 2^6$$
$$= a^{12} + 12\,a^{10} + 60\,a^8 + 160\,a^6 + 240\,a^4 + 192\,a^2 + 64.$$

13. $$(a^2-2\,b)^7 = (a^2)^7 - 7\,(a^2)^6(2\,b) + 21\,(a^2)^5(2\,b)^2 - 35\,(a^2)^4(2\,b)^3 + 35\,(a^2)^3(2\,b)^4 - 21\,(a^2)^2(2\,b)^5 + 7\,(a^2)^1(2\,b)^6 - (2\,b)^7$$
$$= a^{14} - 14\,a^{12}b + 84\,a^{10}b^2 - 280\,a^8b^3 + 560\,a^6b^4 - 672\,a^4b^5 + 448\,a^2b^6 - 128\,b^7.$$

14. $$\left(\frac{a^2+1}{b}\right)^5 = \frac{(a^2+1)^5}{b^5} = \frac{a^{10} + 5\,a^8 + 10\,a^6 + 10\,a^4 + 5\,a^2 + 1}{b^5}.$$

15. $$\left(\frac{a^2-1}{a^3}\right)^6 = \frac{(a^2-1)^6}{a^{18}}$$
$$= \frac{a^{12} - 6\,a^{10} + 15\,a^8 - 20\,a^6 + 15\,a^4 - 6\,a^2 + 1}{a^{18}}$$
$$= \frac{1}{a^6} - \frac{6}{a^8} + \frac{15}{a^{10}} - \frac{20}{a^{12}} + \frac{15}{a^{14}} - \frac{6}{a^{16}} + \frac{1}{a^{18}}.$$

16. $$(a^2+3\,b)^{20} = (a^2)^{20} + 20\,(a^2)^{19}(3\,b)^1 + 190\,(a^2)^{18}(3\,b)^2 + 1140\,(a^2)^{17}(3\,b)^3$$
$$= a^{40} + 60\,a^{38}b + 1710\,a^{36}b^2 + 30{,}780\,a^{34}b^3.$$

17. $$(a^2-3\,b^2)^{10} = (a^2)^{10} - 10\,(a^2)^9(3\,b^2)^1 + 45\,(a^2)^8(3\,b^2)^2 - 120\,(a^2)^7(3\,b^2)^3$$
$$= a^{20} - 30\,a^{18}b^2 + 405\,a^{16}b^4 - 3240\,a^{14}b^6.$$

18. $$\left(\frac{a^2-1}{a}\right)^{30} = \left(a - \frac{1}{a}\right)^{30}$$
$$= a^{30} - 30\,a^{29}\left(\frac{1}{a}\right)^1 + 435\,a^{28}\left(\frac{1}{a}\right)^2 - 4060\,a^{27}\left(\frac{1}{a}\right)^3$$
$$= a^{30} - 30\,a^{28} + 435\,a^{26} - 4060\,a^{24}\cdots.$$

19. $$\left(\frac{a}{b} + \frac{b}{a}\right)^{20} = \left(\frac{a}{b}\right)^{20} + 20\left(\frac{a}{b}\right)^{19}\left(\frac{b}{a}\right)^1 + 190\left(\frac{a}{b}\right)^{18}\left(\frac{b}{a}\right)^2 + 1140\left(\frac{a}{b}\right)^{17}\left(\frac{b}{a}\right)^3$$
$$= \frac{a^{20}}{b^{20}} + \frac{20\,a^{18}}{b^{18}} + \frac{190\,a^{16}}{b^{16}} + \frac{1140\,a^{14}}{b^{14}}.$$

20. $$\left(\frac{2\,x}{y^3} - \frac{y^4}{x^5}\right)^8 = \left(\frac{2\,x}{y^3}\right)^8 - 8\left(\frac{2\,x}{y^3}\right)^7\left(\frac{y^4}{x^5}\right) + 28\left(\frac{2\,x}{y^3}\right)^6\left(\frac{y^4}{x^5}\right)^2 - 56\left(\frac{2\,x}{y^3}\right)^5\left(\frac{y^4}{x^5}\right)^3$$
$$= \frac{256\,x^8}{y^{24}} - \frac{1024\,x^2}{y^{17}} + \frac{1792}{x^4y^{10}} - \frac{1792}{x^{10}y^3}.$$

21. $\left(\dfrac{a^2}{b^3}+\dfrac{2\,b^2}{a^4}\right)^{12}$

$$= \left(\frac{a^2}{b^3}\right)^{12} + 12\left(\frac{a^2}{b^3}\right)^{11}\left(\frac{2\,b^2}{a^4}\right) + 66\left(\frac{a^2}{b^3}\right)^{10}\left(\frac{2\,b^2}{a^4}\right)^2 + 220\left(\frac{a^2}{b^3}\right)^9\left(\frac{2\,b^2}{a^4}\right)^3$$

$$= \frac{a^{24}}{b^{36}} + 24\,\frac{a^{18}}{b^{31}} + 264\,\frac{a^{12}}{b^{26}} + 1760\,\frac{a^6}{b^{21}}.$$

22. $\left(\dfrac{3\,x^5}{y^3}-\dfrac{2\,y^{15}}{9\,x^{12}}\right)^5$

$$= \left(\frac{3\,x^5}{y^3}\right)^5 - 5\left(\frac{3\,x^5}{y^3}\right)^4\left(\frac{2\,y^{15}}{9\,x^{12}}\right)^1 + 10\left(\frac{3\,x^5}{y^3}\right)^3\left(\frac{2\,y^{15}}{9\,x^{12}}\right)^2 - 10\left(\frac{3\,x^5}{y^3}\right)^2\left(\frac{2\,y^{15}}{9\,x^{12}}\right)^3$$

$$= \frac{243\,x^{25}}{y^{15}} - 90\,x^8y^3 + \frac{40\,y^{21}}{3\,x^9} - \frac{80\,y^{39}}{81\,x^{26}}.$$

23. $\left(\dfrac{\sqrt{x}}{y}+\dfrac{\sqrt{y}}{x}\right)^{12}$

$$= \left(\frac{\sqrt{x}}{y}\right)^{12} + 12\left(\frac{\sqrt{x}}{y}\right)^{11}\left(\frac{\sqrt{y}}{x}\right) + 66\left(\frac{\sqrt{x}}{y}\right)^{10}\left(\frac{\sqrt{y}}{x}\right)^2 + 220\left(\frac{\sqrt{x}}{y}\right)^9\left(\frac{\sqrt{y}}{x}\right)^3$$

$$= \frac{x^6}{y^{12}} + \frac{12\,x^{\frac{9}{2}}}{y^{\frac{21}{2}}} + \frac{66\,x^3}{y^9} + \frac{220\,x^{\frac{3}{2}}}{y^{\frac{15}{2}}}.$$

24. $\left(1+\dfrac{1}{n}\right)^n$

$$= 1^n + n\cdot 1^{n-1}\left(\frac{1}{n}\right)^1 + \frac{n\,(n-1)}{2}\cdot 1^{n-2}\left(\frac{1}{n}\right)^2$$

$$+ \frac{n\,(n-1)\,(n-2)}{2\cdot 3}\cdot 1^{n-3}\left(\frac{1}{n}\right)^3$$

$$= 1 + 1 + \frac{n-1}{2\,n} + \frac{(n-1)\,(n-2)}{6\,n^2}.$$

25. $$(a+b)^n = a^n + na^{n-1}b + \frac{n\,(n-1)}{2}\,a^{n-2}b^2$$

$$+ \frac{n\,(n-1)\,(n-2)}{2\cdot 3}\,a^{n-3}b^3$$

$$+ \frac{n\,(n-1)\,(n-2)\,(n-3)}{2\cdot 3\cdot 4}\,a^{n-4}b^4$$

$$+ \frac{n\,(n-1)\,(n-2)\,(n-3)\,(n-4)}{2\cdot 3\cdot 4\cdot 5}\,a^{n-5}b^5.$$

If $n=1$, $(a+b)^n = a^1 + 1\,a^0b + \dfrac{1\cdot 0}{2}\,a^{-1}b^2 = a+b.$

If $n=2$, $(a+b)^n = a^2 + 2\,a^{2-1}b + \dfrac{2\cdot 1}{2}\,a^0b^2 + \dfrac{2\cdot 1\cdot 0}{2\cdot 3}\,a^{-1}b^3$

$$= a^2 + 2\,ab + b^2.$$

If $n = 3$, $(a+b)^n = a^3 + 3a^2b + \frac{3 \cdot 2}{2}a^1b^2 + \frac{3 \cdot 2 \cdot 1}{2 \cdot 3}a^0b^3 + \frac{3 \cdot 2 \cdot 1 \cdot 0}{2 \cdot 3 \cdot 4}a^{-1}b^4$
$= a^3 + 3a^2b + 3ab^2 + b^3.$

If $n = 4$, $(a+b)^n = a^4 + 4a^3b + \frac{4 \cdot 3}{2}a^2b^2 + \frac{4 \cdot 3 \cdot 2}{2 \cdot 3}ab^3 + \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 3 \cdot 4}a^0b^4$
$+ \frac{4 \cdot 3 \cdot 2 \cdot 1 \cdot 0}{2 \cdot 3 \cdot 4 \cdot 5}a^{-1}b^5$
$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4.$

The number of terms in each case is one more than n.

After the $(n+1)$th coefficient a zero factor occurs in the numerator, which makes the coefficient zero.

Where $n = 4$, the sixth term and all terms which follow it in the expansion contain the zero factor $(n - 4)$.

26. $(1.1)^{10} = (1 + .1)^{10}$
$= 1^{10} + 10 \cdot 1^9(.1)^1 + 45 \cdot 1^8 \cdot (.1)^2 + 120 \cdot 1^7 \cdot (.1)^3$
$+ 210 \cdot 1^8(.1)^4$
$= 1 + 1 + .45 + .12 + .021$
$= 2.59 +.$

27. $(5.2)^8 = (5 + .2)^8$
$= 5^8 + 8 \cdot 5^7 \cdot (.2) + 28 \cdot 5^6(.2)^2 + 56 \cdot 5^5 \cdot (.2)^3 + 70 \cdot 5^4 \cdot (.2)^4$
$+ 56 \cdot 5^3 \cdot (.2)^5 + 28 \cdot 5^2 \cdot (.2)^6$
$= 390{,}625 + 125{,}000 + 17{,}500 + 1400 + 70 + 2.24 + .0448$
$= 534{,}597.28 +.$

28. $(1.06)^6 = (1 + .06)^6$
$= 1^6 + 6 \cdot 1^5 \cdot {}^1(.06) + 15 \cdot 1^4 \cdot (.06)^2 +$
$= 1 + .360 + .054 +$
$= 1.41 +.$

29. $(.98)^{11} = (1 - .02)^{11}$
$= 1^{11} - 11 \cdot 1^{10} \cdot (.02) + 55 \cdot 1^9(.02)^2 - 165 \cdot 1^8 \cdot (.02)^3$
$= 1 - .220 + .022 - .00132 +$
$= .80 +.$

30. $(4.9)^9 = (5 - .1)^9$
$= 5^9 - 9 \cdot 5^8 \cdot (.1)^1 + 36 \cdot 5^7(.1)^2 - 84 \cdot 5^6 \cdot (.1)^3 + 126 \cdot 5^5 \cdot (.1)^4$
$- 126 \cdot 5^4(.1)^5 + 84 \cdot 5^3(.1)^6$
$= 1{,}953{,}125 - 351{,}562.5 + 28{,}125 - 1312.5 + 39.375 - .7875$
$+ .0105$
$= 1{,}628{,}413.59 +.$

31. $(2.9)^8 = (3 - .1)^8$
$= 3^8 - 8 \cdot 3^7 \cdot (.1) + 28 \cdot 3^6 \cdot (.1)^2 - 56 \cdot 3^5 \cdot (.1)^3$
$+ 70 \cdot 3^4 \cdot (.1)^4 - 56 \cdot 3^3 \cdot (.1)^5$
$= 6561 - 1749.6 + 204.12 - 13.608 + .567 - .01512$
$= 5002.46.$

Page 459

1. $(1+x)^{\frac{1}{2}} = 1^{\frac{1}{2}} + \frac{1}{2}\cdot 1^{-\frac{1}{2}}\cdot x - \frac{1}{8}\cdot 1^{-\frac{3}{2}}x^2 + \frac{1}{16}\cdot 1^{-\frac{5}{2}}\cdot x^3$
$= 1 + \dfrac{x}{2} - \dfrac{x^2}{8} + \dfrac{x^3}{16}.$

2. $(2+x)^{\frac{1}{2}} = 2^{\frac{1}{2}} + \frac{1}{2}\cdot 2^{-\frac{1}{2}}\cdot x - \frac{1}{8}\cdot 2^{-\frac{3}{2}}x^2 + \frac{1}{16}\cdot 2^{-\frac{5}{2}}\cdot x^3$
$= \sqrt{2} + \dfrac{\sqrt{2}\,x}{4} - \dfrac{\sqrt{2}\,x^2}{32} + \dfrac{\sqrt{2}\,x^3}{128}.$

3. $(3-x)^{\frac{1}{2}} = 3^{\frac{1}{2}} - \frac{1}{2}\cdot 3^{-\frac{1}{2}}x - \frac{1}{8}\cdot 3^{-\frac{3}{2}}x^2 - \frac{1}{16}\cdot 3^{-\frac{5}{2}}x^3$
$= \sqrt{3} - \dfrac{\sqrt{3}\,x}{6} - \dfrac{\sqrt{3}\,x^2}{72} - \dfrac{\sqrt{3}\,x^3}{432}.$

4. $(1+x)^{\frac{1}{3}} = 1^{\frac{1}{3}} + \frac{1}{3}\cdot 1^{-\frac{2}{3}}x - \frac{1}{9}\cdot 1^{-\frac{5}{3}}x^2 + \frac{5}{81}\cdot 1^{-\frac{8}{3}}x^3$
$= 1 + \dfrac{x}{3} - \dfrac{x^2}{9} + \dfrac{5x^3}{81}.$

5. $(2+x)^{\frac{1}{3}} = 2^{\frac{1}{3}} + \frac{1}{3}\cdot 2^{-\frac{2}{3}}x - \frac{1}{9}\cdot 2^{-\frac{5}{3}}x^2 + \frac{5}{81}\cdot 2^{-\frac{8}{3}}\cdot x^3$
$= \sqrt[3]{2} + \dfrac{\sqrt[3]{2}\,x}{6} - \dfrac{\sqrt[3]{2}\,x^2}{36} + \dfrac{5\sqrt[3]{2}\,x^3}{648}.$

6. $(3-x)^{\frac{1}{3}} = 3^{\frac{1}{3}} - \frac{1}{3}\cdot 3^{-\frac{2}{3}}\cdot x - \frac{1}{9}\cdot 3^{-\frac{5}{3}}\cdot x^2 - \frac{5}{81}\cdot 3^{-\frac{8}{3}}\cdot x^3$
$= \sqrt[3]{3} - \dfrac{\sqrt[3]{3}\,x}{9} - \dfrac{\sqrt[3]{3}\,x^2}{81} - \dfrac{5\sqrt[3]{3}\,x^3}{2187}.$

8. $(17)^{\frac{1}{2}} = (16+1)^{\frac{1}{2}}$
$= 16^{\frac{1}{2}} + \frac{1}{2}\cdot 16^{-\frac{1}{2}}\cdot 1^1 - \frac{1}{8}\cdot 16^{-\frac{3}{2}}\cdot 1^2 + \frac{1}{16}\cdot 16^{-\frac{5}{2}}\cdot 1^3$
$= 4 + .125 - .00195$
$= 4.123.$

9. $(28)^{\frac{1}{2}} = (25+3)^{\frac{1}{2}}$
$= 25^{\frac{1}{2}} + \frac{1}{2}\cdot 25^{-\frac{1}{2}}\cdot 3 - \frac{1}{8}\cdot 25^{-\frac{3}{2}}\cdot 3^2 + \frac{1}{16}\cdot 25^{-\frac{5}{2}}\cdot 3^3$
$= 5 + .3 - .009 + .00054$
$= 5.291.$

10. $(38)^{\frac{1}{2}} = (36+2)^{\frac{1}{2}}$
$= 36^{\frac{1}{2}} + \frac{1}{2}\cdot 36^{-\frac{1}{2}}\cdot 2 - \frac{1}{8}\cdot 36^{-\frac{3}{2}}\cdot 2^2$
$= 6 + .166 - .002$
$= 6.164+.$

11. $(78)^{\frac{1}{2}} = (81-3)^{\frac{1}{2}}$
$= 81^{\frac{1}{2}} - \frac{1}{2}\cdot 81^{-\frac{1}{2}}\cdot 3 - \frac{1}{8}\cdot 81^{-\frac{3}{2}}\cdot 3^2$
$= 9 - .1666 - .0015$
$= 8.831.$

12. $(125)^{\frac{1}{2}} = (121 + 4)^{\frac{1}{2}}$
$= 121^{\frac{1}{2}} + \frac{1}{2} \cdot 121^{-\frac{1}{2}} \cdot 4^1 - \frac{1}{8} \cdot 121^{-\frac{3}{2}} \cdot 4^2 + \cdots$
$= 11 + .1818 - .0015 + \cdots$
$= 11.180.$

14. $(79)^{\frac{1}{2}} = (81 - 2)^{\frac{1}{2}}$
$= 81^{\frac{1}{2}} - \frac{1}{2} \cdot 81^{-\frac{1}{2}} \cdot 2 + \cdots$
$= 9 - .1111 + \cdots$
$= 8.888.$

15. $(28)^{\frac{1}{3}} = (27 + 1)^{\frac{1}{3}}$
$= 27^{\frac{1}{3}} + \frac{1}{3} \cdot 27^{-\frac{2}{3}} \cdot 1^1 - \frac{1}{9} \cdot 27^{-\frac{5}{3}} \cdot 1^2$
$= 3 + .0370 - .0004$
$= 3.037.$

16. $(66)^{\frac{1}{3}} = (64 + 2)^{\frac{1}{3}}$
$= 64^{\frac{1}{3}} + \frac{1}{3} \cdot 64^{-\frac{2}{3}} \cdot 2 - \frac{1}{9} \cdot 64^{-\frac{5}{3}} \cdot 2^2 +$
$= 4 + .0416 - .0004$
$= 4.041.$

17. $(30)^{\frac{1}{3}} = (27 + 3)^{\frac{1}{3}}$
$= 27^{\frac{1}{3}} + \frac{1}{3} \cdot 27^{-\frac{2}{3}} \cdot 3 - \frac{1}{9} \cdot 27^{-\frac{5}{3}} \cdot 3^2$
$= 3 + .1111 - .0041$
$= 3.1070.$

18. $(700)^{\frac{1}{3}} = (1000 - 300)^{\frac{1}{3}}$
$= 1000^{\frac{1}{3}} - \frac{1}{3} \cdot 1000^{-\frac{2}{3}} \cdot 300 - \frac{1}{9} \cdot 1000^{-\frac{5}{3}} \cdot 300^2$
$- \frac{5}{81} \cdot 1000^{-\frac{8}{3}} \cdot 300^3 - \frac{10}{243} \cdot 1000^{-\frac{11}{3}} \cdot 300^4$
$- \frac{22}{729} \cdot 1000^{-\frac{14}{3}} \cdot 300^5$
$= 10 - 1 - .1 - .0166 - .0033 - .0007$
$= 8.879.$

Page 460

1. $6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720.$

2. $4! \cdot 3! = 4 \cdot 3 \cdot 2 \cdot 3 \cdot 2 = 144.$

3. $5! \cdot 2! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 2 = 240.$

4. $6! \div 2! = \dfrac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{2} = 360.$

5. $4! - 3! \cdot 2! = 4 \cdot 3 \cdot 2 - 3 \cdot 2 \cdot 2 = 24 - 12 = 12.$

6. $n! \div (n-1)! = \dfrac{n(n-1)(n-2)(n-3) \text{ etc.}}{(n-1)(n-2)(n-3) \text{ etc.}} = n.$

7. $\dfrac{n(n-1)(n-2)\cdots(n-r+2)}{(r-1)!}.$

If $n = 7$, and $r = 5$, $\dfrac{7 \cdot 6 \cdot 5 \cdot 4}{4 \cdot 3 \cdot 2} = 35.$

8. If $n = 15$, and $r = 8$,

$$\frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 6435.$$

9. If $n = 21$, and $r = 12$,

$$\frac{21 \cdot 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 352{,}716.$$

10. If $n = 20$, and $r = 15$,

$$\frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 38{,}760.$$

11. If $n = 18$, and $r = 17$,

$$\frac{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 153.$$

12. If $n = 10$, and $r = 11$,

$$\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 1.$$

Page 461

2. Sixth term of $(a + b)^9 = \dfrac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{5 \cdot 4 \cdot 3 \cdot 2} a^4b^5 = 126\, a^4b^5.$

3. Fourth term of $(a + b)^{20} = \dfrac{20 \cdot 19 \cdot 18}{3 \cdot 2} a^{17}b^3 = 1140\, a^{17}b^3.$

4. Seventh term of $(a - b)^{10} = \dfrac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} a^4b^6 = 210\, a^4b^6.$

5. Eighth term of $(a - b)^{15}$

$$= -\frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} a^8b^7 = -\, 6435\, a^8b^7.$$

6. Fourth term of $\left(a + \dfrac{1}{a}\right)^{30} = \dfrac{30 \cdot 29 \cdot 28}{3 \cdot 2} a^{27} \left(\dfrac{1}{a}\right)^3 = 4060\, a^{24}.$

7. Fifth term of $(a^2 - b)^{20} = \dfrac{20 \cdot 19 \cdot 18 \cdot 17}{4 \cdot 3 \cdot 2} (a^2)^{16}b^4 = 4845\, a^{32}b^4.$

8. Sixth term of $\left(\dfrac{a}{b} - \dfrac{b^2}{a}\right)^{18} = -\dfrac{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14}{5 \cdot 4 \cdot 3 \cdot 2} \left(\dfrac{a}{b}\right)^{13} \left(\dfrac{b^2}{a}\right)^5 = -\, 8568 \dfrac{a^8}{b^3}.$

9. Ninth term of $(x^2 - x)^{16} = \dfrac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} (x^2)^8 x^8$

$$= 12{,}870\, x^{24}.$$

10. Seventh term of $\left(\dfrac{a^2}{b} - \dfrac{2\, b^2}{a}\right)^{14} = \dfrac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} \left(\dfrac{a^2}{b}\right)^8 \left(\dfrac{2\, b^2}{a}\right)^6$

$$= 192{,}192\, a^{10}b^4.$$

11. Fifth term of $\left(\sqrt{x}-\sqrt{\frac{y}{x}}\right)^{15}=\frac{15\cdot 14\cdot 13\cdot 12}{4\cdot 3\cdot 2}(\sqrt{x})^{11}\left(\sqrt{\frac{y}{x}}\right)^4$

$=1365\,x^{\frac{7}{2}}y^2.$

12. If $x^{r-1}=x^5$, $r=6$. The coefficient of $x^5=\frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{5\cdot 4\cdot 3\cdot 2}=252.$

13. If $(x^2)^{r-1}=x^8$, $r=5$. The coefficient of $x^8=\frac{16\cdot 15\cdot 14\cdot 13}{4\cdot 3\cdot 2}=1820.$

14. If $(x^3)^{15-r+1}=x^{15}$, $r=11$. The coefficient of x^{15}

$=\frac{15\cdot 14\cdot 13\cdot 12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6}{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}=3003.$

15. If $(x^2)^{14-r+1}(x^{-1})^{r-1}=x^{10}$, $r=7$. The coefficient of x^{10}

$=\frac{14\cdot 13\cdot 12\cdot 11\cdot 10\cdot 9}{6\cdot 5\cdot 4\cdot 3\cdot 2}=3003.$

Page 463

1. $$\frac{\dfrac{4a^2+a-3}{a^2-1}}{4+\dfrac{4a-1}{a^2-a-2}}=\frac{(4a-3)(a+1)}{(a-1)(a+1)}\times\frac{(a+1)(a-2)}{(2a-3)(2a+3)}$$

$$=\frac{(4a-3)(a+1)(a-2)}{(a-1)(2a-3)(2a+3)}.$$

2. $\frac{1 \text{ kilometer}}{1 \text{ mile}}=\frac{.62}{1}=.62.$

3. $\frac{1 \text{ liter}}{1 \text{ quart}}=\frac{(.001)(39.4)^3}{\frac{231}{4}}=1.05+.$

4. $\frac{100\times 160}{43560}=\frac{400}{1089}.$

5. $\frac{5\frac{15}{16}\times 3\frac{5}{8}}{7\frac{1}{4}\times 4\frac{3}{4}}=\frac{95}{16}\times\frac{29}{8}\times\frac{4}{29}\times\frac{4}{19}=\frac{5}{8}.$

6. Let $5x$ and $7x$ equal the required parts.

$5x+7x=24{,}000.$
$12x=24{,}000.$
$x=2000,\ 5x=10{,}000,$
and $7x=14{,}000.$

7. Let $2x$, $5x$, $7x$, and $9x$ equal the required parts.

$2x+5x+7x+x=690.$
$x=30.$
$2x=60,\ 5x=150,$
and $7x=210, 9x=270.$

8. $\frac{x}{x+2}=\frac{x^2+5x}{(x+2)(x+5)}.$

$\frac{x+3}{x+5}=\frac{x^2+5x+6}{(x+2)(x+5)}.$

Since $x^2+5x<x^2+5x+6,$

then $\frac{x}{x+2}<\frac{x+3}{x+5}.$

9. $\frac{3}{4}=\frac{27}{36}.$

$\frac{7}{9}=\frac{28}{36}.$

Since $\frac{28}{36}>\frac{27}{36},$

then $\frac{7}{9}>\frac{3}{4}.$

CR

Page 466

1. (a) $\frac{3}{m}=\frac{m}{27}.$

$m^2 = 3 \cdot 27.$

$m = \pm 9.$

(b) $\frac{\frac{4}{7}}{m}=\frac{m}{\frac{7}{9}}.$

$m^2 = \frac{4}{9}.$

$m = \pm \frac{2}{3}.$

(c) $\frac{\frac{81}{28}}{m}=\frac{m}{\frac{7}{9}}.$

$m^2 = \frac{9}{4}.$

$m = \pm \frac{3}{2}.$

(d) $\frac{\frac{x}{y}}{m}=\frac{m}{xy}.$

$m^2 = x^2.$

$m = \pm x.$

2. (a) $\frac{9}{6}=\frac{6}{t}.$

$9t = 36.$

$t = 4.$

(b) $\frac{180}{60}=\frac{60}{t}.$

$180t = 3600.$

$t = 20.$

(c) $\frac{216}{36}=\frac{36}{t}.$

$216t = 36^2$

$t = 6.$

3. (a) $\frac{14}{10}=\frac{7}{f}.$

$14f = 70.$

$f = 5.$

(b) $\frac{27}{3}=\frac{36}{f}.$

$27f = 108.$

$f = 4.$

(c) $\frac{96}{12}=\frac{8}{f}.$

$96f = 96.$

$f = 1.$

4. (a) $5x = 9y.$

$5 : 9 = y : x.$

$9 : 5 = x : y.$

$9 : x = 5 : y.$

(b) $(a+3)\cdot 2 = (a+1)\cdot 3.$

$(a+3) : (a+1) = 3 : 2.$

$(a+3) : 3 = (a+1) : 2.$

$(a+1) : (a+3) = 2 : 3.$

(c) $x^2 - y^2 = r^2 - s^2.$

$(x-y)(x+y) = (r-s)(r+s).$

$(x-y) : (r-s) = (r+s) : (x+y)$

$(x-y) : (r+s) = (r-s) : (x+y).$

$(r+s) : (x-y) = (x+y) : (r-s).$

5 (a) $\frac{2}{10} = \frac{3}{15}.$

(b) $\frac{2}{20}=\frac{9}{x}.$

6. (a) $\frac{6}{5} = \frac{12}{10}.$

(b) $\frac{x}{2}=\frac{30}{12}.$

7. $\frac{a}{b}=\frac{c}{d}.$

By alternation, $\frac{a}{c}=\frac{b}{d}.$

8. $\frac{a}{b}=\frac{c}{d}.$

By inversion, $\frac{b}{a}=\frac{d}{c}.$

9. $\frac{a}{b}=\frac{c}{d}.$

Like powers of equal ratios are equal.

$\frac{a^n}{b^n}=\frac{c^n}{d^n}.$

10. $$\frac{a}{b}=\frac{c}{d}.$$

Like roots of equal ratios are equal.

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\frac{\sqrt[n]{c}}{\sqrt[n]{d}}.$$

11. $$\frac{a}{b}=\frac{c}{d}.$$

Adding 1 to each member,

$$\frac{a+b}{b}=\frac{c+d}{d}.$$

By addition.

12. $$\frac{a}{b}=\frac{c}{d}.$$

By inversion,

$$\frac{b}{a}=\frac{d}{c}.$$

Adding 1 to each member,

$$\frac{a+b}{a}=\frac{c+d}{c}.$$

By addition.

13. $$\frac{a}{b}=\frac{c}{d}.$$

Subtracting 1 from each member,

$$\frac{a-b}{b}=\frac{c-d}{d}.$$

By subtraction.

14. $$\frac{a}{b}=\frac{c}{d}.$$

By inversion,

$$\frac{b}{a}=\frac{d}{c}.$$

Subtracting each member from 1,

$$\frac{a-b}{a}=\frac{c-d}{c}.$$

By subtraction.

15. $$\frac{a}{b}=\frac{c}{d}.$$

Adding 1 to each member,

$$\frac{a+b}{b}=\frac{c+d}{d}. \qquad (1)$$

Subtracting each member from 1,

$$\frac{a-b}{b}=\frac{c-d}{d}. \qquad (2)$$

Dividing (1) by (2),

$$\frac{a+b}{a-b}=\frac{c+d}{c-d}.$$

By addition and subtraction.

16. $$\frac{a}{b}=\frac{c}{d}.$$

Multiplying both members by 5,

$$\frac{5a}{b}=\frac{5c}{d}.$$

17. $$\frac{a}{b}=\frac{c}{d}.$$

Multiplying both members by $\frac{2}{7}$,

$$\frac{2a}{7b}=\frac{2c}{7d}.$$

18. $$\frac{a}{b}=\frac{c}{d}.$$

Multiplying both members by $\frac{a}{b}$,

$$\frac{a^2}{b^2}=\frac{ac}{bd}.$$

19. $$\frac{a}{b}=\frac{c}{d}.$$

Multiplying both members by 5,

$$\frac{5a}{b}=\frac{5c}{d}.$$

By addition and subtraction,

$$\frac{5a+b}{5a-b}=\frac{5c+d}{5c-d}.$$

20. $$\frac{a}{b}=\frac{c}{d}.$$

Squaring both members,

$$\frac{a^2}{b^2}=\frac{c^2}{d^2}.$$

By subtraction,

$$\frac{a^2-b^2}{b^2}=\frac{c^2-d^2}{d^2}.$$

21. $$\frac{a}{b}=\frac{c}{d}.$$

Squaring both members, $$\frac{a^2}{b^2}=\frac{c^2}{d^2}.$$

Dividing both members by 7, $$\frac{a^2}{7b^2}=\frac{c^2}{7d^2}.$$

By subtraction, $$\frac{a^2-7b^2}{7b^2}=\frac{c^2-7d^2}{7d^2}.$$

Multiplying both members by 7, $$\frac{a^2-7b^2}{b^2}=\frac{c^2-7d^2}{d^2}.$$

22. Now $$\frac{a}{b}=\frac{c}{d}. \quad (1)$$

By addition, $$\frac{a+b}{a}=\frac{c+d}{c}. \quad (2)$$

By subtraction, $$\frac{a-b}{b}=\frac{c-d}{d}. \quad (3)$$

(2) · (3), $$\frac{a^2-b^2}{ab}=\frac{c^2-d^2}{cd}. \quad (4)$$

(4) · $\frac{1}{2}$, $$\frac{a^2-b^2}{2ab}=\frac{c^2-d^2}{2cd}.$$

23. $$\frac{a}{b}=\frac{c}{d}. \quad (1)$$

Squaring, $$\frac{a^2}{b^2}=\frac{c^2}{d^2}. \quad (2)$$

Multiplying by $\frac{7}{3}$, $$\frac{7a^2}{3b^2}=\frac{7c^2}{3d^2}. \quad (3)$$

By subtraction, $$\frac{7a^2-3b^2}{7a^2}=\frac{7c^2-3d^2}{7c^2}. \quad (4)$$

By alternation, $$\frac{7a^2-3b^2}{7c^2-3d^2}=\frac{7a^2}{7c^2}=\frac{a^2}{c^2}. \quad (5)$$

Writing (1) by alternation and then multiplying by $\frac{a}{c}$,
$$\frac{a^2}{c^2}=\frac{ab}{cd}. \quad (6)$$

From (6), $$\frac{a^2}{c^2}=\frac{5ab}{5cd}. \quad (7)$$

From (5) and (7), $$\frac{7a^2-3b^2}{7c^2-3d^2}=\frac{5ab}{5cd},$$

or $$\frac{7a^2-3b^2}{5ab}=\frac{7c^2-3d^2}{5cd}.$$

24. $$\frac{a}{b}=\frac{c}{d}. \quad (1)$$

Cubing both members, $$\frac{a^3}{b^3}=\frac{c^3}{d^3}. \quad (2)$$

By addition and subtraction, $$\frac{a^3+b^3}{a^3-b^3}=\frac{c^3+d^3}{c^3-d^3}. \quad (3)$$

By addition and subtraction in (1), $$\frac{a+b}{a-b}=\frac{c+d}{c-d}. \quad (4)$$

(3) ÷ (4), $$\frac{a^2-ab+b^2}{a^2+ab+b^2}=\frac{c^2-cd+d^2}{c^2+cd+d^2}.$$

By inversion and alternation, $$\frac{a^2+ab+b^2}{c^2+cd+d^2}=\frac{a^2-ab+b^2}{c^2-cd+d^2}.$$

25. In a series of equal ratios the sum of the antecedents is to the sum of the consequents as any antecedent is to its consequent.

26. $$\frac{a}{b}=\frac{c}{d}=\frac{5}{6}. \quad (1)$$

$$\frac{a}{b}=\frac{5}{6}. \quad (2)$$

Applying the theorem of Exercise 25 to (2),

$$\frac{a+5}{5}=\frac{b+6}{6}. \quad (3)$$

Then from (3) and (1),

$$\frac{a+5}{b+6}=\frac{5}{6}=\frac{c}{d}.$$

27. $2x:(x+8)=10:3.$

$$6x=10x+80.$$
$$x=-20.$$

28. $$25:x=x:169.$$
$$x^2=169\cdot 25.$$
$$x=\pm 5\cdot 13=\pm 65.$$

29. $$5:4=(x-3):(x-4).$$
$$5x-20=4x-12.$$
$$x=8.$$

30. $(15+x):(15-x)=13:17.$

By addition and subtraction,

$$\frac{30}{2x}=\frac{30}{-4}.$$
$$x=-2.$$

31. $$(10+x):(20+3x)=(10-x):-3x.$$
$$200+10x-3x^2=-30x-3x^2.$$
$$40x=-200.$$
$$x=-5.$$

32. $$\frac{\sqrt{x}+2}{\sqrt{x}-2}=\frac{x+1}{x-7}.$$

By addition and subtraction, $$\frac{2\sqrt{x}}{4}=\frac{2x-6}{8}.$$

$$4\sqrt{x}=2x-6.$$
$$x-2\sqrt{x}-3=0.$$
$$(\sqrt{x}-3)(\sqrt{x}+1)=0.$$

Then $$x=9 \text{ or } 1. \text{ (Root 1 rejected.)}$$

33. Let x and y be the first and third terms, respectively, of the progression.

Then
$$\frac{x}{m} = \frac{m}{y}.$$
$$m^2 = xy.$$
$$m = \pm\sqrt{xy}. \qquad (1)$$

Let $a, ar, ar^2, \cdots,$ be a geometrical progression.

If $a = x$ and $ar^2 = y$, then $r^2 = \frac{y}{x}.$

$$r = \pm\sqrt{\frac{y}{x}} = \pm\frac{1}{x}\sqrt{xy}.$$

Also
$$ar = x \cdot \pm\frac{1}{x}\sqrt{xy} = \pm\sqrt{xy}. \qquad (2)$$

From (1) and (2),
$$m = ar.$$

Page 468

1. (a) $\frac{S_1}{S_2} = \frac{4\pi R_1^2}{4\pi R_2^2} = \frac{R_1^2}{R_2^2}.$

(b) $\frac{S_1}{S_2} = \frac{4\pi(\frac{1}{2}D_1)^2}{4\pi(\frac{1}{2}D_2)^2} = \frac{D_1^2}{D_2^2} = \frac{(2R_1)^2}{(2R_2)^2} = \frac{R_1^2}{R_2^2}.$

2. $\frac{R_1}{R_2} = \frac{1}{10}.$

$\frac{S_1}{S_2} = \frac{4\pi 1^2}{4\pi 10^2} = \frac{1}{100}.$

3. $\frac{S \text{ of earth}}{S \text{ of Mars}} = \frac{7920^2}{4230^2} = \frac{88^2}{47^2} = \frac{7744}{2209}.$

4. $\frac{S \text{ of moon}}{S \text{ of earth}} = \frac{2160^2}{7920^2} = \frac{3^2}{11^2} = \frac{9}{121}.$

5. $\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3} = \frac{\frac{4}{3}\pi(\frac{1}{2}D_1)^3}{\frac{4}{3}\pi(\frac{1}{2}D_2)^3} = \frac{R_1^3}{R_2^3} = \frac{D_1^3}{D_2^3}.$

6. $\frac{\text{Volume of sun}}{\text{Volume of earth}} = \frac{109^3}{1^3} = \frac{1295029}{1}.$

7. $\frac{\text{Volume of Mars}}{\text{Volume of earth}} = \frac{4230^3}{7920^3} = \frac{47^3}{88^3} = \frac{103823}{681472}.$

8. $\frac{\text{Volume of earth}}{\text{Volume of moon}} = \frac{7920^3}{2160^3} = \frac{11^3}{3^3} = \frac{1331}{27}.$

9. $\frac{12^2}{20^2} = \frac{90}{x}.$

$x = 250.$

10. $$\frac{10^2}{x^2} = \frac{147}{300}.$$
$$147x^2 = 30{,}000.$$
$$49x^2 = 10{,}000.$$
$$x^2 = \frac{10000}{49}.$$
$$x = \frac{100}{7} = 14\tfrac{2}{7}.$$

11. $$\frac{AB^2}{AK^2} = \frac{\triangle ABC}{\triangle AKR}.$$
$$\frac{15^2}{AK^2} = \frac{225}{81}.$$
$$AK^2 = \frac{15^2 \cdot 81}{225} = 81.$$
$$AK = 9.$$

12. $$\frac{BC^2}{KR^2} = \frac{\triangle ABC}{\triangle AKR}.$$
$$\frac{13^2}{7^2} = \frac{845}{\triangle AKR}.$$
$$\triangle AKR = \frac{845 \cdot 49}{169} = 5 \cdot 49$$
$$= 245.$$

13. $$\frac{\triangle AKR}{\triangle ABC} = \frac{AR^2}{AC^2}.$$
$$\frac{4}{25} = \frac{AR^2}{15^2}.$$
$$25AR^2 = 15^2 \cdot 4.$$
$$5AR = 15 \cdot 2, \text{ or } AR = 6.$$
$$AC - AR = RC = 15 - 6 = 9.$$

14. $$\frac{\triangle AKR}{\triangle ABC} = \frac{AR^2}{AC^2}.$$
$$\frac{1}{6} = \frac{AR^2}{15^2}.$$
$$AR^2 = \frac{15^2}{6}.$$
$$AR = \tfrac{15}{6}\sqrt{6}$$
$$= 6.12 \text{ inches.}$$

15. $$\frac{\triangle AKR}{\triangle ABC} = \frac{AK^2}{AB^2}.$$
$$\frac{1}{2} = \frac{10^2}{AB^2}.$$
$$AB = 10\sqrt{2} = 14.14.$$
$$KB = AB - AK$$
$$= 14.14 - 10$$
$$= 4.14.$$

16. $$\frac{45}{4\frac{1}{2}} = \frac{p}{8}.$$
$$4.5p = 360.$$
$$p = 80 \text{ feet.}$$

17. Let $x =$ the shorter segment.

Then $24 - x =$ the longer segment.
$$\frac{x}{24 - x} = \frac{15}{25} = \frac{3}{5}.$$
$$5x = 72 - 3x.$$
$$x = 9.$$
$$24 - x = 15.$$

18. $$\frac{D-KRL}{D-ABC} = \frac{DS^3}{DH^3}.$$
$$\frac{8}{27} = \frac{DS^3}{15^3}.$$
$$\frac{2}{3} = \frac{DS}{15}.$$
$$DS = 10 \text{ inches.}$$

19. $$\frac{D-KRL}{D-ABC} = \frac{DS^3}{DH^3}.$$
$$\frac{1}{3} = \frac{DS^3}{18^3}.$$
$$DS^3 = \frac{18^3}{3}.$$
$$DS = \tfrac{1}{3} \cdot 18\sqrt[3]{9}$$
$$= 6 \cdot 2.080$$
$$= 12.48 \text{ inches}$$
and two imaginary roots, which are rejected.

20. (*a*)
$$\frac{D-KRL}{D-ABC}=\frac{DS^3}{DH^3}.$$
$$\frac{1}{8}=\frac{DS^3}{16^3}.$$
$$DS^3=\frac{16^3}{8}.$$

$DS=8$ and two imaginary roots, which are rejected.

(*b*)
$$\frac{1}{8}=\frac{DS^3}{x^3}.$$
$$DS^3=\frac{x^3}{8}.$$
$$DS=\frac{x}{2}.$$

21. Let $x=$ the upper segment of DH.

Then $75-x=$ the lower segment of DH.

$$\frac{D-KRL}{D-ABC}=\frac{x^3}{75^3}.$$
$$\frac{1}{2}=\frac{x^3}{75^3}.$$
$$x=\frac{75}{\sqrt[3]{2}}=\frac{75}{2}\sqrt[3]{4}$$
$$=59.51+.$$

$75-(59.51+)=15.48+$ and two imaginary roots, which are rejected.

22.
$$\frac{V_1}{V_2}=\frac{e_1^3}{e_2^3}=\frac{2^3}{3^3}=\frac{8}{27}.$$

23.
$$\frac{V_1}{V_2}=\frac{R_1^3}{R_2^3}.$$
$$\frac{125}{27}=\frac{R_1^3}{R_2^3}.$$
$$\frac{5}{3}=\frac{R_1}{R_2}.$$

Page 472

2. $x \propto y.$
$$\frac{8}{10}=\frac{15}{y}.$$
$$y=18.75.$$

3. $x \propto y.$
$$\frac{h}{r}=\frac{k}{y}.$$
$$y=\frac{rk}{h}.$$

4. $x \propto y.$
$$x=Ky.$$
$$2=5K.$$
$$K=\tfrac{2}{5}.$$

Page 474

2. $x \propto \frac{1}{y}.$
$$1:x=\tfrac{1}{25}:\tfrac{1}{10}.$$
$$x=2\tfrac{1}{2}.$$

3. $y \propto \frac{1}{z}.$
$$h:y=\frac{1}{k}:\frac{1}{r}.$$
$$y=\frac{hk}{r}.$$

4. $m \propto \frac{1}{n}.$
$$2:m=3:\tfrac{1}{12}.$$
$$m=\tfrac{1}{18}.$$

5. $t \propto \frac{1}{n}.$
$$2:8=\frac{1}{8}:\frac{1}{n}.$$
$$n=2.$$

6. $w \propto \frac{1}{d}.$
$$\frac{100}{w}=\frac{\frac{1}{4000}}{\frac{1}{5000}}=\frac{5000}{4000}.$$
$$w=80.$$

7. $t \propto \frac{1}{r}.$
$$t=K\cdot\frac{1}{r}.$$
$$4=\frac{K}{25}.$$
$$K=100.$$

8. $w \propto \frac{1}{d}.$
$$w=K\frac{1}{d}.$$
$$200=K\tfrac{1}{4000}.$$
$$K=800{,}000.$$

Page 476

2. $x \propto yz$.

$$\frac{10}{x} = \frac{15 \cdot 6}{9 \cdot 8}.$$
$$x = 8.$$

3. $A \propto hb$.

$$\frac{30}{A} = \frac{5 \cdot 12}{7 \cdot 10}.$$
$$A = 35.$$

4. $A \propto hb$.

$$A = Khb.$$
$$48 = K \cdot 8 \cdot 12.$$
$$K = \tfrac{1}{2}.$$

5. $x \propto \frac{y}{z}$.

$$\frac{10}{x} = \frac{\frac{5}{27}}{\frac{12}{36}}.$$
$$x = 18.$$

6. $V \propto \frac{T}{P}$.

$$\frac{80}{40} = \frac{\frac{300}{30}}{\frac{400}{P}}.$$
$$P = 80.$$

Page 477

1. $W \propto d$.

(a) $$\frac{172}{W} = \frac{4000}{3000}.$$
$$W = 129.$$

(b) $$\frac{172}{W} = \frac{4000}{1000}.$$
$$W = 43.$$

(c) $$\frac{W}{172} = \frac{0}{4000}.$$
$$W = 0.$$

2. $d \propto t$.

$$\frac{1100}{d} = \frac{1}{5\frac{2}{5}}.$$
$$d = 5940.$$

3. The less the number of cubic inches, the greater the pressure; therefore the variation is inverse.

If $p \propto \frac{1}{v}$, then

$$24 = K \cdot \tfrac{1}{30},$$

and $$20 = K \tfrac{1}{36}.$$

In each case $K = 720$.

$$42 = K\frac{1}{v} = 720\frac{1}{v}.$$
$$v = 17\tfrac{1}{7} \text{ cubic inches.}$$

4. $d \propto t^2$.

(a) $$\frac{16}{d} = \frac{1^2}{3^2}.$$
$$d = 144.$$

(b) $$\frac{16}{d} = \frac{1^2}{10^2}.$$
$$d = 1600.$$

5. $I \propto \frac{1}{d^2}$.

Since the intensity varies inversely as the square of the distance, the page appears brighter when nearer, or at 3 feet.

$$I_1 = K\frac{1}{3^2}.$$
$$I_2 = K\frac{1}{6^2}.$$
$$\frac{I_1}{I_2} = \frac{\frac{1}{9}}{\frac{1}{36}} = \frac{4}{1}.$$

The light is four times as bright at 3 feet.

6. $I \propto \dfrac{1}{d_1^2}$. $\dfrac{I}{2I} = \dfrac{\frac{1}{5^2}}{\frac{1}{d_1^2}}$.

$d_1^2 = \frac{25}{2}$.

$d_1 = \pm \frac{5}{2}\sqrt{2} = \pm 3.54$ feet.

(Root -3.54 rejected.)

$\dfrac{I}{4I} = \dfrac{\frac{1}{5^2}}{\frac{1}{d_2^2}}$.

$d_2^2 = \frac{25}{4}$.

$d_2 = \pm \frac{5}{2} = \pm 2\frac{1}{2}$ feet.

(Root $-\frac{5}{2}$ rejected.)

7. $A \propto r^2$. $\dfrac{154}{616} = \dfrac{7^2}{x^2}$.

$x^2 = 196$.

$x = \pm 14$.

(Root -14 rejected.)

8. $A \propto d^2$. $\dfrac{40^2}{15^2} = \dfrac{10^2}{x^2}$.

$x = \pm 3\frac{3}{4}$ feet.

(Root $-3\frac{3}{4}$ rejected.)

9. $W \propto \dfrac{1}{d^2}$.

(a) $\dfrac{172}{W_1} = \dfrac{5000^2}{4000^2}$.

$W_1 = 110.08$.

(b) $\dfrac{172}{W_2} = \dfrac{7000^2}{4000^2}$.

$W_2 = 56.16$.

(c) $\dfrac{172}{W_3} = \dfrac{9000^2}{4000^2}$.

$W_3 = 33\frac{79}{81} = 33.97$.

10. $W \propto \dfrac{1}{d^2}$.

$\dfrac{150}{100} = \dfrac{x^2}{4000^2}$.

$x = \pm 4898.9$.

(Root -4898.9 rejected.)

$4898.9 - 4000 = 898.9$, the number of miles above the surface of the earth.

11. $w \propto R^3$. $\dfrac{8}{w} = \dfrac{3^3}{5^3}$.

$w = 37$ pounds.

12. $t \propto \sqrt{l}$. $\dfrac{1}{t} = \dfrac{\sqrt{100}}{\sqrt{81}}$.

$t = \frac{9}{10}$ seconds.

13. $t \propto \sqrt{l}$.

$\dfrac{1}{2} = \dfrac{\sqrt{100}}{\sqrt{l_1}}$.

$\sqrt{l_1} = 20$.

$l_1 = 400$ centimeters.

$\dfrac{1}{7} = \dfrac{\sqrt{100}}{\sqrt{l_2}}$.

$\sqrt{l_2} = 10 \cdot 7 = 70$.

$l_2 = 4900$ centimeters.

14. $p \propto av^2$.

$\dfrac{.9}{72.9} = \dfrac{1 \cdot 15^2}{9 \cdot x^2}$.

$x = \pm 45$ miles per hour.

(Root -45 rejected.)

15. $p \propto a \cdot d$.

$\dfrac{62.5}{x} = \dfrac{1 \cdot 1}{154 \cdot 10}$.

$x = 96{,}250$ pounds.

16. $c \propto l \cdot \dfrac{1}{d}$.

$\dfrac{1320}{c} = \dfrac{1 \cdot \frac{1}{2}}{20 \cdot \frac{1}{2\frac{1}{4}}}$.

$c = \$23{,}466.67$.

Page 488

1. 3.5593.	**5.** 2.8108.	**9.** 2.8485.	**13.** 1.4782.
2. 2.6672.	**6.** 1.9167.	**10.** .4782.	**14.** .4971.
3. 1.7220.	**7.** 1.0043.	**11.** $\bar{3}.4564$.	**15.** .4343.
4. 1.6309.	**8.** 2.6993.	**12.** $\bar{4}.8908$.	**16.** $\bar{3}.3703$.

Page 489

1. 35.67.	**4.** 373.3.	**7.** .6803.	**10.** 5.928.
2. 241.1.	**5.** .1805.	**8.** .009148.	**11.** .001276.
3. 4.776.	**6.** .002276.	**9.** .1847.	**12.** .0001705.

Page 490

2.
$$\begin{aligned} \log 37 &= 1.5682 \\ \log 28 &= \underline{1.4472} \\ \log(28 \times 37) &= 3.0154 \\ \text{antilog } 3.0154 &= 1036. \end{aligned}$$

3.
$$\begin{aligned} \log 29 &= 1.4624 \\ \log 9 &= \underline{.9542} \\ \log(29 \times 9) &= 2.4166 \\ \text{antilog } 2.4166 &= 261. \end{aligned}$$

4.
$$\begin{aligned} \log 9.8 &= .9912 \\ \log 6 &= \underline{.7782} \\ \log(9.8 \times 6) &= 1.7694 \\ \text{antilog } 1.7694 &= 58.8. \end{aligned}$$

5.
$$\begin{aligned} \log 42 &= 1.6232 \\ \log 3.3 &= \underline{.5185} \\ \log(42 \times 3.3) &= 2.1417 \\ \text{antilog } 2.1417 &= 138.6. \end{aligned}$$

6.
$$\begin{aligned} \log 386 &= 2.5866 \\ \log 27 &= \underline{1.4314} \\ \log(386 \times 27) &= 4.0180 \\ \text{antilog } 4.0180 &= 10,420. \end{aligned}$$

7.
$$\begin{aligned} \log 432 &= 2.6355 \\ \log 263 &= \underline{2.4200} \\ \log(432 \times 263) &= 5.0555 \\ \text{antilog } 5.0555 &= 113,600. \end{aligned}$$

8.
$$\begin{aligned} \log 589 &= 2.7701 \\ \log 375 &= \underline{2.5740} \\ \log(589 \times 375) &= 5.3441 \\ \text{antilog } 5.3441 &= 220,900. \end{aligned}$$

9.
$$\begin{aligned} \log 4326 &= 3.6361 \\ \log 497 &= \underline{2.6964} \\ \log(4326 \times 497) &= 6.3325 \\ \text{antilog } 6.3325 &= 2,150,000. \end{aligned}$$

10.
$$\begin{aligned} \log 2870 &= 3.4579 \\ \log 3754 &= \underline{3.5745} \\ \log(2870 \times 3754) &= 7.0324 \\ \text{antilog } 7.0324 &= 10,780,000. \end{aligned}$$

11.
$$\begin{aligned} \log 286.7 &= 2.4575 \\ \log 2.391 &= \underline{.3786} \\ \log(286.7 \times 2.391) &= 2.8361 \\ \text{antilog } 2.8361 &= 685.7. \end{aligned}$$

12.
$$\begin{aligned} \log 3.412 &= .5330 \\ \log 2.526 &= \underline{.4024} \\ \log(3.412 \times 2.526) &= .9354 \\ \text{antilog } .9354 &= 8.618. \end{aligned}$$

14.
$$\begin{aligned} \log 385 &= 2.5855 \\ \log .647 &= \underline{\bar{1}.8109} \\ \log(385 \times .647) &= 2.3964 \\ \text{antilog } 2.3964 &= 249.1. \end{aligned}$$

15.
$$\begin{aligned}\log 571 &= 2.7566\\ \log .073 &= \bar{2}.8633\\ \log(571 \times .073) &= 1.6199\\ \text{antilog } 1.6199 &= 41.68.\end{aligned}$$

16.
$$\begin{aligned}\log 37.6 &= 1.5752\\ \log .00865 &= \bar{3}.9370\\ \log(37.6 \times .00865) &= \bar{1}.5122\\ \text{antilog } \bar{1}.5122 &= .3252.\end{aligned}$$

17.
$$\begin{aligned}\log .0476 &= \bar{2}.6776\\ \log 673 &= 2.8280\\ \log(.0476 \times 673) &= 1.5056\\ \text{antilog } 1.5056 &= 32.04.\end{aligned}$$

18.
$$\begin{aligned}\log .07325 &= \bar{2}.8648\\ \log 6.354 &= .8031\\ \log(.07325 \times 6.354) &= \bar{1}.6679\\ \text{antilog } \bar{1}.6679 &= .4654.\end{aligned}$$

19.
$$\begin{aligned}\log .6381 &= \bar{1}.8049\\ \log .01897 &= \bar{2}.2781\\ \log(.6381 \times .01897) &= \bar{2}.0830\\ \text{antilog } \bar{2}.0830 &= .01211.\\ .6381 \times (-.01897) &= -.01211.\end{aligned}$$

20.
$$\begin{aligned}\log 675 &= 2.8293\\ \log .0286 &= \bar{2}.4564\\ \log(675 \times .0286) &= 1.2857\\ \text{antilog } 1.2857 &= 19.30.\\ 675 \times (-.0286) &= -19.30.\end{aligned}$$

21.
$$\begin{aligned}\log .437 &= \bar{1}.6405\\ \log .0046 &= \bar{3}.6628\\ \log(437 \times .0046) &= \bar{3}.3033\\ \text{antilog } \bar{3}.3033 &= .002010.\end{aligned}$$

Page 492

2.
$$\begin{aligned}\log 96 &= 1.9823\\ \log 32 &= 1.5051\\ \log(96 \div 32) &= .4772\\ \text{antilog } .4772 &= 3.001.\end{aligned}$$

3.
$$\begin{aligned}\log 888 &= 2.9484\\ \log 47 &= 1.6721\\ \log(888 \div 47) &= 1.2763\\ \text{antilog } 1.2763 &= 18.89.\end{aligned}$$

4.
$$\begin{aligned}\log 976 &= 2.9894\\ \log 361 &= 2.5575\\ \log(976 \div 361) &= .4319\\ \text{antilog } .4319 &= 2.703.\end{aligned}$$

5.
$$\begin{aligned}\log 439 &= 2.6425\\ \log 27.1 &= 1.4330\\ \log(439 \div 27.1) &= 1.2095\\ \text{antilog } 1.2095 &= 16.20.\end{aligned}$$

6.
$$\begin{aligned}\log 3860 &= 3.5866\\ \log 4.32 &= .6355\\ \log(3860 \div 4.32) &= 2.9511\\ \text{antilog } 2.9511 &= 893.5.\end{aligned}$$

7.
$$\begin{aligned}\log 4627 &= 3.6653\\ \log 281 &= 2.4487\\ \log(4627 \div 281) &= 1.2166\\ \text{antilog } 1.2166 &= 16.47.\end{aligned}$$

8.
$$\begin{aligned}\log 9896 &= 3.9954\\ \log 52.78 &= 1.7224\\ \log(9896 \div 52.78) &= 2.2730\\ \text{antilog } 2.2730 &= 187.5.\end{aligned}$$

9.
$$\begin{aligned}\log 6732 &= 3.8281\\ \log 7.81 &= .8927\\ \log(6732 \div 7.81) &= 2.9354\\ \text{antilog } 2.9354 &= 861.8.\end{aligned}$$

11.
$$\begin{aligned}\log 2.35 &= 10.3711 - 10\\ \log .0683 &= \bar{2}.8344\\ \log(2.35 \div .0683) &= 11.5367 - 10\\ \text{antilog } 1.5367 &= 34.41.\end{aligned}$$

12.
$$\begin{aligned}\log 4.86 &= 10.6866 - 10\\ \log .751 &= \bar{1}.8756\\ \log(4.86 \div .751) &= 10.8110 - 10\\ \text{antilog } .8110 &= 6.471.\end{aligned}$$

13.
$$\begin{aligned}\log .0635 &= \bar{2}.8028\\ \log .277 &= \underline{\bar{1}.4425}\\ \log(.0635 \div .277) &= \bar{1}.3603\\ \text{antilog } \bar{1}.3603 &= .2293.\end{aligned}$$

14.
$$\begin{aligned}\log .2674 &= \bar{1}.4271\\ \log 3.66 &= \underline{.5635}\\ \log(.2674 \div 3.66) &= \bar{2}.8636\\ \text{antilog } \bar{2}.8636 &= .07305.\end{aligned}$$

15.
$$\begin{aligned}\log .07882 &= \bar{2}.8966\\ \log 68.72 &= \underline{1.8371}\\ \log(.07882 \div 68.72) &= \bar{3}.0595\\ \text{antilog } \bar{3}.0595 &= .001147.\end{aligned}$$

16.
$$\begin{aligned}\log 356 &= 2.5514\\ \log 392 &= \underline{2.5933}\\ \log(356 \times 392) &= 5.1447\\ \log 128 &= \underline{2.1072}\\ &\ 3.0375\\ \text{antilog } 3.0375 &= 1090.\end{aligned}$$

17.
$$\begin{aligned}\log 347 &= 2.5403\\ \log 625 &= \underline{2.7959}\\ \log(347 \times 625) &= 5.3362\\ \log 346 &= \underline{2.5391}\\ &\ 2.7971\\ \text{antilog } 2.7971 &= 626.7.\end{aligned}$$
-626.7. *Ans.*

18.
$$\begin{aligned}\log 473.2 &= 2.6751\\ \log 4.78 &= \underline{.6794}\\ \log(473.2 \times 4.78) &= 3.3545\\ \log 68.3 &= \underline{1.8344}\\ &\ 1.5201\\ \text{antilog } 1.5201 &= 33.12.\end{aligned}$$
-33.12. *Ans.*

19.
$$\begin{aligned}\log 9.63 &= .9836\\ \log .0892 &= \underline{\bar{2}.9504}\\ \log(9.63 \times .0892) &= \bar{1}.9340\\ \log .00635 &= \underline{\bar{3}.8028}\\ &\ 2.1312\\ \text{antilog } 2.1312 &= 135.3.\end{aligned}$$

Page 493

2.
$$\begin{aligned}\log 6.32 &= .8007.\\ \log(6.32)^4 &= 4(.8007)\\ &= 3.2028.\\ \text{antilog } 3.2028 &= 1595.\end{aligned}$$

3.
$$\begin{aligned}\log 34.26 &= 1.5348.\\ \log(34.26)^2 &= 3.0696.\\ \text{antilog } 3.0696 &= 1174.\end{aligned}$$

4.
$$\begin{aligned}\log 6.715 &= .8271.\\ \log(6.715)^3 &= 2.4813.\\ \text{antilog } 2.4813 &= 302.9.\end{aligned}$$

6.
$$\begin{aligned}\log .362 &= \bar{1}.5587.\\ \log(.362)^4 &= \bar{2}.2348.\\ \text{antilog } \bar{2}.2348 &= .01717.\end{aligned}$$

7.
$$\begin{aligned}\log .0972 &= \bar{2}.9877.\\ \log(.0972)^2 &= \bar{3}.9754.\\ \text{antilog } \bar{3}.9754 &= .009450.\end{aligned}$$

8.
$$\begin{aligned}\log .003597 &= \bar{3}.5559.\\ \log(.003597)^5 &= \overline{13}.7795.\\ \text{antilog } \overline{13}.7795 &= .000{,}000{,}000{,}000{,}601{,}9.\end{aligned}$$

9.
$$\begin{aligned}\log(486.2)^2 &= 2(2.6868) = 5.3736\\ \log(3.85)^3 &= 3(.5855) \ \ = \underline{1.7565}\\ \log\ (486.2)^2(3.85)^3 &= 7.1301\\ \text{antilog } 7.1301 &= 13{,}490{,}000.\end{aligned}$$

10.
$$\begin{aligned}\log(.375)^5 &= 5(\bar{1}.5740) = \bar{3}.8700\\ \log(62.5)^4 &= 4(1.7959) = \underline{7.1836}\\ \log(.375)^2(62.5)^4 &= 5.0536\\ \text{antilog } 5.0536 &= 113{,}100.\end{aligned}$$

11.
$$\begin{aligned}\log(2.25)^4 &= 4(.3522) = 1.4088\\ \log(1.232)^3 &= 3(.0906) = \underline{.2718}\\ \log(2.25)^4 \div (1.232)^3 &= 1.1370\\ \text{antilog } 1.1370 &= 13.71.\end{aligned}$$

Page 496

2. $\log \sqrt[3]{783} = \frac{1}{3}(2.8938)$
$= .9646.$
antilog $.9646 = 9.218.$

3. $\log \sqrt[5]{1435} = \frac{1}{5}(3.1569)$
$= .6314.$
antilog $.6314 = 4.280.$

4. $\log \sqrt[4]{3421} = \frac{1}{4}(3.5341)$
$= .8835.$
antilog $.8835 = 7.647.$

6. $\log \sqrt{.0756} = \frac{1}{2}(8.8785 - 10)$
$= 4.4393 - 5.$
antilog $\bar{1}.4393 = .2750.$

7. $\log \sqrt[5]{.0007624} = \frac{1}{5}(6.8822 - 10)$
$= 1.3764 - 2.$
antilog $\bar{1}.3764 = .2379.$

8. $\log \sqrt[4]{.005679} = \frac{1}{4}(1.7542 - 4)$
$= .4386 - 1.$
antilog $\bar{1}.4386 = .2745.$

9. $\log (38.4)^{\frac{2}{3}} = \frac{2}{3}(1.5843)$
$= 1.0562.$
antilog $1.0562 = 11.38.$

10. $\log (4.925)^{\frac{3}{2}} = \frac{3}{2}(.6924)$
$= 1.0386.$
antilog $1.0386 = 10.93.$

11. $\log (6.387)^{\frac{5}{3}} = \frac{5}{3}(.8053)$
$= 1.3422.$
antilog $1.3422 = 21.99.$
$- 21.99.$ *Ans.*

12. $\log 283 = 2.4518$
$\log 7.627 = .8824$
$\log (283)(7.627) = 3.3342$
$\log (3.423)^3 = 1.6032$
log of radicand $= 1.7310$
log of radical $= .8655.$
antilog $.8655 = 7.337.$
$- 7.337.$ *Ans.*

13. $\log (43.56)^2 = 3.2782$
$\log 7.984 = .9022$
log of numerator $= 4.1804$
$\log (7.623)^3 = 2.6466$
log of quotient $= 1.5338$
log of expression $= \frac{1}{2} \cdot 1.5338$
$= .7669.$
antilog $.7669 = 5.846.$

14. $\log \sqrt[11]{269} = \frac{1}{11}(2.4298)$
$= .2209.$
antilog $.2209 = 1.663.$

15. $\log \sqrt[4]{463} = .6664$
$\log 87 = 1.9395$
log of product $= 2.6059$
log of expression $= 1.3030.$
antilog $1.3030 = 20.09.$

16. $\frac{25}{381}\sqrt{\frac{137}{67}}\sqrt[3]{972}$

$$= \frac{25}{381} \cdot \frac{(137)^{\frac{1}{2}}}{(67)^{\frac{1}{2}}} \cdot (972)^{\frac{1}{6}}.$$

$\log 25 = 1.3979$
$\log (137)^{\frac{1}{2}} = 1.0684$
$\log (972)^{\frac{1}{6}} = .4980$
log of numerator $= 2.9643$
$\log 381 = 2.5809$
$\log (67)^{\frac{1}{2}} = .9131$
log of denominator $= 3.4940$
log of expression $= \bar{1}.4703.$
antilog $\bar{1}.4703 = .2953.$

17. $\log 31 = 1.4914$
$\log \sqrt[3]{.07241} = \bar{1}.6199$
log of numerator $= 1.1113$
$\log 426 = 2.6294.$
log of expression $= \bar{2}.4819.$
antilog $\bar{2}.4819 = .03033.$

18. For any number n, if $\log_{10} n = x$,

then $10^x = n.$ (1)

For the same number n, if $\log_{100} n = y$,

then $100^y = n.$ (2)

From (1) and (2), $10^x = 100^y$,

whence $10^x = 10^{2y}$ or $y = \frac{x}{2}$.

Therefore $\log_{100} n = \frac{1}{2} \log_{10} n.$

$\log_{10} 5732 = 3.7583$ and $\log_{100} 5732 = 1.8792.$
$\log_{10} 573.2 = 2.7583$ and $\log_{100} 573.2 = 1.3792.$
$\log_{10} 57.32 = 1.7583$ and $\log_{100} 57.32 = .8792.$
$\log_{10} 5.732 = .7583$ and $\log_{100} 5.732 = .3792.$

Page 498

1. No. No real power of a positive real number is a negative number. Hence no logarithm can be found for a negative number to a positive base.

2. Let $\log_3 9 = x.$
$3^x = 9 = 3^2.$
$x = 2.$

3. Let $\log_2 8 = x.$
$2^x = 8 = 2^3.$
$x = 3.$

4. Let $\log_8 2 = x.$
$8^x = 2^{3x} = 2^1.$
$x = \frac{1}{3}.$

5. Let $4 \log_9 27 = x.$
$\log_9 27 = \frac{x}{4}.$
$9^{\frac{x}{4}} = 3^{\frac{x}{2}} = 3^3.$
$\frac{x}{2} = 3,$
or $x = 6.$

6. Let $5 \log_{27} 9 = x.$
Then $\log_{27} 9 = \frac{x}{5}.$
$(27)^{\frac{x}{5}} = 9.$
$3^{\frac{3x}{5}} = 3^2.$
$\frac{3x}{5} = 2.$
$x = \frac{10}{3}.$

7. Let $\log_4 8 = x.$
$4^x = 8.$
$x = \frac{3}{2}.$
Now let $3 \log_8 4 = y.$
$\log_8 4 = \frac{y}{3}.$
$8^{\frac{y}{3}} = 4.$
$2^y = 2^2.$
$y = 2.$
$\log_4 8 + 3 \log_8 4 = x + y$
$= \frac{3}{2} + 2 = 3\frac{1}{2}.$

8. $2 \log_{27} 81 - 4 \log_{81} 27 = 2\left(\frac{4}{3}\right) - 4\left(\frac{3}{4}\right) = -\frac{1}{3}.$

9. $3 \log_{25} 125 + 2 \log_5 25 - 2 \log_{125} 5 = \frac{9}{2} + 4 - \frac{2}{3} = 7\frac{5}{6}.$

10. $4 \log_3\left(\frac{1}{3}\right) - 5 \log_9\left(\frac{1}{27}\right) + 2 \log_{27} 9 = 4(-1) - 5\left(-\frac{3}{2}\right) + 2\left(\frac{2}{3}\right) = 4\frac{5}{6}.$

11. $\log \frac{5}{8} + \log \frac{24}{25} = \log\left(\frac{5}{8} \cdot \frac{24}{25}\right) = \log \frac{3}{5}.$

12. $\log \frac{7}{32} - \log \frac{35}{64} = \log(\frac{7}{32} \cdot \frac{64}{35}) = \log \frac{2}{5}$.

13. $\log \frac{25}{4} + \log \frac{36}{50} - \log \frac{3}{4} = \log(\frac{25}{4} \cdot \frac{36}{50} \cdot \frac{4}{3}) = \log 6$.

14. $2 \log 3 + 3 \log 2 = \log(3^2 \cdot 2^3) = \log 72$.

15. $3 \log 4 + 4 \log 3 - 2 \log 6 = \log(4^3 \cdot 3^4 \div 6^2) = \log 144$.

16.
$$\begin{aligned} \log\left(a - \frac{x^2}{a}\right) &= \log \frac{a^2 - x^2}{a} \\ &= \log \frac{(a+x)(a-x)}{a} \\ &= \log(a+x) + \log(a-x) - \log a. \end{aligned}$$

17.
$$\log\sqrt{a^2 - x^2} = \log(a^2 - x^2)^{\frac{1}{2}} = \tfrac{1}{2}[\log(a+x) + \log(a-x)].$$

18.
$$\log\sqrt{s(s-a)} = \log[s(s-a)]^{\frac{1}{2}} = \tfrac{1}{2}[\log s + \log(s-a)].$$

19.
$$\begin{aligned} \log\sqrt{\frac{s(s-b)(s-c)}{s-a}} &= \log\left(\frac{s(s-b)(s-c)}{s-a}\right)^{\frac{1}{2}} \\ &= \tfrac{1}{2}[\log s + \log(s-b) + \log(s-c) - \log(s-a)]. \end{aligned}$$

20. (*a*)
$$\begin{aligned} 2\pi R &= 2\,(85)(3.1416). \\ \log 2 &= .3010 \\ \log 85 &= 1.9294 \\ \log 3.1416 &= \underline{.4971} \\ \text{log of product} &= 2.7275 \\ \text{antilog } 2.7275 &= 534 \text{ inches.} \end{aligned}$$

(*b*)
$$\begin{aligned} 3281 &= 2\pi r. \\ r &= 3281 \div 2\pi. \\ \log 3281 &= 3.5160 \\ \log 2\pi &= \underline{.7981} \\ \text{log of quotient} &= 2.7179 \\ \text{antilog } 2.7179 &= 522.3 \text{ centimeters.} \end{aligned}$$

21. (*a*)
$$\begin{aligned} \pi R^2 &= 3.1416(5.672)^2 \\ \log 3.1416 &= .4971 \\ \log(5.672)^2 &= \underline{1.5074} \\ \text{log of product} &= 2.0045 \\ \text{antilog } 2.0045 &= 101.0 \text{ square feet.} \end{aligned}$$

(*b*)
$$\begin{aligned} \pi R^2 &= 67.37. \\ R &= \sqrt{67.37 \div 3.1416}. \\ \log 67.37 &= 1.8285 \\ \log 3.1416 &= \underline{.4971} \\ \text{log of quotient} &= 1.3314 \\ \text{log of expression} &= .6657. \\ \text{antilog } .6657 &= 4.631 \text{ feet.} \end{aligned}$$

22. (*a*) $$4\pi R^2 = 4(3.1416)(3958.79)^2.$$

$$\begin{aligned} \log 4 &= .6021 \\ \log 3.1416 &= .4971 \\ \log (3958.79)^2 &= \underline{7.1952} \\ \log \text{ of product} &= 8.2944 \end{aligned}$$

$$\text{antilog } 8.2944 = 196{,}900{,}000 \text{ square miles.}$$

(*b*) $$2\pi R = 2(3.1416)(3958.79).$$

$$\begin{aligned} \log 2 &= .3010 \\ \log 3.1416 &= .4971 \\ \log 3958.79 &= \underline{3.5976} \\ \log \text{ of product} &= 4.3957 \end{aligned}$$

antilog $4.3957 = 24{,}870$ miles.

23. (*a*) $$\frac{4\pi R^3}{3} = \frac{4 \cdot 3.1416\, R^3}{3} = 86.$$

$$R = \sqrt[3]{\frac{86 \cdot 3}{4 \cdot 3.1416}}.$$

$$\begin{aligned} \log 86 &= 1.9345 \\ \log 3 &= \underline{.4771} \\ \log \text{ of numerator} &= 2.4116 \\ \log 4 &= .6021 \\ \log 3.1416 &= \underline{.4971} \\ \log \text{ of denominator} &= 1.0992 \end{aligned}$$

$$\begin{aligned} \log \text{ of fraction} &= 1.3124. \\ \tfrac{1}{3} \log \text{ of fraction} &= .4375. \\ \text{antilog } .4375 &= 2.738 \text{ feet.} \end{aligned}$$

(*b*) Here $$47 = \tfrac{4}{3}\pi R^3 \text{ or } R = \sqrt[3]{\frac{47 \cdot 3}{4 \cdot 3.1416}}.$$

$$\begin{aligned} \log 47 &= 1.6721 \\ \log 3 &= \underline{.4771} \\ \log \text{ of numerator} &= 2.1492 \\ \log 4\pi &= \underline{1.0992} \\ \log \text{ of fraction} &= 1.0500 \end{aligned}$$

$$\log R = \tfrac{1}{3} \log \text{ of fraction} = .3500.$$

$$\text{antilog } .3500 = 2.239.$$

The diameter is $2R$, or 4.478 inches.

CR

24. (*a*)

$$c + a = 377 + 288 = 665.$$
$$c - a = 377 - 288 = 89.$$
$$\begin{aligned} \log 665 &= 2.8228 \\ \log 89 &= \underline{1.9494} \\ \text{log of product} &= 4.7722 \\ \text{log of expression} &= 2.3861 \\ \text{antilog } 2.3861 &= 243.3, \text{ the side.} \end{aligned}$$

(*b*)

$$c + a = 1285 + 924 = 2209.$$
$$c - a = 1285 - 924 = 361.$$
$$\begin{aligned} \log 2209 &= 3.3442 \\ \log 361 &= \underline{2.5575} \\ \text{log of product} &= 5.9017 \\ \text{log of expression} &= 2.9509 \\ \text{antilog } 2.9509 &= 893.0, \text{ the side.} \end{aligned}$$

25.

$$\frac{s^2}{4}\sqrt{3} = \frac{(34.23)^2}{4}\sqrt{3}.$$
$$\begin{aligned} \log (34.23)^2 &= 3.0688 \\ \log \sqrt{3} &= \underline{.2386} \\ \text{log of numerator} &= 3.3074 \\ \log 4 &= \underline{.6021} \\ \text{log of expression} &= 2.7053 \\ \text{antilog } 2.7053 &= 507.3 \text{ square inches} \\ &= 3.523 \text{ square feet.} \end{aligned}$$

26.

$$\begin{aligned} 3^x &= 25. \\ x \log 3 &= \log 25. \\ x &= \frac{\log 25}{\log 3} \\ &= \frac{1.3979}{.4771} = 2.92. \end{aligned}$$

27.

$$\begin{aligned} 64^x &= 4. \\ 4^{3x} &= 4^1. \\ 3x &= 1. \\ x &= \tfrac{1}{3}. \end{aligned}$$

28.

$$\begin{aligned} 16^x &= 1024. \\ 2^{4x} &= 2^{10}. \\ 4x &= 10. \\ x &= 2.5. \end{aligned}$$

29.

$$\begin{aligned} (-2)^x &= 64. \\ (-2)^x &= (-2)^6. \\ x &= 6. \end{aligned}$$

30.

$$\begin{aligned} 3 &= (1.04)^x. \\ x \log 1.04 &= \log 3. \\ x &= \frac{\log 3}{\log 1.04} \\ &= \frac{.4771}{.0170} = 28.06. \end{aligned}$$

31. $2^x = 64 = 2^6.$

$x = 6.$

32. $4^{2x+1} = 84.$

$$(2x+1)\log 4 = \log 84.$$

$$x = \frac{\log 84 - \log 4}{2 \log 4}$$

$$= \frac{1.9243 - .6021}{1.2041}$$

$$= \frac{1.3222}{1.2041}$$

$$= 1.09 +.$$

33. $3^{x+3} = 6561.$

$$(x+3)\log 3 = \log 6561.$$

$$x = \frac{\log 6561 - 3\log 3}{\log 3}$$

$$= \frac{3.8170 - 1.4313}{.4771}$$

$$= 5.0.$$

34. $10^{x-1} = 3.$

$$(x-1)\log 10 = \log 3.$$

$$x - 1 = \log 3.$$

$$x = 1 + \log 3 = 1.4771.$$

35. $8^{x+2} = 6.$

$$(x+2)\log 8 = \log 6.$$

$$x = \frac{\log 6 - 2\log 8}{\log 8}$$

$$= \frac{.7782 - 1.8062}{.9031}$$

$$= -1.13.$$

36. $(.3)^{-x} = 5.$

$$-x\log .3 = \log 5.$$

$$x = -\frac{\log 5}{\log .3} = -\frac{.6990}{\bar{1}.4771}$$

$$= -\frac{.6990}{-.5229} = 1.33 +.$$

37. $(.07)^x = 9.$

$$x = \frac{\log 9}{\log .07} = \frac{.9542}{\bar{2}.8451}$$

$$= \frac{.9542}{-1.1549}$$

$$= -.82.$$

38. (*a*) $\log (3)^{52} = 52 \log 3$

$$= 52(.4771)$$

$$= 24.8092.$$

25 digits. *Ans.*

(*b*) $\log 2^{340} = 340(.3010)$

$$= 102.3400.$$

103 digits. *Ans.*

(*c*)

$$\begin{aligned} \log 2^9 &= 2.7090 \\ \log 3^8 &= 3.8168 \\ \log 5^7 &= \underline{4.8930} \\ \text{log of product} &= 11.4188 \end{aligned}$$

12 digits. *Ans.*

40. $(1.06)^x = 2.$

$$x = \frac{\log 2}{\log 1.06}$$

$$= \frac{.3010}{.0253}$$

$$= 11.89+ \text{ years.}$$

41. $(1.04)^x = 3.$

$$x = \frac{\log 3}{\log 1.04}$$

$$= \frac{.4771}{.0170}$$

$$= 28.06+ \text{ years.}$$

42. $(1.035)^x = 2.$

$$x = \frac{\log 2}{\log 1.035}$$

$$= \frac{.3010}{.0149}$$

$$= 20.20+ \text{ years.}$$

43.

$$4000(1.05)^x = 7360.80.$$

$$x = \frac{\log 7360.80 - \log 4000}{\log 1.05}$$

$$= \frac{3.8669 - 3.6021}{.0212}$$

$$= 12.49 + \text{years}.$$

44. Let

$$x = \text{amount in dollars}.$$

$$24(1.04)^{300} = x.$$

$$300 \log 1.04 + \log 24 = \log x.$$

$$5.1000 + 1.3802 = \log x.$$

$$6.4802 = \log x.$$

$$x = \$3{,}021{,}000.$$

45.

$$(1.015)^x = 2.$$

$$x = \frac{\log 2}{\log 1.015} = \frac{.3010}{.0065} = 46.3.$$

$$\frac{46.3}{2} = 23.15 \text{ years}.$$

46. The amount of P dollars in one year at $r\%$ is $P(1+r)$.

The amount of P dollars in two years at $r\%$ is $P(1+r)^2$.

Hence the amount of P dollars in t years at $r\%$ is $P(1+r)^t$.

The amount of P dollars in one year at $r\%$ (semiannual interest) is $P\left(1+\frac{r}{2}\right)\left(1+\frac{r}{2}\right)$.

The amount of P dollars in two years at $r\%$ (semiannual interest) is $P\left(1+\frac{r}{2}\right)^4$.

The amount of P dollars in t years at $r\%$ (semiannual interest) is $P\left(1+\frac{r}{2}\right)^{2t}$.

In like manner for the others.

47. (*a*)

$$5000(1.04)^4 = A.$$

$$\log A = \log 5000 + 4 \log 1.04$$

$$= 3.6990 + .0680$$

$$= 3.7670.$$

$$A = \$5848.$$

(*b*)

$$
\begin{aligned}
5000(1.02)^8 &= A. \\
\log A &= \log 5000 + 8 \log 1.02 \\
&= 3.6990 + .0688 = 3.7678. \\
A &= \$5859.
\end{aligned}
$$

(*c*)

$$
\begin{aligned}
5000(1.01)^{16} &= A. \\
\log A &= \log 5000 + 16 \log 1.01 \\
&= 3.6990 + .0688 = 3.7678. \\
A &= \$5859.
\end{aligned}
$$

(Four-place tables do not show a difference here between semiannual and quarterly payment of interest. Seven-place tables show the amount for semiannual payments to be \$5858.30, and for quarterly payments to be \$5862.90.)

48.

$$
\begin{aligned}
A &= 4.12(1.01)^{20}. \\
\log A &= \log 4.12 + 20 \log 1.01 \\
&= .6149 + .0860 \\
&= .7009. \\
A &= 5.022.
\end{aligned}
$$

49.

$$
\begin{aligned}
5 &= P(1.01)^{20}. \\
\log P &= \log 5 - 20 \log 1.01 \\
&= .6990 - .0860 \\
&= .6130. \\
P &= 4.102.
\end{aligned}
$$